INDUSTRIAL PRODUCTIVITY MEASUREMENT

(A Survey of the Main Methods)

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A productivity measure is the ratio between output and input both specifically and comparably defined. This ratio can be expressed in terms of a single factor input or selected inputs or all inputs combined. Thus the following broad concepts are introduced.

$$\text{A. Partial Productivity} = \frac{\text{total output}}{\text{one factor input}}$$

$$\text{B. Total Productivity} = \frac{\text{total output}}{\text{total inputs}}$$

Within these two frame-works various authors have defined productivity in different ways and consequently several different measurements have been suggested. These methods of measurement are discussed in the following pages.

A. Partial Productivity Measurement

This measures the relation between output and one factor input. In this way it is possible to speak of the productivity of capital, raw materials, or labour. The usefulness of this measure depends upon the type of data utilised in the measurement : it is more useful if the units are measured in physical quant ities. The use of value elements would introduce the impact of price changes, which have little bearing on productive efficiency and hence would give a distorted picture of the actual efficiency, while the advantage of the partial physical measure is that additive quantities can be obtained. For example, it can be shown that a unit of a given prod uct is arrived at by adding together certain consumed quantities of labour, raw materials etc· Thus, as they are calculated from physical quantities, concepts of partial physical productivities have the advantage of being extremely significant and easily understood by the workers actually engaged on the job.

The indexes thus obtained should, however, be interpreted with discrimination, as the increase in the productivity of the specific factors most usually considered (productivity of labour, raw materials) may in some cases coincide with a rise in production costs, even if the factor costs have not increased. This may occur if the specific productivity of certain other factors less frequently taken into account has unexpectedly fallen : for instance, the substitution of a machine for manual work may in some cases cause an increase in the unit cost of the product, and, consequently, a drop in the productivity of labour to a very marked extent. This may occur if the machine has to stand idle for a long time or if the cost of running the plant equals or exceeds the saving of labour per unit produced.

However, it must be noted that the most usual meaning of productivity, and in many ways the most important, is productivity of labour. The importance of labour productivity stems from the fact that it provides a general measurement of the economy and efficiency in the use of labour. The computations of labour productivity indexes do not imply that labour is the only relevant factor. Such an index, like other partial indexes, must not, however, be interpreted causally. It reflects the productivity of all factors involved in production, including labour.

Labour productivity may be measured in a variety of ways, each to suit a certain purpose. These measures fall into three general categories. First, there are the physical productivity measures which show changes in the labour time required to produce a fixed composite of goods and services. These measure changes in technical efficiency alone. Secondly, there are the gross productivity measures which take into account shifts in the relative importance of component sectors with different levels of output per man, as well as changes in output per man within sectors. Thirdly, there are the net output per man measures which reflect, in addition to changes in physical productivity and shifts between sectors, changes in material requirements per unit of output. Since they reflect the effect of changes in resource allocation, the gross and net measures may record changes even when there is no movement in the productivity of component sectors. Each of the categories specified requires different data. A summary of the main types of labour productivity measurement is shown in the chart below.

Chart 1

Labour Productivity Indexes

(Man-hours)

I. PHYSICAL OUTPUT			
Output per man-hour		Man-hour per unit of output	
Current year composite	Base year composite	Current year composite	Base year composite
II. NET OUTPUT			
Output per man-hour		Man-hour per unit of output	
Current year composite	Base year composite.	Current year composite	Base year composite

As can be seen from the above chart labour productivity can be measured in more than one way· For a firm producing a single product the technique of measurement is simple ; productivity in this case can be expressed as follows :

$P = \frac{Q}{L}$, where Q is the quantity or units of output and L denotes labour input. A change in productivity during two periods of time can be written as :

$\frac{Ql}{Ll} / \frac{Qo}{Lo}$, where suffix l and o denote the current and base year respectively. But measurement of labour productivity is usually not as simple as this. For in many cases a firm or industry produces a number of different products expressed in different physical units. Labour inputs also vary depending on whether skilled, unskilled, etc. This, however, is overcome by the introduction of a unit value element in combining the variety of output and various types of labour. Hence, two types of indexes are calculated : physical and value. The calculation of each type depends on the usefulness of the measure to the problem at hand and according to the availability of data.

If output is homogenous then it is easy to construct a production index. But when output is varied then a weighted index must be constructed. Similarly, in calculating the index of labour, weights should be used in combining the various types of labour. This weighting procedure is, however, not required when the indexes of output and labour are calculated in value units. Prices of output and various types of labour serve as weight co-efficients. [1]

B. Total Productivity Measurement

(A Survey of the Main Methods)

Numerous attempts at measuring total productivity in physical terms have not yet met with much success. It is sometimes impractical to take all elements of inputs and add them up in terms of man-hours and the result is always difficult to interpret. [2] On the other hand, if all elements are measured and added up in monetary terms one usually ends up by measuring something other than productivity, such as profitability. Interpretation is again made difficult by including, in addition to productivity factors, the effect of all factors which determine prices. If they are summed up on a constant price basis there are difficulties with price indexes and in particular data of quantities used as well as prices for each are needed.

Many of the problems concerning the measurement of factor inputs apply equally to the measurement of output· However, these problems have not stopped authors from introducing certain measurements, the object of which is to assess the overall efficiency and also to improvise on existing methods. In the following pages a summary of the main methods introduced into this field is presented.

The subject of total productivity measurement has been greatly developed through production function analysis. The techniques of production function have been developed largely at the theoretical level, and it is only very recently that some progress at the practical level has been made through the use of various models with the help of computers.

In most production function studies a period of more than 10 years is usually chosen. This is done to eliminate short-run influencess and to establish the trend of development in productivity.

During the last few decades two production functions have occupied a prominent place in quantitative economics. The first is the Cobb-Douglas type, with constant returns to scale. This is expressed in the following formula :

$O_t = A_t \, L_t^{b} \, K_t^{1-b}$, where O_t is output in year t, k_t and L_t are capital and labour inputs respectively in year t ; b and 1-b are the elasticities of output in relation to labour and capital respectively. A_t is an index of total factor productivity and represents the contribution of productivity due to quality improvements of both labour and capital. The improvements are attributed to better skills and application of technological developments. The Cobb-Douglas function assumes neutral technological effects ; the contribution of factor inputs to output depends on their elasticities with respect to output. After allowing for the contribution of labour and capital to output a residual is arrived at which represents the contribution of total factor productivity, i.e. A_t , to output. [3]

The other function is a more elementary one of the form :

$P = f(K)$ or $P = aK$, where $\underline{a}$ may be interpreted either as the output-capital ratio, or in some contexts as the reciprocal of the accclerator. This is commonly found in growth models of the kind made popular by Harrod and Domar. [4] This function being a one factor function, can be viewed as a special and limiting case of the function :

$$P = a K^{1-b} \cdot L^{b} \text{, where } b = 0$$

M. Frankel introduced a method for reconciling the Cobb-Douglas and the Harrod-Domar functions. The object, as he puts it, "is to introduce a formula that retains desirable properties of both functions." [5] This formula is expressed as :

$$Pi = a H K_i^{\beta} L_i^{\alpha}$$

, where subscript i denotes ith enterprises, and P, K, L, and a are defined as above. H is referred to as the "development modifier", and is intended to denote the development of the economy in which the enterprise operates and is, for the enterprise, a parameter. β and α are the elasticities of factor inputs, labour and capital, with respect to output. Enterprises in advanced countries are able to produce more with given inputs of capital and labour than enterprises in relatively underdeveloped economies. [6] H, the development modifier, is measured as

$\left(\frac{K}{L}\right)^{v}$, where the exponent v is a parameter and gives the expression a more general form. The aggregate function may be written as follows :

$$P = a \left(\frac{K}{L}\right)^{v} K^{\beta} L^{\alpha}$$

$$= a K^{\beta+v} L^{\alpha-v}$$

The final step to complete the synthesis is based on the assumption that $v = \alpha$. Then the aggregate function reduces to

$$P = a K.$$ [7]

The above analysis is a brief summary of the main types of the production function technique. It is worth noting that improved models of the production function have been introduced by various authors. The main two models are those of R. Solow and E. Denison. Solow's focuses attention on the impact of improvements in the quality of capital stock on output ; Denison's draws attention to the impact of improvements in the quality of labour input. The object of both models is to make allowances for the impact of quality changes of labour and capital, over the years, that would result in a higher output level. [8] These allowances are deducted from the total productivity index A_t in the Cobb-Douglas function.

In the remaining part of this section a brief survey of other total productivity measurements are presented. These methods, which have been used by various authors, may be summed up as follows :

1. The inverse of unit total costs at constant prices. [9]
2. The inverse of unit total costs at current factor prices. [10]
3. The inverse of unit total costs expressed in wage equivalents. [11]
4. Output divided by the quantity of each factor weighted by the quantities of labour employed in making it. [12]
5. The weighted average, in physical terms, of the productivities of inputs. [13]
6. Output divided by total inputs, both in terms of standard prices. [14]
7. Total productivity measured in terms of progress. [15]
8. Total weighted productivity at constant prices, and the combined productivity of the partial productivities. [16]

Brief as this catalogue is, it clearly indicates the large number of definitions used in the measurement of productivity. In the following paragraphs a brief comment on each is made.

Method (1) presupposes the availability for each input of the physical quantities used as well as their prices. Method (2) is similar to that used in ordinary costing. However, any index based on unit costs at current prices or on wage equivalents is unsuitable as a measure of technical productivity, since it already includes, in addition to technical efficiency, the effect of factors that determine the value of output and input, i·e. price changes.

The expression of costs in labour units, as in method (3) and (4), raises problems of its own. The essence of method (3), which was put forward by Smith and Beeching, suggests that depreciation, purchased services and certain changes in raw materials, i.e. stocks, should be converted to equivalent manpower and added to the manpower employed by the firm to be used as the denominator of a productivity ratio. Depreciation and other inputs bought, i.e. raw materials, would be converted to equivalent manpower by dividing their value by the average annual income of industrial employees of all kinds in the industry in which the firm's' is located.

Method (4) is similar to Method (3) but suggests using a firms own cost of labour. The transformation of costs, other than those of labour, to man-hour equivalents is arrived at by dividing these costs by the average hourly rate paid by the firm.

However, Method (3) and (4) raise many problems. For instance, how are the services of capital goods which have been partly amortised to be expressed in terms of man-hours ? And how are we to allow for the fact that labour employed is not homogeneous ? In practice it is difficult to overcome these problems without the availability of detailed data. And even if we assume that it is possible to overcome most of these problems, it remains difficult to interpret the results of such a productivity measurement. To wrap up many things in one common unit conceals many important facts which this common unit, i.e. labour equivalent, attempts to represent. The results tell us nothing about changes unless we first know the partial productivities of factor inputs, because the effects of various factors are cumulative and interacting. Without additional information we cannot impute what part of the change is due to variations in technical productivity and what part to a change in the size of factor input utilisation.

Method (5) is slightly different from the previous two, but amounts to much the same thing as inverse unit costs at constant prices.

Method (6) involves the presentation of the revenue accounts at constant prices and these will be related to revalued output and inputs, and the latter, i.e. inputs, are determined according to certain average prices.

Method (7) was introduced by Smith and Reddaway in recent years. This measure was carried out for 14 major groups of British manufacturing industries between 1948 and 1954. The overall change in productivity was considered to measure progress during this period. The authors used a measure of the combined inputs of labour and capital. The basic formula is :

$$\frac{\dfrac{O_2}{O_1}}{\dfrac{I_2}{I_1}} - 1$$

The outputs (0) in the formula are measured net of raw materials and services obtained from outside, and also allow for depreciation, Labour inputs represent man-hours worked by salaried employees and workers on own account. Capital is taken to be the quantity of real resources, building, machinery, vehicles and stocks, measured at a constant price replacement cost, net of depreciation. These inputs are weighted with their respective prices, i.e. labour (L) with wage rate(W), capital (C) with the return or charge to a unit of capital (r) ; P_1 stands for prices of output at 1948 prices. The formula then becomes :

$$\text{Progress} = \frac{P_1 \ O_2 / P_1 \ O_2}{\dfrac{W_1 \ L_2 + r_1 \ C_2}{W_1 \ L_1 + r_1 \ C_1}} - 1$$

If it is assumed that labour and capital could exhaust net output in the base year :

$W_1 \ L_1 + r_1 \ C_1 = P_1 \ C_1$, then the formula simplifies to :

$$\frac{P_1 \quad C_2}{W_1 \quad L_2 + r_1 \ C_2}$$

Measures of capital stock were not readily available. The authors, therefore, used the absolute changes between 1948 and 1954 for all variables, so that

$$r_1 \ C_2 = r_1 \ C_1 + r_1 \ \triangle C,$$

and capital formation can be substituted for stock of capital. Employing a uniform notation throughout we get :

$$\frac{P1 \quad 0\ 2}{P_1 \ O_1 + W_1 \ \triangle L + r_1 \ \triangle C} - 1,$$

since $P_1 \ O_2 = P_1 \ O_1 + P_1 \ \triangle O$, the final equation becomes :

$$\frac{P_1 \ \triangle O - (W_1 \ \triangle L + r_1 \ \triangle C)}{P_1 \ O_1 + W_1 \ \triangle L + r_1 \ \triangle C}$$

Thus progress can be interpreted as :

$$\frac{\text{Increase in output, less allowance for extra factors}}{\text{Output which would have been attained with unchanged productivity}}$$

Method (8) has been introduced by various authors. The main studies in this field are those of J. Kendrick and S· Fabricant.

J. Kendrick measured the total productivity of 33 industry groups, and also the private domestic economy of the United States during the period 1889-1953. Factor inputs were classified into two broad classes : labour and capital. The labour input used in this study denotes man-hours worked in various industries by all types of persons engaged in productive activities (including proprietors), weighted by base period average hourly earnings. He thus takes into account differences in the quality of labour. With regard to capital, a constant dollar value of stock of real capital-land, equipment, plant, and inventories- was used. The capital input was weighted according to the rate of return in the base year. The value of the plant and equipment net of depreciation was used. Thus the method implies that capital services move proportionately with capital stock.

Almost the same method has been used by S. Fabricant in his study "Basic Facts on Productivity Change" (17). He used weighted and unweighted man-hours, and unweighted and weighted input of tangible capital. Thus he arrives at two estimates of total input :

(a) the weighted combination of an unweighted man-hours index and an unweighted capital input index, and

(b) the weighted combination of a weighted man-hours index and a weighted capital input index.

He also had two estimates of total productivity ; the first estimate is based on the unweighted total input index and the second on the weighted total input index. In addition to total productivity indexes, Fabricant also gives partial productivity indexes of both labour and capital.

In conclusion the foregoing analysis of productivity measurement is recapitulated. Productivity has been defined as a ratio of output to input (s). This in a broad sense covers all productivity ratios that can be calculated by relating output to either one factor input or to all factors used in the production process· The index of productivity attempts to measure the contribution of input (s) to output, and productivity indexes over the years would indicate the efficiency of factor inputs using different production techniques.

The type of productivity index used is dependent upon two main factors :

(a) the usefulness of the concept of productivity adopted in the country concerned ; and

(b) practical feasibility for measurement.

Sources and Notes

1. For a study of the types of index-numbers and the choice of the

weighting procedure see U.N., *"Index-Numbers of Industrial Production"*, Studies in Methods, No. 1, New York, Sept. 1950. For calculation of capital input see H. A. Suleiman, *"Factors Affecting British Industrial Productivity, 1948-61"*, M.A. Thesis, Unpublished, Sheffied University, (England), 1964. For other procedural calculations of productivity indexes see : H. A. Suleiman, *"Industrial Productivity in Iraq with Special Reference to Selected Firms : 1953-63"*, Ph.D. Thesis, Unpublished, London University, 1967.

2. See comment on Method 4 p. 43.

3. Cf. H. Phelps Brown, "The Meaning of the Fitted Cobb-Douglas Function", *Quarterly Journal of Economics*, Nov. 1957 ; P. H. Douglas, "Are there Laws of Production ?", *American Economic Review*, March 1948 ; G. T. Gunn and P. H. Douglas, "Further Measures of the Marginal Productivity", in *"Quarterly Journal of Economic"*, May 1940 ; and R. R. Nelson, "Aggregate Production Functions and Medium Rate Growth Projections", in *"American Economic Review"*, Sept. 1964.

4. Cf. R. F. Harrod, *"Towards a Dynamic Economics"*, London 1948; E. D. Domar, "Capital Expansion, Rate of Growth and Employment", in *"Econometrica"*, April, 1946. Also for a brief assessement of these two functions see : M. Frankel, "The Production Function in Allocation and Growth: A Synthesis", in *"American Economic Review"*, Dec. 1962 ; and R. R. Nelson, *op. cit.*

5. *Ibid*, p. 997.

6. This is attributed to the fact that productive capacities of factor inputs, i.e. labour and capital, are higher in advanced countries compared with underdeveloped. This is because of better skills, attributed to a higher levele of education, and because of the use of more up-to-date machinery and equipment.

7. The sum of the co-efficients : $\beta + \alpha + \nu - \nu$, remains equal to unity. But this constraint could be eliminated by putting the

modifier into the yet more general form : $K^{v/v}$. For details on this aspect see M· Frankel's article, *ibid*, especially Section IV.

8. For details on these models see R. R. Nelson, *op. cit.*

9. Hiram S. Davis, "*Productivity Accounting*", Phil., 1955, pp. 6-8.

10. M. J. Farrel, "The Measurement of Productive Efficiency", in "*Journal of the Royal Statistical Society*, Series A, Part III, 1957, esp. 253.

11. Sir Ewart Smith and Dr. R. Beeching, "Measurement of the Effectiveness of the Productive Unit", *British Institute of Management*, Winter Proceedings, 1948-49, No· 4.

12. This method has been introduced by W. Alderson. See H. S. Davis, *op. cit.*, pp. 8-9.

13. G. J. Stigler, "*Trends in Output and Employment*", NBER, New York, 1947.

14. The techniques of standard costing are determined on a variety of basis· For example, historical costs may be used in this respect, and in this case present achievements are judged by past standards ; or a standard costing may be determined on budgetary accounting ; or an average standard may be determined by enginnering methods or by adding historical costs to a certain standard. These techniques are adopted in most firms in advanced countries, and are explained in most sources in the field of accounting and management. See for example, "*Cost Accounting and Productivity*", Report by a Group of European Experts, OEEC, 1952.

15. E. Smith and W. B. Reddaway, "Progress in British Manufacturing Industries in the period 1948-54", in "*Economic Journal*", March 1960.

16. Cf. J. Kendrick, "*Productivity Trends in the United States*", NBER, 1961 ; S. Fabricant, "Basic Facts on Productivity Change", NBER, 1958.