THE CHANGES IN INPUT-COEFFICIENTS

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1. Introduction

The stability of input-output coefficients over time can be considered under certain theoretical conditions. The simple input-output model in which the input-coefficients are stable over time is constructed assuming the following :

1. The technique of production remains the same over time.

2. The inputs purchased by each industry are proportional to the level of output of that industry.

3. The output of each industry is a homogeneous commodity, and there is only one production function by which it can be produced. Joint production by an industry of a commodity also produced in another industry does not exist ; otherwise the inputs absorbed by a given output would depend on the varying combination of two production functions of two industries.

The idea of stable input-coefficients is the main assumption in the Leontief input-output model. It enables input-output analysis to be employed in the econometric field ; yet the main criticism directed against the input-output analysis is the fact that the input-coefficients do change. In any input-output table, the three assumptions put forward do not exist strictly, because of the practical limitations. This makes it hard to accept the stability assumption of the input-coefficients.

Many studies have been undertaken to examine the stability of

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the input output tables. Most of these studies prove that the input coefficients are subject to variation. In the next section we discuss some of these studies.

The causes of variation in input-coefficients differ from economy to economy. Although the main aspects of the causes are the same, the degree of influence of each cause varies according to the economy : that is, whether the economy in question is advanced, developing, or backward. In section 3 we discuss the causes of variation in the input-coefficients, and attempt to compare how each cause works in the input-output table of various economies.

2. Tests of constancy

Tests of constancy have been constructed in two main types. The first type uses an input-output table observed at a certain date to compute gross-outputs from the final demands of another date and compares the resulting gross-outputs with the actual one or with gross-outputs derived from other models. The naive estimate of gross outputs in this type of tests have been made variously (i) by multiple regressions of outputs on G.N.P. and time, (ii) by assuming a constant ratio between each industry's output and G.N.P. (the "G.N.P. blow-up" method), or (iii) by assuming a constant ratio between each industry's output and the final demand placed upon it (the "final demand blow-up" method). This type of test does not examine the changes in the input-coefficients, but only the reflection of these changes on the gross-outputs. The second type of test compares a time series of input-coefficients. This test can be applied to the whole input-output martix if a time series of tables is available. We discuss some of the tests which have been carried out by each method.

A. *Tests of the first type*

The main characteristic of the test is the use of an input-output table for a certain period and observed or estimated final demand and real outputs for different periods. Using the input-output table, calculated outputs can be derived. These calculated outputs can be compared with actual outputs or outputs estimated in other ways.

Leontief (10) used a 13-order 1939 table for a backward forecast of the years 1919 and 1929, the results being slightly better than those of the naive models which he used for comparison purposes. According to Bodenhorn (5), Leontief's comparison between the input-output model and the other models is made with regard to the prediction of outputs for nine out of thirteen industries. He found that the prediction errors in the input-output and the other models are of approximately equal size with regard to the outputs of the thirteen industries. Hoffenberg (reported in 7) computed forecasts for the odd years in the period 1929-1937 on the basis of the 38-order table for 1939. He found little difference from the results of the naive models. Further, for the input-output model particularly, larger discrepancies were found for years in the more distant past than for years close to 1939, which is expected.

Barnett (4) used the 1939 table to make predictions for 1950. He tested them against regression predictions according to which the production of each sector is described as a function of G.N.P. in 1939 real outputs. The regression proved to be superior for the prediction of value levels but inferior for forecasting in terms of real outputs. Final demands blow-up gave nearly the same results, but G.N.P. blow-up was worse. Arrow (reported in 9) tested the same input-output table as did Barnett, uisng the observed data final demands. His comparison showed that the final demands projection yielded about the same results as the input-output model, which was slightly inferior to the time series projection of output on G.N.P. The G.N.P. projection was again inferior to any one of the others.

Hatanaka (9) used the 1947 input-output table to test its workability relative to a time series projection of output on G.N.P. and the final demands projection in the short and long-range projection. In this test the 1947 U.S. input-output table demonstrates its definite superiority to both the time series projection of output on G.N.P. and the final demands projection in the short-range projection, but the superiority is not so marked in the long-range. Hatenaka also performed statistical significance tests on the differences between errors in the various predictions in the gross outputs of individual industries. The input-output projections were superior to projections from multiple regressions at 25% significance level in both short and long-range tests, but showed no clear margin of superiority over alternative naive projections. As a result of the tests, Hatanaka suggested that in the input-output model, the effects of gradual replacement of the old production line by the new can adequately be taken into account. Further-more, some information on the behaviour of industries must be introduced in order to determine the speed of the replacement of the old production lines and the changes in the rate of operation of each production line. If a new model is set forth, its workability must be compared with the workability of the known input-output model. If it is superior to the input-output model, Hatanaka suggests this new model as an alternative to the input-output.

B. *Tests of the second type*

The test of the first type reveals the need for a detailed study in the input-output field. The study would be effective if it dealt with the input-coefficients directly instead of studying their effect on the final demand or the gross national products. A time series of input-output tables is not usually avaiable, so the test of constancy for the whole table has rarely been applied. However, a test of separate input-coefficients has been carried out several times. The test is characterized by a study of the movement in observed coefficients and an examination of the effect of proposed variables on them. The main study of this type of test is by Arrow and Hoffenberg which we will explain in some detail after elucidating the main features of some other studies.

Two similar tests for U.S. and Japan input-output tables have been done (reported in 7). The first by Leontief, comparing the 1919, 1929 and the 1939 U.S, tables. The second by the Japanese government, comparing the 1951 and 1953 input-output tables. According to Chenery (7) neither of these tests provides a criterion as to whether the observed changes are satisfactorily small. Helzner (raported in 7) examined five coefficients of inputs into the steel industry over a nine years period for significance in the sense that a constancy assumption of these coefficients would lead to excessive errors in gross-output estimates.

Cameron (6) carried out a test of constancy by using fifty-two Australian manufacturing industries in which the products are relatively homogeneous. The ratios of inputs to outputs have been obtained for whatever periods data were available. The test is divided into two parts, examining respectively the employment coefficients, and the material coefficients. For the employment coefficients Cameron concluded that "the level of output and a linear trend factor appear to account for virtually all significant movement in the coefficients in nearly all industries examined". For material coefficients Cameron found that on the whole they tend to be approximately constant over a short period of time, and for half the examined industries, the major coefficients at least are approximately constant for a longer period of ten years or more. Hatanaka (9) says that such a test is not a test of constancy of input-coefficients in an input-output model, because the ratios of real inputs to real outputs were tested by Cameron for specific industries only. In the input-output model there is a certain industrial classification which differs from specific portions of industries, and in addition Cameron tested a non-aggregated input-output model which cannot be compiled for the whole economy.

A test carried out by Savalson (12) in which he puts forward some explanatory variables for the changes in the input-coefficients. These variables relate to :

1. technological changes

2. the output level of the using industry
3. prices
4. other economic variables
5. changes in the product mix of the using industry and
6. observation errors.

Savalson used the Norwegian cork and woodpulp industries to examine the variables put froward, and he concluded that the variation in the input-coefficients can be explained by these variables.

Arrow and Hoffenberg (1) constructed two models to study the variation in the input-output tables. The authors possessed four input-output tables for the U.S., but their industrial classification is not comparable, and even four comparable tables are not enough to examine Arrow and Hoffenberg's models. So the authors examined the movement in the total intermediate outputs instead of individual coefficients. For intermediate and gross-outputs Arrow and Hoffenberg were able to derive a series running from 1929 to 1950. The coefficients whose changes were to be estimated were those of the 200-order U.S. table for 1947.

In the first model a distinction was made between flows of materials physically embodied in the output of the consuming industry, and those not embodied but consumed in some way or another by the equipment of production. The usage rate of the embodied flows should be basically constant. However, rapid increases in output rate are associated with wastage and scrapping of embodied materials due to difficulties in organization, which is characteristic of the less capital intensive industries. So the embodied inputs are taken as dependent upon output and upon the rate of change of output, (if positive). In the more capital intensive industries, the embodied input flows are taken as dependent upon output and upon the intensity of use of capital. Though a non-embodied input might be economized through better capital equipment, a certain amount is needed to maintain capital equipment which is independent of the level of output, so the flow of a non-embodied input is considered to be a linear function of output instead of a simple proportion. Both types of inputs are made to depend upon product-mix variations. This model has been rejected by the authors, for it relies heavily on a massive amount of data, not all of which is available.

In the second model, Arrow and Hoffenberg represent some explanatory variables which cause the changes in the input coefficients. These are :

1. Quality variation due to changes in income : the real per capita disposal income is taken as a measure of this effect.

2. Variations of product-mix due to war conditions : the ratio of defence expenditure on goods to private gross national product is taken as a measure.

3. Trend variations to explain the changes in technology and the changes in consumer tastes. Time is taken as a variable.

4. Learning effect, which is similar to the one represented in the first model. The ratio of the excess of output in any year over the highest previous peak to the output of the year is taken as a measure of this variable.

In their model, Arrow and Hoffenberg omitted two variables, relative prices and lags of production. The relative prices have been omitted for four reasons :

1. In the short run, methods of production cannot respond to price variations.

2. The vertical aggregation in the industrial classification reduce the price substitution effect among the aggregate commodities.

3. During part of the period of study, the war made price motivations irrelevant.

4. Some of the factors that give rise to relative price variations are partially incorporated into the other explanatory variables on the model. For example, the change in consumer tastes which is one of the causes of the trend variation as in 3 above, can be attributed partially to changes in prices.

Lags of production are omitted because, when annual data is used, the lags are unimportant if the current output is considered as a reasonable forecast of a planned output, and there will be little error in ignoring the existence of lags. Clearly, this statement could be acceptable if the lag periods are short. However, lags in production might exist over longer periods to the extent that ignoring it would violate such statement.

The authors attempted to estimate the balance equations for a small number of inputs in which most of the structural coefficients are assumed to be zero on a priori grounds. The remaining structural coefficients were estimated according to both the simultaneous and the linear programming methods. The former method proved unusable. The linear programming estimates have been chosen to minimize the sum of absolute variations in the balance equations, subject to the conditions which ensure that for any plausible values of the explanatory variables, the input-output ratios computed from the estimated structural coefficients will be neither negative nor unreasonably high. Arrow and Hoffenberg were able to attach little reliability to the results of the linear programming. The authors concluded that the explanatory variables put forward are not sufficient to explain the changes in the input-coefficients. The complexity of the model makes it difficult for the authors to see where it is wrong or incomplete.

The tests which have been discussed stress the need for more comprehensive study in the field of input-output analysis. The superiority of the input-output model over naive methods is not so marked, and none of the tests suggests any development on the input-output model to make it really superior to the other tools.

3. Factors affecting the changes in the input-coefficients

In the last section we discussed some of the studies which examine the stability of the input-coefficients. The stability hypothesis has failed to be achieved and the weaknesses in it can be attributed to the following causes :

I. Technological change.

II. Product-mix and the aggregation problem.

III. Relative price substitution.

IV. Level of production.

Each of these causes will be discussed in this section, and an attempt will be made to show how it works in different types of economy. It will be concluded from this discussion that the input-output table of a developing economy is subject to more frequent changes than that of an advanced economy.

I. *Technological change*

Technological change may happen in two forms : reduction of the amount of input per unit of output or changes in the qualities of products. The reduction of the amount of input per unit of output can be achieved by substitution. There may be a substitution of capital for labour, capital for material, or material for material. Obviously, such changes will be reflected in the input structure of the industries. The substitution of capital for labour occurs through new investment aiming at this sort of substitution, and takes the form of machinery and equipment. Introducing more machinery and equipment usually affects the fuel demand substantially. This may be more effective in the case of a developing economy. The developing economy is usually marked by the existence of a plan to convert the economy from underveloped to advanced. The economic plan is usually characterized by more investment, and the more machinery and equipment the less labour force, and the more fuel input per unit of output. To clarify this point, we consider the input of all fuel to agriculture and textile industries in the U.K. as an advanced economy and the U.A.R. as a developing economy. These input-coefficients are shown in table (1).

TABLE (1)

	U.K.	U.A.R.
All fuel into agriculture	.054	.012
All fuel into textile	.019	.010

More capital equipment aims either for fuel saving, e.g. purchasing equipment to be run with cheaper fuel, or for labour saving, e.g. the use of tractors which consequently means more use of fuel. In U.K. it is not solely the case that equipment enables substitution of capital for labour. It often means technological progress, either to improve the properties or to economize the fuel consumption. So it is possible that there may be no substantial change in the use of fuel into agriculture and textile in the U.K. The case is different in the U.A.R. : there, more equipment means less labour force and more fuel per unit of output. So if the U.A.R. is applying a developing programme, one expects the use of fuel in agriculture and textiles to increase substantially. So it is expected that the rate of change in these two coefficients is likely to be more than the rate of change in the U.K. figures and it is expected that the two sets of figures will approach each other if the developing economy is progressing well.

The substitution of capital for material or material for material as a technological change is often less effective than the substitution of capital for labour. On the other hand, these two sorts of substitution affect the input structure in the advanced economy more than in a developing one, where these effects are more or less negligible. This can be attributed to the fact that in an advanced economy the capital is not as scarce as in a developing one, and the more scarce the capital the less likely for the substitution of capital for material to occur. The same idea can be applied in the substitution of material for material. Usually this sort of substitution is accompanied by a change in the process of production and the quality of commodities. It may need some change in equipment which in turn needs capital. The effect of substitution of capital or material for material will be discussed more in the effect of price substitution on the input-coefficients.

Technological change may be considered as the changes of the quality of commodities which occur through invention and innovation. The changes in the quality of products make it difficult to express the quantities of these products in terms of a common unit throughout a period during which these industries have undergone considerable technological progress. The real outputs which are expressed in terms of prices in a base year are expected to represent the quantities of the same products over different periods in terms of a common unit. However, it may not be possible to represent these quantities in terms of a common unit when the quantities of the products undergo significant changes due to technological progress. Consequently, over different periods technological progress may make it impossible to regard the input-coefficients as the ratios of the quantities of the same factors of production to the quantities of the same products. In a developed economy there is a continuous attempt to change the quality of commodities and when changes occur the input-coefficients are expected to change. In a developing economy, the growth programmes are usually started by enlarging the existing industries by increasing the number of firms. The new firms work at the most advanced technique available at the time. So at the period of construction, the input-coefficients are expected to change sharply. Having constructed these new firms, there will be a period of stagnation due to no further application of a more advanced technique. Through this period the input-coefficients are likely to be stable which in an advanced economy they change almost continuously.

II. *Product-mix and the aggregation problem*

The aggregation problem in the input-output model has been subject to a thorough investigation since the model was created. The input-coefficients are considered stable assuming the three conditions put forward in section 1. One of these conditions is that the output of each industry is a homogenous commodity which can be produced by one production function, and it is a definition of the sector in the input-output table put by Leontief that it is composed of plants producing a single homogenous product by a similar technique. This definition is practically impossible to be achieved because of statistical limitations and so some consideration must be added to the sector's definition to relieve its rigidity. Practically, the main problem is aggregating all activities into sectors ; how should aggregation be determined, on what principle, and what is the best degree of aggregation ?

The degree of aggregation is subject to different considerations. The choice of a certain degree of aggregation is decided, considering two points ; the degree of interdependence between the existing industries, and the functions which the input-output table is going to serve. For the first point we consider the cases of advanced and underdeveloped economics. In the case of advanced economy we should distinguish two types ; an economy which practises all types of industrial activities, e.g. U.K. and U.S.A. and an economy which lays stress on some particular activities, e.g. Switzerland. One would expect the first to be highly interdependent and a highly disaggregated input-output table would be more useful to describe such economy. For the second type, it is preferable to disaggregate the most important activities and aggregate the less important ones.

Considering the underdeveloped economies, we can as well distinguish between conomies where there is hardly any interdependence and others where the economy depends heavily on complementary imports. As an example of the first type, the input-output table in Tanganyika contains eighteen sectors which make 306 cells. Out of these cells it was possible to fill 23 cells only. Moreover, the total inputs figure is £ 8.3 m. compared with deliveries of £ 182 m. to final demands consists mainly of unprocessed agricultural products (11). This shows that there is hardly any interdependence between industries and it is a common character of the underdeveloped economies which rely heavily on imports for final demands. For such economies if there exist some industries dependent on each other, it is preferable to disaggregate them and aggregate the industries which have no significant interdependence between them. However, in some underdeveloped economies, the input-output table seems condensed, but usually the existing industries do not provide most of the intermediate demands, which are provided through imports. As an example, the input-output table for the U.A.R. for 1954 contains 1,056 cells out of which there are 542 non-zero elements (8). However, many of these inputs are maintained through imports. Table (2) shows some input-coefficients and the percentage of imports included in them.

TABLE (2)

	Input-Coefficient	Percentage of imports
Fertilizers into agriculture	.041	71%
Petroleum into transport	.104	40%
Metals into construction	.080	53%

For such an economy there is a need for fine aggregation and separate cells for intermediate imports if it is possible to know the exact figure of such imports.

There is no doubt that highly disaggregated input-output tables serve all the functions much better than less disaggregate ones. However, in a developing economy a fine aggregate is needed especially if the country is applying a developing programme.

In Leontief theoretical input-output model, the sector is assumed to be composed of plants producing a single homogeneous product by similar techniques, this means grouping together only plants in which both outputs and inputs structures are similar. A close approximation to this concept would be impossible for the variety of products produced by the typical plants. According to Barna (3), Baldereton and Whiten (4), and Chenery (7), aggregation can be based on one of the following basis :

(a) Aggregation based on similarity of input structure.

(b) Aggregation based on the use of outputs of several processes in fixed proportions.

(c) Aggregation of substitutes.

Each of these bases has its advantage and disadvantage. In method (a) a change in composition of output will have no effect on the input required from other sectors, but on the other hand, there is a considerable variation in the commodities produced by a sector. The condition for case (b) is most likely to be met in successive stages of processing of the same material. This is rarely perfect for there are some demands for semi-finished products. Sectors aggregated according to the third basis will have unstable input-coefficients unless the productive processes also have the similar inputs.

III. *Relative prices substitution*

Alternative techniques can be applied to produce the same output from different types of inputs. The choice between alternative inputs is decided largely by their relative prices. The type of substitution depends mainly on the production processes. Some production processes can accept an alternative input without changes in the process or the quality of the product. This has a marked effect on the input-coefficients, especially in the short-range. Some substitutions cannot be applied directly, and these may require a change in the capital equipment and the process of production, and these changes cannot be expected to occur over a short period. So the changes in input-coefficients due to such type of substitution are likely to happen over longer periods during which the substitution exists. In the economies which depend to a great extent on imports as intermediate products, the changes in input-coefficients will not only depend on the relative prices of the domestically produced commodities but also on the import prices. These prices are subject to world supply and demand.

IV. *Level of production*

The assumption in the input-output model is that there is a linear relation between the inputs and the outputs of an industry. This is not always the case as it may happen that, by increasing the level of production, more of particular inputs than is indicated by their average coefficients will be needed. If the production of an industry changes without change in the number of firms, one can expect that certain input-coefficients might change. In a developing economy industries may initially be set below their optimum size because of limitation of markets, or for other reasons. The expansion of these industries may bring about a change in the input-coefficients. Certainly this can be applied to all types of economies, but it is more noticeable in a developing economy.

4. Conclusinn

1. The overall tests do indicate, in conjunction with direct study of input-coefficients, that modifications of strict input-output assumption are clearly desirable.

2. The degree of stability of input-coefficients depends in part on the way sectors are selected and in part on the underlying properties of the productive system.

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