SAVINGS WHICHARE IN EXCESS OF ONE'S NEEDS*

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The purpose of this paper is to investigate the behavior of a society in which no consumer is all owed to keep more the he needs. To prepore for discussion and analysis, the following model will be adopted.

The Model :

It is assumed (for Simplicity) that owr society consists of two individuals (individual number (I) and individual number (II). Both individwals are producing the same product (good) and this product is homogenous. Individual number (I) produces (X_1)and the other produces (X_2). The total production of the society is given by,

$$X = X_1 + X_2 \qquad (1)$$

The amount consumed by individual number (I) is givn by (y_1) and (y_2) is the amount consumed by individual number (II).

* According to part two of the Green Book "Savings which are in excess of one's needs are another person's share of the weolth of the Society.

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If $(X_1 - y_1) = \epsilon_1$, where (ϵ_1) is the amount that exceeds the needs of individual number (I), (ϵ_1) can be considered as the other individual's share and should be given to him.

Similarty, if $(X_2 - y_2) = \epsilon_2$, where (ϵ_2) is the amount that exceeds the needs of individual number (I), (ϵ_2) can be considered as individual number (I)'s share and should be given to him.

Obviously $\epsilon_1 \gtreqless 0$ according to $X_1 \gtreqless y_1$

and also $\epsilon_2 \gtreqless 0$ according to $X_2 \gtreqless y_2$

Without violating the rule we can have

$$\epsilon_1 + \epsilon_2 = 0 \qquad (2)$$

Which implies that if somone has to give, the other has to receive. This will also support the equivilibrium rule

$$X_1 + X_2 = y_1 + y_2 \qquad (3$$

Which means that total consumption equals total production,

Since (X) is constant each time, we can qive (as a' constraint) ;

$$X \geqq y_1 + y_2 \qquad (4)$$

We assume that (W) is the total welfare of the society which is given as;

$$W = W_1 + W_2 \qquad (5)$$

where,

$W_1 = W_1 (y_1)$ is the welfare of individual number (I)

$$\frac{\partial W_1}{\partial y_1} > 0 \qquad \frac{\partial^2 W_1}{\partial y_2^2} < 0 \quad \text{for any } y_1 > 0$$

$W_2 = W_2 (y_2)$ is the welfare of individual number (II)

$$\frac{\partial W_2}{\partial y_2} > 0 \qquad \frac{\partial^2 W_2}{\partial y_2^2} < 0 \quad \text{for any } y_2 > 0$$

Hence,

$$W = W (y_1 \ \ y_2) = W_1 (y_1) + W_2 (y_2) \qquad (5')$$

Problem and Solution :

The problem now can be regarded as that of maximiging (W), Subject to the following constraints :

$$\epsilon_1 + \epsilon_2 \geqq 0 \qquad (2')$$

$$X \geqq y_1 + y_2 \qquad (4)$$

And hence, the Lagrange expression (L) con be given as;

$$L = W_1(y_1) + W_2(y_2)$$
$$- \lambda_1 (y_1 + y_2 - X)$$
$$- \lambda_2 (\epsilon_1 + \epsilon_2)$$

Our necessary conditions for a maximum are given by the following set of equations* :

$$\frac{\partial L}{\partial y_1} y_1 = \left(\frac{\partial W_1}{\partial y_1} - \lambda_1 \right) y_1 = 0 \qquad (6)$$

$$\frac{\partial L}{\partial y_2} y_2 = \left(\frac{\partial W_2}{\partial y_2} - \lambda_1 \right) y_1 = 0 \qquad (7)$$

$$\frac{\partial L}{\partial \lambda_1} \lambda_1 = (y_1 + y_2 - X) \lambda_1 = 0 \qquad (8)$$

* SeeM. D. Intriligator, Mathematical optimization and Economic Theory, Prentice - Hall, 1971, PP. 24—60.

$$\frac{\partial L^{1}}{\partial \lambda^{2}} \lambda_{2} = (\epsilon_{1} + \epsilon_{2}) \lambda_{2} = 0 \qquad (9)$$

$$\frac{\partial L^{2}}{\partial \epsilon_{1}} \epsilon_{1} = \left(\frac{\partial W_{1}}{\partial y_{1}} - {}_{1} \right) \frac{\partial y_{1}}{\partial \epsilon_{1}} - {}_{2} \Big) \epsilon_{1} = 0 \qquad (10)$$

$$\frac{\partial L}{\partial \epsilon^{2}} \epsilon_{2} = \left(\frac{\partial W_{2}}{\partial y_{2}} - \lambda_{1} \right) \frac{\partial y_{2}}{\partial_{2}} - \lambda_{2} \Big) \epsilon_{2} = 0 \qquad (11)$$

Since $y_{1} > 0 \ , \ y_{2} > 0 \ , \lambda_{1} \quad 0, \ _{2.} \quad 0$

the above conditions can be reducel to ;

$$\frac{\partial W_{1}}{\partial y_{1}} - \lambda_{1} = 0 \qquad (6')$$

$$\frac{\partial W_{2}}{\partial y_{2}} - \lambda_{1} = 0 \qquad (7')$$

$$y_{1} + y_{2} - X = 0 \qquad (8')$$

$$\epsilon_{1} + \epsilon_{2} = 0 \qquad (9')$$

$$\lambda_2 \epsilon_1 = 0 \qquad (10')$$

$$\lambda_2 \epsilon_2 = 0 \qquad (11')$$

To illustrate the solution of this problem two cases will be considered. The first case is that when $W_1 = W_2$ and the second is that when $W_1 \neq W_2$

The first case :

Let us prepare aur first rule of maximigation which can be obtained from equations (6') and (7') as follows ;

$$\frac{\partial W_1}{\partial y_1} = \frac{\partial W_2}{\partial y_2} \qquad (12)$$

This means that if total welfare of the society as a whole has to be at a maximum, the marginal welfare of every individual in the society should be the same. When $W_1 = W_2$, this implies that $y_1 = y_2$ (i. e. the total amount Consumed by individual number (I) should equal the total amount consumed by individual number (II).

If for one reason or another $y_1 \neq y_2$), our first rule will be violated and hence the total welfare is not maximiged. A special arrangment should be made in order to achieve equal distribution of consumption between the first and the socond individual.

Assume that ($X_1 < X_2$) i.e. the first consumer produces less than the second. This means that individual (II) will be able to consume more than individual (I), if there is no interference, hence $y_1 < y_2$ and

$$\frac{\partial W_1}{\partial y_1} > \frac{\partial W_2}{\partial y_2}$$

which means that the total welfare of the society can be taken to a ligher level, if we transfer part of the production of the second Consumer for consumption by the first, without violating our first rule y_1 can be given as

$$y_1 = X_1 + \epsilon_2$$

where, $\epsilon_2 = (X_2 - X_1) / 2$

which implies that

$$_1 = (X_1 - X_2) / 2$$

This means that total production should be shared equally by the first and the second consumer*.

* This can be seen by rearranging $y_1 = X_1 + {}_2$ as
$y_1 = X_1 + (X_2 - X_1) / 2 =$
$(2 X_1 - X_1 - X_2) / 2 = (X_1 + y_2) / 2,$
which can obtained also from $y_2 = X_2 + \epsilon_1$.

Now we are in a position to study the implications of conditions (10') and (11') which provide our second rule.

$$\lambda_2 \epsilon_1 = \lambda_2 \epsilon_2 = 0 \qquad (13)$$

This can be maintained when,

$$\lambda_2 = 0, \epsilon_1 \neq 0, \epsilon_2 \neq 0$$

and also when,

$$\lambda_2 = 0, \epsilon_1 = \epsilon_2 = 0$$

Which implies that ($\epsilon_1 + \epsilon_2 = 0$) can not be considered as a constraint because the Shadow price $\lambda_2 = 0$ (no cost is involved in violating the constraint), a case which is logically excluded.

The only rase which is left in order to Satisfy our second rule is that when

$$\lambda_2 \neq 0 \quad \text{but} \quad \epsilon_1 = 0, \epsilon_2 = 0$$

which implies either,

(i) Each individual should be allowed to use up his total production completyl, which means that

$$y_1 \gtreqless y_2$$

$$\frac{\partial W_1}{\partial y_1} \lesseqgtr \frac{\partial W_2}{\partial y_2}$$

and our first rule can be violated, or

(ii) Every individual should be forced (by some other arrangements) to produce the same amount of production as any other individual.

This leads to the following theorem.

Theorem '1' :

"Any society (where each individual has the same welfare function) can maximize its total welfare if and only if each individual in this society is forced to produce the same amount for consumption ".

The Second Case :

When $W_1 \neq W_2$ our first rule

$$\frac{\partial W_1}{\partial y_1} = \frac{\partial W_2}{\partial y_2}$$

implies that $y_1 \neq y_2$

i.e. the total amount consumed by individual (I) should not be the same as that Consumed by individual (II). One individual should consume more than the other in order to satisfy the first rule of maximization.

This rule can be satisfied when any of the following main catogories are true :

Catogory (i)

$$y_1 \neq y_2 \,,\; X_1 \neq X_2 \,,\; y_1 = X_1 \,,\; y_2 = X_2$$

Catogory (ii)

$$y_1 \neq y_2 \quad X_1 \neq X_2 \quad y_1 = X_1 \quad y_2 < X_2$$

$$y_1 \neq y_2 \quad X_1 \neq X_2 \quad y_1 < X_1 \quad y_2 = X_2$$

$$y_1 \neq y_2 \quad X_1 \neq X_2 \,,\; y_1 < X_1 \quad y_2 < X_2$$

Catogory (iii)

$$y_1 \neq y_2 \quad X_1 = X_2 \quad y_1 = X_1 \quad y_2 < X_2$$

$$y_1 \neq y_2 \quad X_1 = X_2 \quad y_1 < X_1 \,,\; y_2 = X_2$$

$$y_1 \neq y_2 \,,\; X_1 = X_2 \,,\; y_1 < X_1 \,,\; y_2 < X_2$$

Catogory (i) may indicat ea state where all the conditions (and all the rules) are sadisfied, consumption equals production and no transfer should be made by any consumer to another consumer (i.e $\epsilon_1 = \epsilon_2 = 0$).

Catogory (ii) and catogory (iii) state the case where consumption does not equal total production even when the same amount is produced by each individual. This implies that not all the conditions are satisfied.

Let us concentrate on catogory (ii). Starting from a point where,

$$y_1 = X_1 \quad y_2 < X_2$$

meons that

$$(X_2 - y_2) = \epsilon_2 > 0$$

$$(X_1 - y_2) = \epsilon_1 = 0$$

which implies that the second rule i.e.

$$\lambda_2 \epsilon_1 = \lambda_2 \epsilon_2 = 0 \qquad (13)$$

is not satisfied (if $\lambda_2 \mp 0$) and also

$$\epsilon_1 + \epsilon_2 \mp 0$$

$$y_1 + y_2 \quad X \mp 0$$

Those two constraints are not exhausted.

Hence, Welfare is not at a maximum.

In order to achieve maximum welfare all production should be consumed to satisfy

$$y_1 + y_2 - X = 0 \qquad (8')$$

and part of (X_2) should be transfered to individual (I) to sotisfy the following condition.

$$\epsilon_1 + \epsilon_2 = 0 \qquad (9')$$

Our first rule can also be satisfied if Consumption is arranged in order to make

$$\frac{\partial W_1}{\partial y_1} = \frac{\partial W_2}{\partial y_2}$$

However, the second rule i.e

$$\lambda_2 \epsilon_2 = \lambda_2 \epsilon_1 \mp 0 \quad (\text{if} \quad _2 \mp 0)$$

because, $\epsilon_2 = - \epsilon_1 > 0$

This implies that in order to achieve maximum welfore, no one should be allowed to produce more than he needs ($\epsilon_1 = \epsilon_2 = 0$) that is, to hit catogory (i) right from the beginning. All other case will lead to the same conclusion.

However, it should be noted that when,

$$\epsilon_1 = \epsilon_2 = 0$$

$$X = y_1 + y_2$$

$$\lambda_2 \epsilon_1 = \lambda_2 \epsilon_2 = 0$$

$$y_1 \mp y_2, X_1 \mp X_2 \; y_1 = X_1, y_2 = X_2$$

The first rule is not necessarity catsified i.e.

$$\frac{\partial W_1}{\partial y_1} \qquad \frac{\partial W_2}{\partial y_2}$$

Which means we will not be sure that aur welfare is maximized. All that we can say i sthat in order to be at a maximum (provided that the first rule is satisfied), no consumer should be allowed to produce more than he needs (for consumption). In other words, no individual should expect any help from the other i.e ($\epsilon_1 = \epsilon_2 = 0$). This, leasd to the followirg theorem.

Theorem (2) :

"Any Society (where every in dividual has a rifferent welfare fanction) conot be sure that its total welfare is maximized when every individuol consumes all that he produces ".

Conclusion :

Fro mthe above discussion we have seen that two important rules should be satisfied in order to maximize the society welfare function; when no individual in this society is allowed to keep more then he neels for consumption. The first rule insures that the marginal welfare of every individual in this society should be at the same level. The second rule insures that everything produced by any individual should be consumed by the same individual.

We discovered that the only case for which a definite solutionn can be obtained is that case where individual welfare functions are identical everykhere. When individual welfare functions are different everywhere, we can be sure that the second rule is satisfied, but there is no guarantee that the first rule is also satisfied. This means that we cannot be sure that the society's total welfare is maximized.

However, a challenging question is facing us : "Is there a policy which insures that every individual in this society gets exactly what he needs for maintaining total welfare at its maximum level ?! " .

References

(1) Intriligator, M.D., Mathematical Optimization and Economic Theory, Prentice-Hall, 1971.

(2) Qadhafi, M., The Green Book, Part two, Martin Brian & O'weege, Landon, 1978.