THE NATURE OF OPERATIONAL RESEARCH

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This paper discusses the nature of operational research and the concept of modelling. The paper then looks at a simplified information flow of a typical company as a basis for indicating some of the main tactical decision areas in the control of production systems. These will include the problems of demand forcasting and inventory control and production planning and control. In each of these areas digital computers have made the use of new methods possible. Selected problem areas such as shop scheduling where computer simulation has given encouragement to research are then discussed and the paper concludes with the description of a simplified model of the industrial dynamics type.

1. Introduction

Operations research is not necessarily associated with computers but there are several sets of interactions.

(a) technical, in that computers enable one to solve problems insoluble or uneconomic by other means.

(b) historical, in that some companies introduced O.R. as an adjoint to computer data processing systems which provide a good base for the development of more sophisticated methods.

(c) organisational, in that, following from (b), their work interacts and O.R. commonly forms part of a management services group in a company.

2. What is Operational Research ?

Operational research is essentially problem orientated and what-

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ever the methods used whether based on simple arithmetic, simple statistics or sophisticated mathematics the investigations should be essentially practical in nature.

Many definitions of O.R. have been suggested, The important two are :

1. O.R. is the application of scientific method, techniques, and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solutions to the problems. (Churchman, Ackoff, and Arnoff, 1957) [1].

2. The O.R. society definition :—
O.R. is the attack of modern science on complex problems arising in the direction and management of large systems of men, machines, material, and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system, incorporating measurement of factors such as chance and risk, with which to predict and compare the outcome of alternate decisions, strategies and control. The purpose is to help management to determine its policy and actions scientifically.

The second definition brings out several of the essential features. First, that O.R. is concerned with problems of organization and allocation of resources of men, machines, materials, and money to achieve some objective. Secondly, it emphasises the idea of building a model of the system. Thirdly, it says that it is a service which is trying to help management. Fourthly, it implies that there must be some measure of effectiveness if one is to be able to compare the outcome of alternate decisions.

If one then combines these points with the fact that one often has inter-desciplinary team of people working on the project one has a description of what many O.R. groups are doing and how they are composed. In other words O.R. is not the set of well known techniques associated with it such as queueing theory and linear programming. These, and others, are the tools of the trade used by the operational researcher when he carries out research into operations. O.R· is a scientific method applied to operational problems.

To provide the advantages of specialization and subdivision of labour a company's organization is usually split into functional areas such as marketing, production, finance, research, and development. But this is achieved at the price of having to co-ordinate these areas to achieve the best overall result. Conflict often arises and, even when the co-operative intent is there, it may not be possible to obtaine " best " solutions because of the complexity of operations and the difficulty of obtaining objective information through the normal channels of communication.

As an example of conflicting interests, let us consider what stock level should be held. The more stock that is held the better the service that can be provided but the more it costs in stock holding charges. It is common to get a curve of the form shown in fig. 1. i·e., the cost of holding stock increases rabidly to brovide the last few percentage of service. If poor service is given one loses goodwill and potential profit; on the other hand too good service may cost more than is justified. The problem is, what service level and what stock holding should be chosen in order to maximize the overall performance, e.g. by minimizing cost ?

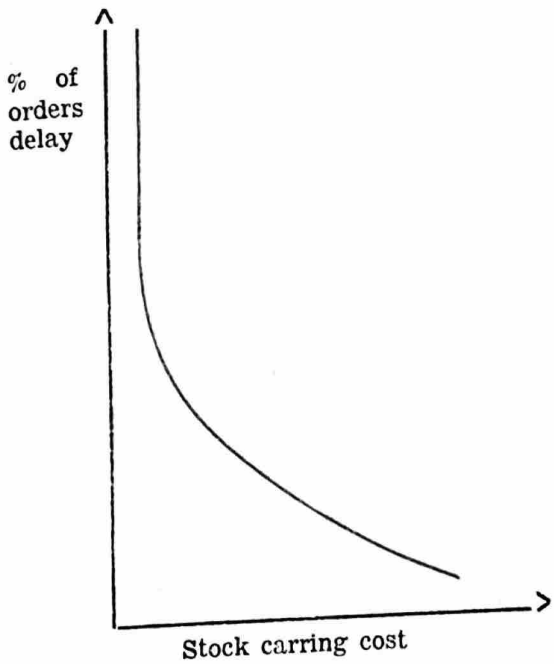


FIG. 1.

Expressed in a more general form we can say that the result of an O.R. study is to produce cost or some other measure of effectiveness as a function of the variable or variables under consideration, this is illustrated for one variable in fig. 2.

As a result of an O.R. study one might recommend, according to whether the results are sensitive or relatively insensitive to the variable under consideration,

(a) the value of the variable which should be chosen (B), or

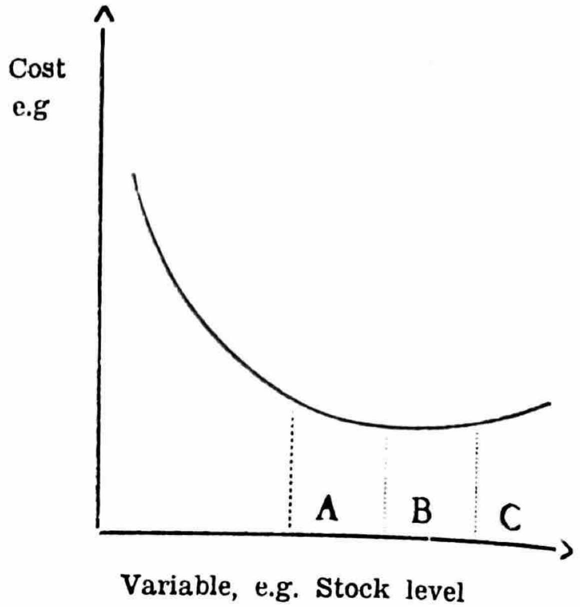


FIG. 2

(b) the range (A-B) within which the variable might be without greatly affecting the result.

Expressed mathematically one can say that one tries to construct an effectiveness function $E = f(x_i, y_i)$ where x_i are the controllable variables and y_i are the variables not under control.

Three final comments. First, that the problem area chosen should be one where worthwhile saving are possible. A preliminary survey often indicates that some problems are not worth tackling and that others could be enlarged to include potentially more rewording studies. Secondly, there is, sometimes, difficulty in deciding which of several possible measures of effectiveness should be used, e.g. whether one should maximize profit, minimize cost, maximize return on capital or something else. Thirdly, most published work concentrates on the model building and techniques. This tends to mask the fact that much of the work consists of identifying the problem, collecting data and implementing results.

3. The O.R. Procedure

O.R. tackles its problems by a series of steps which might be called the O.R. procedure. This has been expressed in several slightly different forms but basically it is as follows :

1. formulate the problem.
2. obtain quantitative information relevant to the problem.
3. analyse the information.
4. formulate a hypothesis (build a model).
5. test the hypothesis (test the model).
6. determine the effects of alternatives to enable the executives to make a decision.
7. a) ensure that the decision is implemented

   b) establish control over the solution.

The first five of these are the commonly accepted steps of the scientific method, the last two are concerned with the implementation stage. The steps are of course not necessarily tackled in this sequence, e.g. the problem might not be dear until a lot of information had been analysed, and sometimes there will be some cyclic steps, e.g. the testing of the hypothesis could give a negative result sending one back to an earlier step. Let us look at each step in more detail.

Step 1. *Formulation*

This is the most important and difficult step in the procedure. As indicated by churchman and others [1], there are basically two problems :

(a) the consumer, i.e. the person for whom the work is being carried out.

(b) the researcher. His problem is the determination of the effect that alternative courses of action have on the measure of effectiveness.

Goode and Machol [2], indicate that the correct formulation of the problem is the essence of the solution and that one can specify necessary elements in the correct statement, but cannot be certain about sufficient elements. The four essential elements in problem formulation are given as :—

1. Environments : The present system, administrative lines, political relationships etc.
2. View points : e.g. The director manager and functional manager will have different objectives.
3. Permissible solutions. Limitations, state of arts, knowledge, vested interests etc.
4. Measures of effectiveness : e.g. are trying to manufacture something at least cost, most profit etc. ?

Ideally we would like the "best" solution relative to as large a portion of the total organization as is possible. Preferably we want to consider the whole system not just a part. We want to optimize, not sub-optimize. Thus, e.g. in dealing with an inventory control problem we want to consider as far as possible the effects on production, purchase difficulties, organisational effects, costing, alternative investments, etc. But, practical considerations usually require one to deal with the parts in sequance and to adjust later. It is usually not possible to understand the whole problem. Results are often needed quickly, data may not be available or may be inaccurate. Often there are too many possibilities and sometimes a lack of techniques available for reducing the number of choices to manageable proportions. Therefore most projects start with problems of restricted scope and later enlarge, and usually the results are in the form of an answer which is better than previously but not necessarily the best possible.

Step 2 and 3 :

Obtaining and analysing information relevant to the problem.

This is usually remarkably difficult, e.g. to find the proportion of specials in a factory production system will often yield remarkably different answers according to the purpose of investigation. Numerically it might be small, its nuisance value might be high, its worth to the company in terms of good customers service might be high or greatly overrated. Analysis of information often requires the use of statistical methods and the construction of frequency distributions, e.g. when deciding on safety stock one will probably want to analyse delivery times of raw material.

Step 4 :

Formulate a hypothesis or construct a model.

This is the construction of the effectiveness function $E = f(x_i, y_i)$ mentioned earlier.

Generally one wishes to maximize or minimize E. It is sometimes possible to do this analytically, e.g. by calculus, but generally one needs to adopt a numeric approach. The main methods for this are :

1. List all possibilities, e.g· if there are n choices with values E_i $(i = 1, \ldots, n)$, select the choice with the largest value.

This system is often not possible because of the large number of choices, e.g· assigning 10 jobs to 10 machines has 10 ! possibities.

2. Trial and error.

   Look at several and select best.

3. A formalised sort of random sampling called Mont Carlo simulation. We shall look at this in more detail.

4. Iterative methods. These are methods in which one has a procedure which successively produces results closer to an optimum solution and one has a set of rules to define an optimum when one gets there. An example of this is linear programming.

5. Search procedures based on heuristics. A heuristics is a common sence rule which seems likely but is not guaranteed to give a good answer, e.g. schedule the job which is most late first.

Step 5 :

Test the hypothesis (model).

This is deciding how well the model predicts the effects of a change in the value of a variable or overall system effectiveness. Even if it does not prodict well, the model may be useful in terms of the understanding that it gives, but a new model will need to be built for predictive purposes·

Step 6 :

Use to predict and compare the outcomes of alternative decisions.

This is providing a service to the manager to enable him to make a decision and it requires that the results should be expressed in a form that the manager can understand and should indicate the sensitivity of results to different decision.

Step 7 :

Put the solution to work and establish controls over the solution. The operation of the system will require operating instructions saying, who does what and where ; and probably some training of the personnel involved. Secondly, because business systems are dynamic, one needs to monitor the system to know when significant changes have occured. The new system must itself be in control.

4. Information and Material Flow

The main steps in information and material flow which result from the receipt of a customer's order are outlined (in a simplified form) in fig. 3.

FIG. 3

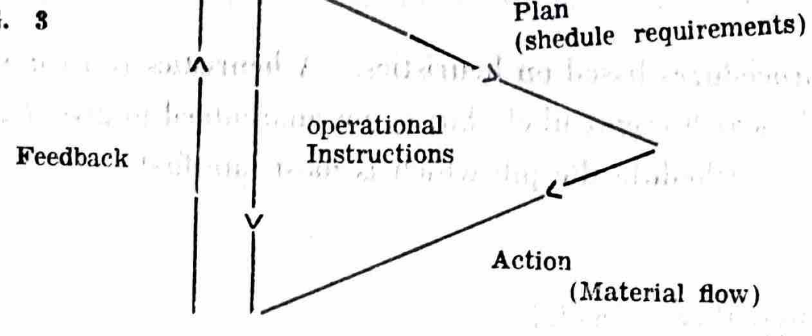


Fig. 3 is a schematic diagram, showing how the information and material flow is represented in the following outlined steps. The steps are

1. The customers orders are accumulated to form the order book.
2. If necessary a demand forcast is produced.
3. A company programme is produced, usually in 2 parts, a sales programme and a production programme showing respectively what is planned to be sold and manufactured in some period in the future. These programmes are based on such things as the demand forcast, desired stock levels, current stock levels factory capacity, and management knowledge.
4. The production programme is exploded to give successively gross sub-assembly requirements, parts requirements and material requirements. These gross requirements together with the current stock position and inventory control rules are used to produce net requirements and hence the ordering requirements on the shop, sub-contractors and material suppliers.
5. As a result of the ordering one plans to trigger off a series of events such as (with some possible documentation alongside).

   a) receive raw materials (goods received note)

   b) issue raw materials (material requisitions)

   c) manufacture part (operation card)

   d) assemble part (assemply orders)

   e) despatch parts from warehouse to customers (despatch note)

6. The data generated as a result of the events are used to control the system, e.g. stock records will be adjusted on material receipt, lack of receipt may be the bases of expediting action and rejections from manufacture may initiate rectification work or additional manufacture.

In other words one has a forward (planning) information flow and a reverse (feed back or control) information flow which is used to update records on the current state of the shop and, suitably filtered to control subsequent operations on the shop. This information flow is of course now being produced on a routine basis in some companies by their computer system and once the data is available and the administrative problems of obtaining it on a routine basis have been overcome there is great scope for more sophisticated approaches to be used.

Among the problems which can be identified in the production planning and control loop are demand forcasting, sales planning and control, quality planning and control, production planning and control, and inventory planning and control. In a sense these are all the tectical problems, i.e. problems looking at how best to use present resources, and typical results if one had a computer might be to use exponential smoothing methods for demand forcasting, to use inventory control analyses to determine re-order levels and re-order quantities or to use priority rules in a job-shop scheduling situation.

But what will be the outcome of using a proposal method ? To find the answer to that, it is common to carry out a simulation. We shall therefore briefly describe simulation and illustrate it with some examples

5. Simulation

Simulation is now an accepted and fashionable tool of O.R. Simulation is the process of setting up a model of a situation and then performing experiment on the model. The model may be either deterministic or statistical and the word simulation is used to describe the model building, the model itself and the experimentation of the model.

Some suggested definitions of simulation are :

1. The development of a model of a system and its use to investigate alternatives without interfering with the real system.
2. The setting up of a stochastic model of a real situation and then

performing sampling experiments upon the model. The feature which distinguishes a simulation from a mere sampling experiment in the classical sense is that of the stochastic model. Whereas a classical sampling experiment in statistics is most often performed directly upon real data, a simulation entials first of all the construction of an abstract model of the system to be studied. (ref. Harling [12]).

But whatever the definition, the main use of simulation is to enable one to investigate situations in which it is not practical or worth while to obtain results by other means, in particular situations where one does not wish to interfere with the real system. Thus one might test the performance of a forcasting system in response to a given pattern of demand data, or investigate the service provided by an inventory control system in response to given demand patterns. In effect simulation enable one to study certain aspects of business problems in the laboratory. But this very fact may be its weakness ; it is emphasising decision making and model building and ignoring the sociological implications.

Simulation Example of the Job-Shop Sheduling

Giffler in 1966 [10] defines job shop sheduling as the ordering of the operations on jobs at the machines subject to routing constraints, so that the best value is obtained for some measure of effectiveness appropriate to the system.

Except for the trivially simple situations of 2 or 3 machines with simplifying assumption such as jobs once started must be completed (Johnson (1954) [3] and Lomnicki (1965) [4]) there are no analytic formulations of the problem which are computationally feasible. The only practical method of investigation is therefore by simulation.

Many simulations of the job-shop sheduling problem have been constructed. These vary in complexity in the object of the investigation. Early papers describing job-shop scheduling simulation were by Baker and Dzielinski (1960) [5]. Le Grand (1963) [6] used actual operating data covering 115 machine groups and 47 labour classes and examined the effect of 6 different priority rules on factory performance which are :

1. Minimum process time per operation.
2. Minimum slack time per operation.
   slack = (due date — present date) / no. of operations.
3. First come first served.
4. Minimum planned start date per operation. The theoretical operation start date has been calculated previously by a scheduling procedure.
5. Minimum due date per order.
6. Random selection.

Factory performance is determined by a variety of factors such as in-process inventory, labour utilisation, number of orders delivered on time and so on. Le Grand's simulation studied the effect of the six priority rules on the following measures of effectiveness (criteria).

1. Number of orders completed.
2. Percent of orders completed late.
3. Mean of distribution of completions.
4. Standard deviation of the distribution of completions.
5. Average number of orders waiting in the shop.
6. Average waiting time of orders.
7. Yearly cost of carrying orders in queue.
8. Ratio of inventory carrying cost while waiting to inventory cost on machines.
9. Percent of labour utilised.

10. Percent of machine capacity utilised.

The simulation showed that using minimum process time per operation gave the best performance overall, if all criteria are equally weighted. Where greater weight was given to completing an order on time, slack time (rule 2) produced the best performance but even here minimum process time per operation produced good results.

The interest of this simulation is its practical nature and the fact that it was later used as the basis of sequencing order on the Hughes Aircraft Co., as discussed further in the paper by Steinhoff (1964) [7] and Bulkin, Cotly and Steinhoff (1965) [8].

At the other end of the range of complexity Eilon and Hodgson (1967) [9] examine the performance of a single queue with 2 services in parallel with Poisson arrival and exponential service time subject to random fluctuations. Five priority rules were studied and their results were presented in easily understood graphical form.

In this brief review we can conclude two uses of simulation of job-shop scheduling :

1. investigation of a problem in general way, in order to obtain an understanding of the effect of rules for scheduling on the schedule produced.
2. investigation of practical job shops in order to make specific decision relating to the condition on the shop.

Simulation Example on Industrial Dynamics

The second simulation illustrates a method of simulating larger systems which might include several factories, warehouses, retailers, production and inventory control, demand forcasting capacity and financial considerations etc. We have already seen the feedback nature of business control system and therefore it is not surprising that the approaches to simulation of these complex problems also strongly illustrate the feedback nature. In general one is interested in the behaviour of a system as a function of time. There are two approaches,

1. to represent time as a continuous variable and produce a set of differential equations to represent the system. Some models of this sort are described in papers by Tustin (1953) [11], Simon (1960) [13] and Bane (1965) [14].

2. to represent the system by a model in which time is stepped on by increments. This produces sets of difference equations. This idea has been developed in depth by Forrester and he called the work "industrial dynamics" [15]· Basically he considers six main factors in a system ; information, money, orders, materials, personnel, and capital equipment but not all of these factors need be present.

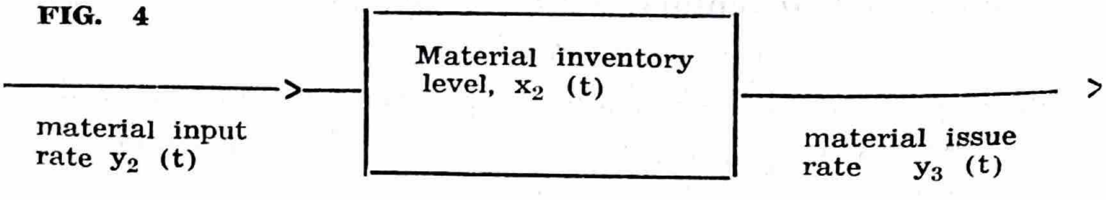
The basic idea is that one consider a system composed primarily of :

(I) levels, such as inventory levels.

(II) rates of flow in and out of the inventory.

These are related by (III) decisions and (IV) delays. Using the notations as included in Clough (1963) [16] one may take x (t) to represent a level at time t and y (t) to represent the rate of flow during the time interval $\triangle$t taking one from (t-l) to t i.e., (t-l) + $\triangle$ = (t). For simplicity take t = 1, e.g., 1 week. In general $\triangle$t has to be small enough to assume constant rate of flow over the period and this is related primarily to the delay in the system. We will distinguish between different rates and levels by suffices, e.g. as in fig. 4., from which one can get the equations of the form :

FIG. 4



1. representing levels :

level at end of interval = level at begining + flow in — flow out

$$x_2 (t) = x_2 (t-1) + y_2 (t). \Delta t - y_3 (t) \cdot \Delta t.$$

2. representing rates :

One controls the input according to decision rules e.g. if one re-ordered on the basis of a max-min stock control system based on physical stock,

$$y_2 (t + 1) = k \quad \text{if } x_2 (t) < \text{Re-order level.}$$
$$= o \quad \text{other wise}$$

Notice that one determines the rate of flow in the next time interval in terms of past values of levels. In this case there are no time delays. If there were a delivery delay of 1 week the equation would become

$$y_2 (t + 1) = k \quad \text{if } x_2 (t-1) < \text{Re-order level}$$
$$= o \quad \text{otherwise}$$

One may thus, given the initial values of level and rates, produce the new levels from which in turn one produces the rates for the next time interval.

Conclusion

In conclusion one can say that the objective of O.R. is to aid decision making. O.R. has had a high success ratio and we have indicated, by means of a simphified model of material and information flow, some of the commonly tackled problems within a company. In particular we have considered simulation which is one tool of O.R. which as well as an apportunity to investigate tactical problems also, as in the industrial dynamics approach, gives the opportunity of looking at some problem of strategic decision making.

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