ELASTICITY OF DEMAND ON THE TWO SIDES OF THE RECTANGULAR HYPERBOLA

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While explaining elasticity of demand most of the modern books on economics refer to the important relationships which exist among elasticity, price changes and the total amount which the consumers spend on a certain commodity. If a decrease in price raises the quantity so much as to increase the total outlay of the consumers, then

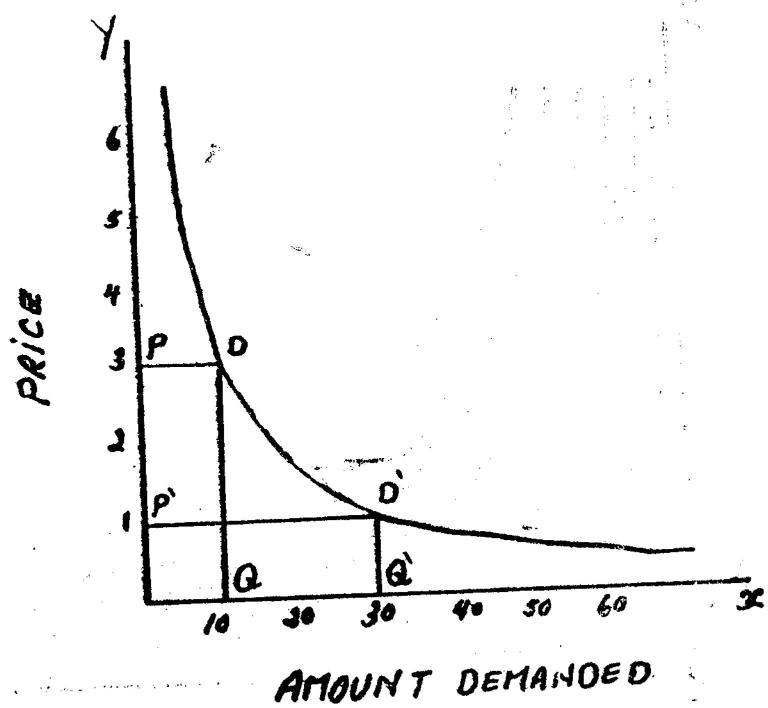

Fig. 1

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demand is said to be elastic and elasticity of demand will be greater than unity. If a decrease in price results in a less proportionate increase in the quantity sold so as to reduce the total outlay of the consumers, then demand is said to be inelastic. If, on the other hand, demand and price change in exactly equal proportions, the total expenditure of the consumers will remain unchanged. Demand in this case is said to be of unit elasticity and the demand curve will take the shape of a Rectangular Hyperbola as shown in Fig. 1.

A question now arises concerning the elasticities of all the points on the right side, as well as all the points on the left side of the Rectangular Hyperbola. Professor Cairncross put an answer to this question by saying, "On one side of the line, elasticity will be greater,

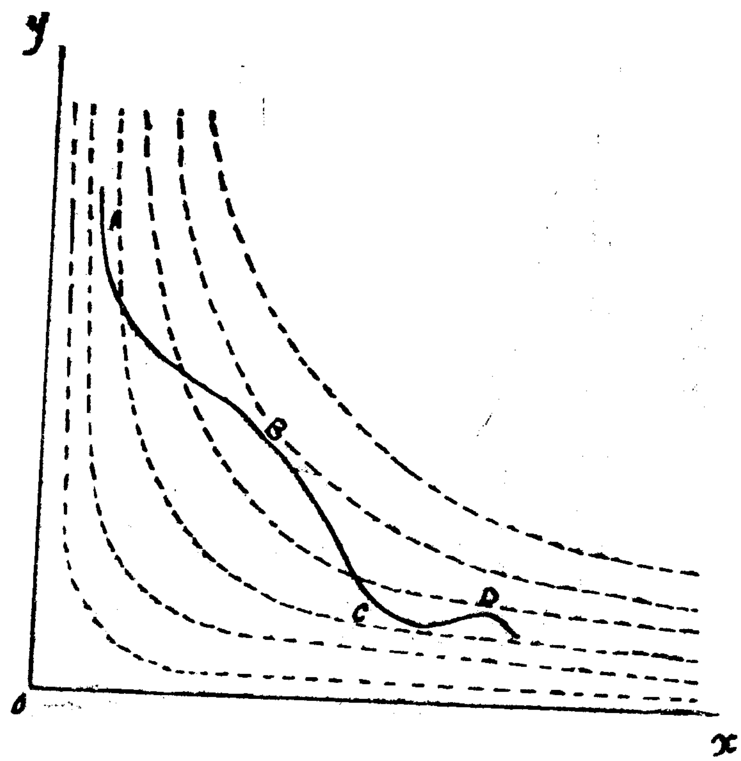

Fig. 2.

and, on the other side, less than unity"[1].

However, Professor Cairncross did not specify which side of the line will have elasticity which is greater than unity and which side will have elasticity which is less than unity, although it may be well understood. It was Professor Marshall who put the following figure and dwelt more on the point under consideration. We quote :—

" ... it may, for instance, be seen at once that the demand curve in the figure represents at each of the points A, B, C, and D an elasticity about equal to one ; between A and B, and again between C and D, it represents an elasticity greater than one ; while between B and C it represents an elasticity less than one ...."[2]

We consider that the mathematical proof could be the final word as to elasticities of the points on either side of the Rectangular Hyperbola (greater or less than unity). Despite consulting many text books on economics in general, and on price theory or economic theory in particular, we found that the mathematical proof was still in abeyance. In order to justify our treatise, three books were chosen for comments.

Professor Robert Dorfman[3] mentions, " If a commodity has unit elasticity (elasticity of demand equal to one), a one percent fall in price will cause a one percent increase in sales. Therefore, the dollar value of the sales will not change when price changes though, of course, the physical volume will change. Unit elasticity is a convenient dividing point."

Professor Ryan in his book, *Price Theory*[4], mentions, " We see that the revenue curve will rise while the demand is relatively elastic, reach a maximum when demand has unit elasticity and fall when demand is relatively inelastic". " If the demand for each commodity has unit elasticity throughout its length, then the revenue contours will coincide with one another."[5]

1 A. Cairncross, **Introduction to Economics** 3rd. ed., (London ; 1960), p. 228.
2 A. Marshall, **Principles of Economics**, 8th. ed., (London : 1956), p. 691.
3 Robert Dorfman, **The Price System**, (Englewood Cliffs, N.J. : Prentice Hall, Inc., 1964), p. 68.
4 W. J. L. Ryan **Price Theory**, (London : 1964), p. 204.
5 **Ibid.** p. 373.

Stonier and Hague [6] write "... the situation represents a dividing line where demand is neither elastic nor inelastic ... and this is true whichever points on the curves one takes. Total outlay is always constant and elasticity of demand equals one at all prices. This type of curve is called a Rectangular Hyperbola by mathematicians. ... It is asymptotic (i.e. it approaches but never touches) the X - and Y - axes. If it did touch them, the inscribed rectangles would vanish."

From these discussions it is clear that the Rectangular Hyperbola divides the price-demand plane into two sections. From geometrical consideration one may, therefore, conclude that the perpendicular from all points above this hyperbola will have a positive sign and from all points below the curve the perpendicular on it will have negative sign. Interpreted in terms of price elasticity of demand it means that the elasticity in the former section will be greater than one and in the latter section it will be less than one. However, the nature of the relationship between quantity demanded and price restricts the region in which we need consider the magnitude of the elasticity. For students of economics it may be useful to have an explicit proof of the property, viz, that the elasticity will be greater than unity above the Rectangular Hyperbola and less than unity below this curve. We have not found an explicit proof in most of the standard text-books on economics. The present proof is based on simple trigonometry.

Let us consider a simple static demand function with unit elasticity at every point. Such a curve may be presented in the form.

$$d = Ap^{-1} \text{ or } \log d = \log A - 1 \log p. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Graph of (1) on double logarithmic scale will be a straight line AB as shown in the Fig. 3. AB will make an angle of 135° with the log p axis and tan 135° is equal to — 1 (the elasticity). Now let us consider any point x_1 above AB, which is the same as (1). Through x_1 an infinite number of demand curves can pass, one of them is a straight line parallel to AB and is shown as MN in the Figure. Equation of MN will be

6 Alfred W. Stonier and Douglas C. Hague, A Text-book of Economic Theory, 3rd. ed., (London : 1967), p. 24.

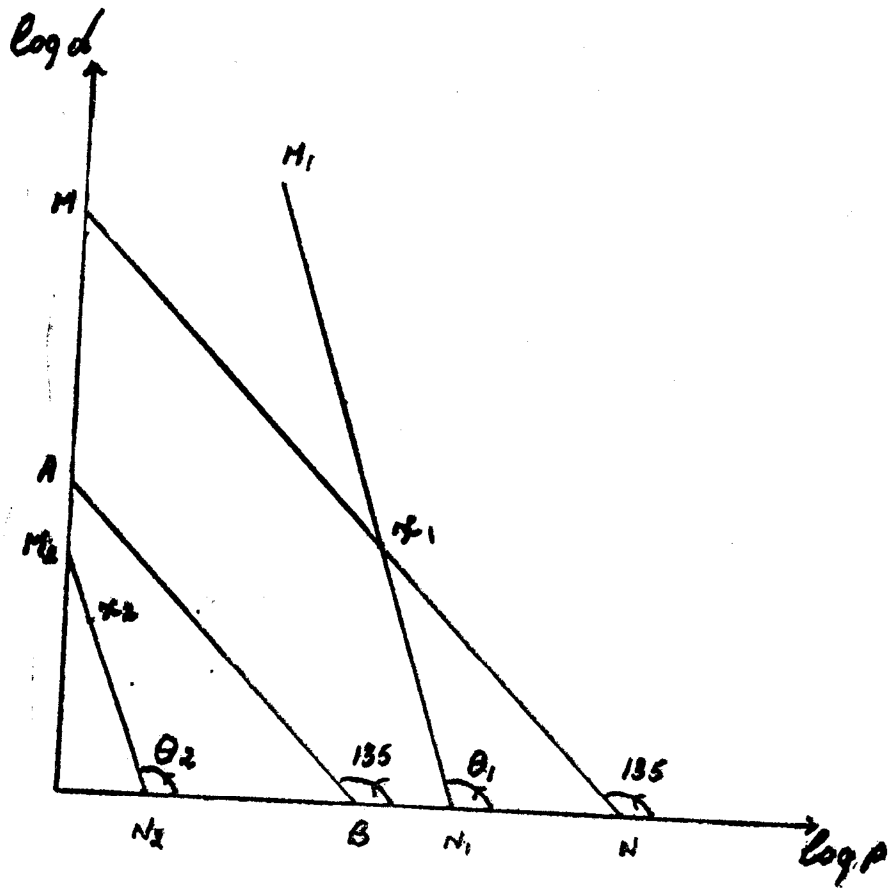

Fig. 3

$$\log d = \log A_1 - 1 \log p. \quad \ldots\ldots\ldots\ldots \quad (2)$$

Thus here elasticity has not changed from — 1 but $\log A_1 > \log A$. This can happen due to one or more of the reasons given below :

(a) Income of the group goes up without a change in the proportion of income spent on the commodity.

(b) Income remains stable but the proportion of income spent on the commodity increases. That is, there is a change in taste of the consumers.

(c) There has been a change in the prices of substitute goods.

But such situations violate the *cetris paribus* condition of the static demand equation ; hence an equation of the form (2), though theoretically possible, is not a logically valid equation. Any other line passing through x_1 and representing a demand equation will have an angle Θ_1 less than 135°. One such line $M_1 N_1$ has been shown in Figure 3.

$\Theta_1 < 135°$ will mean $\Theta_1 = 180° - \alpha°$ where α has any value between 45+ and 90 degrees. α cannot be greater than 90° because then demand will become an increasing function of price.

$\tan \Theta_1$ = the elasticity of the new demand function passing through x_1

$$= \frac{\text{Sin } \Theta_1}{\text{Cos } \Theta_1} = \frac{\text{Sin }(180 - \alpha)}{\text{Cos }(180 - \alpha)} = - \tan \alpha$$

But as α increases from 45° to 90°, Cos α decreases and Sin α increases, so the rate Sin α / Cos α increases continuously as α moves in the range 45^+ to 90. This means that $|\tan \Theta_1| > 1$. In other words the elasticity will be greater than unity. As x_1 is any point above AB we may conclude that all demand curves passing through points above AB will have elasticity greater than unity.

Similarly for any point below AB (x_2 is one such point and $M_2 N_2$ is a demand curve passing through x_2) and barring the case of a line parallel to AB but below AB, we can see that

$$\Theta_2 > 135° = 180° - 45°$$

This means $\Theta_2 = 180° - \alpha$ where α is now between 0 and 45 degrees. In this case $\tan \Theta_2 = \frac{\text{Sin }(180 - \alpha)}{\text{Cos }(180 - \alpha)} = - \tan \alpha =$ the elasticity. But as $0 < \alpha < 45$, tan α will lie between 0 and 1. In other words demand curves passing through points below AB will have elasticity less than unity.