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Magnetohydrodynamic (MHD) Waves in Plasmas

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Abstract

The model for a magnetoplasma is given by the MHD equations, so the first aim is to give a full list of MHD equations, with the criteria of their applicability for wave propagation. The validity conditions under which the MHD equations are used require the wave frequency to be, $\omega \ll \omega_{ci}$ and the seven eigenvectors are obtained. In MHD, the magnetic fields are frozen into the fluid and are elastic; displacing fluid elements causes magnetic restoring forces to switch on. This action appears as distorted magnetic field lines due to torsional and compressional Alfve'n waves.

Keywords: Alfven wave, Shear viscosity, magnetoacoustic wave.

الموجات الهيدروديناميكية المغناطيسية في الحالة الرابعة للمادة (بلازما)

الملخص:

يعطي نموذج الحالة الرابعة للمادة بواسطة معادلات الهيدروديناميكية المغناطيسية (MHD)، لذا فإنّ الهدف الأول هو إعطاء قائمة كاملة من معادلات (MHD) ، مع معايير قابليتها للتطبيق لانتشار الموجات. تتطلب شروط الصلاحية التي يتم بموجبها استخدام معادلات (MHD) أن يكون تردد الموجة ، $\omega \ll \omega_{ci}$ ويتم الحصول على المتجهات الذاتية السبعة في الهيدروديناميكية المغناطيسية، يدل ذلك علي تجميد المجالات المغناطيسية في المائع ويكون له نوع من المرونة تؤدي إزاحة عناصر المائع إلى امبعاث قوى مغناطيسية لسيطرة علي الوضع، و يظهر ذللك كخطوط مجال مغناطيسي مشوهة بسبب موجات ألفاين الالتوائية والتضاغطية.

1. Introduction

In some cases the macroscopic variables change on time and length scales which are long enough for an adequate description of a magnetoplasma to be provided by the usual one-fluid equations, normally used in plasma. In ordinary fluid dynamics, the basic dependent variables are; the density (ρ), the temperature (T) and the velocity ($\vec{v}$) with the addition to the magnetic field induction ($\vec{B}$). For the transport of energy, momentum and charge the constitutive laws are required. This group of equations defined what is meant by 'Magnetohydrodynamic' (MHD).The simplest model for a magnetoplasma is given by the MHD equations, so our first aim is to give a full list of MHD equations, with the criteria of their applicability for wave propagation in Section.3 [9]. For the validity of the conditions under which the MHD equations are used, we require the values of the collision interval τ_e for the electron-component test particles moving through a back-ground of ion-component particles. If the electron-component particles are initially in a non-equilibrium distribution, and if perturbation influence is removed, the electron-components will reach equilibrium with themselves within a time τ_{ee}. The ion-components will reach equilibrium among each other within a time a bit later τ_{ii}. Finally the thermal equilibrium between the electron and the ion-components will be reached in a time τ_{ei}. These times scale a

$\tau_{ee} : \tau_{ii} : \tau_{ei} = 1 : \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} : \frac{m_i}{m_e}$; they are about two orders of magnitude apart [2,7].

The fluid representation demands that the macroscopic time scale $\mathcal{T}$ be much longer than relaxation for the components in the process. The length scales constraints depend on the orientation of the magnetic field $(\overrightarrow{B})$ and they are:

$L_\parallel \gg \lambda$, λ- mean free path.

$L_\perp \gg r_L$, r_L- Larmor radius.

Length scales parallel and perpendicular to the magnetic field $\vec{B}$ respectively.

Another important constraint in MHD is that the diffusion velocity be small such that;

$|\vec{v}_e - \vec{v}_i| \ll c_{th} \Longrightarrow |\vec{j}| \ll en_e c_{th} = en_e \left(\frac{2kT_e}{m_e}\right)^{\frac{1}{2}}$. Where c_{th} is the thermal speed for the electrons in the plasma, $\vec{j}$ is the current density and k is the Boltzmann constant respectively. This will reduce the streaming velocity of electrons past the ions to low subsonic speeds. The final constraint is imposed by the neglect of charge separation, for any wave of frequency ω, to be with a macroscopic time scale

$\omega^{-1} = \omega_{pe}^{-1}$, where ω_{pe} is the plasma electrons oscillation frequency. Furthermore restrictions are imposed by the form adopted for Ohm's law used and the non-relativistic particle speed, [1,5] and [9].

Small amplitude wave propagation is a topic of considerable importance for many practical reasons: first, it gives a relatively simple method of comparing theory with experiment; secondly, if theory is confirmed, this offers a way that waves can be used for diagnosing plasma and may be used to heat it. Thirdly; waves do occur naturally in the ionosphere and the turbulence they generate by their growth unstable lead to phenomena known as plasma transport.

Huge numbers of propagation modes exist, [6] and dispersion relation of quite high order can be given, especially in fluid mixture when viscosity and other dissipative effects are included. However, our concern will be only with the three basic MHD modes, known as: The Alfven waves and the slow and fast magnetosonic waves [8], also known as magnetoacoustic waves. In this work the dissipation is treated as a small effect, superimposed on the basic mode.

2. The MHD Equations

The MHD equations can be classified into four categories:

a) The electromagnetic equations:

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad , \quad \text{and} \quad \vec{\nabla} \cdot \vec{B} = 0$$

(1a)

$$\vec{\nabla} \times \vec{B} - \mu_0 \vec{j} = 0 \quad , \quad \text{and} \quad \vec{\nabla} \cdot \vec{j} = 0$$

(1b)

b) The conservation equations:

$$\mathcal{D}\rho + \rho \vec{\nabla} \cdot \vec{v} = 0$$

(2a)

$$\rho \mathcal{D} \vec{v} + \vec{\nabla} \cdot \bar{\bar{\mathcal{P}}} - \vec{j} \times \vec{B} - \rho \vec{\mathcal{F}} = 0$$

(2b)

Eq.(2a-2b) are the mass conservation with ρ is the mass density and the momentum conservation with $\bar{\bar{\mathcal{P}}}$ is the total momentum tensor respectively, and the total derivative $\mathcal{D} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$.

The fluid outside the volume $\mathcal{V}$ (enclosed by surface $\mathcal{S}$) exerts inward force per unit volume (force density) on the fluid inside the volume within $\mathcal{S}$ as; $-\vec{\nabla} \cdot \bar{\bar{\mathcal{P}}}$, see Fig.(1). Hence it is convenient to divide $\bar{\bar{\mathcal{P}}}$ into two parts: $\bar{\bar{\mathcal{P}}} = p\bar{\bar{\delta}} + \bar{\bar{\pi}}$, where $\bar{\bar{\pi}}$ is the so called the viscous stress tensor, which nonzero if the fluid possesses shear, otherwise would be zero, p is the normal scalar fluid pressure and the diagonal unit tensor or a delta tensor $\bar{\bar{\delta}}$ which is more perspicuous in component form $\bar{\bar{\delta}} = \hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath} + \hat{k}\hat{k}$ with trace $\bar{\bar{\delta}} : \bar{\bar{\delta}} = 3$, has a lot of properties normally used in tensor calculus applications,[Brunetti,2020]. One of its properties applied to scalar function $\varphi(r)$ is $\vec{\nabla} \cdot (\varphi \bar{\bar{\delta}}) = \vec{\nabla}\varphi$ used in $-\vec{\nabla} \cdot \bar{\bar{\mathcal{P}}} = -\vec{\nabla} \cdot (p\bar{\bar{\delta}} + \bar{\bar{\pi}}) = -\vec{\nabla} \cdot (p\bar{\bar{\delta}}) - \vec{\nabla} \cdot \bar{\bar{\pi}} = -\vec{\nabla}p - \vec{\nabla} \cdot \bar{\bar{\pi}}$, which is reduced to $-\vec{\nabla} \cdot \bar{\bar{\mathcal{P}}} = -\vec{\nabla}p$ when the fluid has no shear viscosity, hence $\bar{\bar{\mathcal{P}}} = p\bar{\bar{\delta}}$. The $\rho\vec{\mathcal{F}}$ is the force density acting on the body of the fluid, proportional to the gravitational gradient, which $\vec{\nabla}g \ll \vec{\nabla}p$ and it can be dropped from Eq.(2b).

$$\rho \mathcal{D} u + \rho p \mathcal{D} \rho^{-1} + \bar{\bar{\pi}} : \vec{\nabla}\vec{v} + \vec{\nabla} \cdot \vec{q} - \vec{j} \cdot (\vec{E} + \vec{v} \times \vec{B}) = 0$$

(2c)

is the energy conservation equation excluding the radiation energy. Where $u = c_{\mathcal{V}}T$, the internal energy with $c_{\mathcal{V}}$ is the specific heat at constant volume and $\vec{q}$ is the heat flux vector. The third term in Eq.(2c) defined as $\bar{\bar{\pi}} : \vec{\nabla}\vec{v} \equiv \Phi$, is known as the viscous dissipation function. With the help of $m = \rho d\mathcal{V}$ and Eq. (2a), we can relate the term $\rho p \mathcal{D}\rho^{-1}$ to velocity divergence of the fluid using

$$\frac{1}{\rho}\mathcal{D}\rho = -\bar{\bar{\delta}}:\vec{\nabla}\vec{v} \equiv -\vec{\nabla}\cdot\vec{v} = \frac{d\mathcal{V}}{m}\mathcal{D}\left(\frac{m}{d\mathcal{V}}\right) = -\frac{1}{d\mathcal{V}}\mathcal{D}(d\mathcal{V}) = -\frac{\rho}{m}\mathcal{D}\left(\frac{m}{\rho}\right) = -\rho\mathcal{D}\rho^{-1} \longrightarrow p\vec{\nabla}\cdot\vec{v} = p\rho\mathcal{D}\rho^{-1}$$

as required. It can be easily shown what ρ^{-1} is the specific volume using $\frac{d\mathcal{V}}{m} = \frac{d\mathcal{V}}{\rho d\mathcal{V}} = \rho^{-1}$, If our infinitesimal volume $d\mathcal{V}$ is a convicted point P_c with the fluid, than the first term in Eq. (2c) is the change in the internal energy at P_c , the second term is the reversible work done by P_c on the surrounding fluid, the third term is the irreversible work supplied to P_c by viscosity, the fourth term is the heat (energy) conducted into the point P_c , the last term is the Ohmic heat supplied by

P_c into the surrounding fluid, see,Fig.(1).

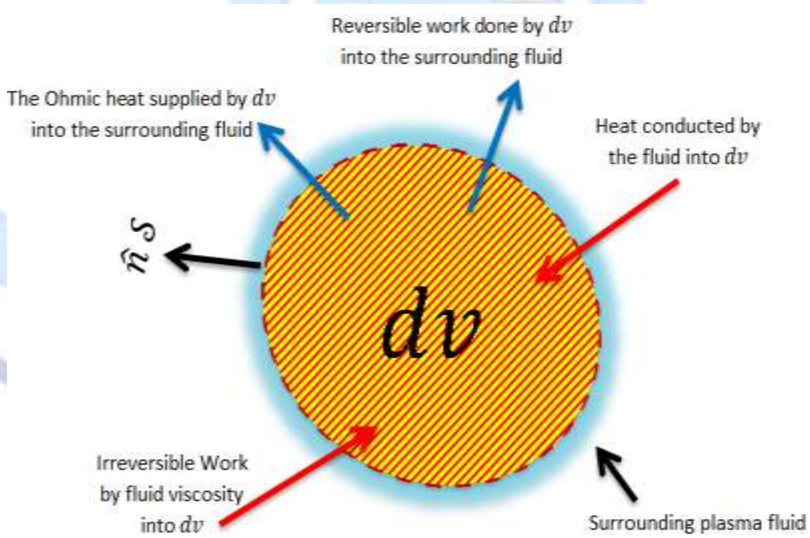

Figure.(1): A volume element dv enclosed by surface $\mathcal{S}$ Convicted though the plasma fluid.

c) The thermodynamic relations with (s) is the entropy of the fluid system

$T\mathcal{D}s = \mathcal{D}u + p\mathcal{D}\rho^{-1}$, and $u = \frac{3}{2}\frac{p}{\rho} = c_v T$
(3a)

$p \propto \rho^{\gamma} exp^{\left(\frac{s}{c_v}\right)}$, and $p = nkT$
(3b)

Where, $\gamma = \frac{5}{3}$ and n is the plasma density and k is the Boltzmann constant.

d) The simplified constitutive equations

$$\vec{j} = \bar{\bar{\sigma}} \cdot \vec{E}' \quad , \quad \vec{q} = -\bar{\bar{\kappa}} \cdot \vec{\nabla} T \quad \text{and} \quad \bar{\bar{\mathcal{P}}} = p\bar{\bar{\delta}} + \bar{\bar{\pi}} \tag{4a}$$

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B} - \frac{1}{en_e}\vec{j} \times \vec{B} + \frac{1}{en_e}\vec{\nabla} p_e \quad \text{and} \quad \bar{\bar{\pi}} = -2\bar{\bar{\mathbb{u}}} : \vec{\nabla}^0 \vec{v} \tag{4b}$$

Where $\bar{\bar{\sigma}}$ is the plasma electric conductivity and $\bar{\bar{\kappa}}$ is the plasma thermal conductivity with both quantities been second-order tensors. The constitutive equations are termed simplified, because we have dropped the thermoelectric terms from $\vec{j}$ and $\vec{q}$ in Eq. (4a). The term $\frac{1}{en_e}\vec{j} \times \vec{B}$ in Eq. (4b) is called the 'Hall' or the 'gyroscopic' term. Also in the second set of Eq.(4b), $\bar{\bar{\mathbb{u}}}$ is a fourth-order tensor, laterally isotropic, shear viscosity tensor, called laterally isotropic because it is un affected by rotation about $\hat{b}$ the direction of the magnetic field $\vec{B}$ and $\vec{\nabla}^0 \vec{v}$ is called deviatoric rate of strain, representing pure straining motion without volume change. It plays a central role in fluid transport theory, [3], When the length scales introduced earlier $\lambda \nabla_{\parallel} = \frac{\lambda}{L_{\parallel}} \ll 1$ and $r_L \nabla_{\perp} = \frac{r_L}{L_{\perp}} \ll 1$ are sufficiently well-satisfied the constitutive equations can be simplified further by ignoring the gradient terms in Eq.(4a-b), to reduced it to:

$$\vec{j} = \bar{\bar{\sigma}} \cdot \left(\vec{E} + \vec{v} \times \vec{B} - \frac{1}{en_e}\vec{j} \times \vec{B}\right) \quad , \quad \bar{\bar{\vec{q}}} = 0 \quad \text{and} \quad \bar{\bar{\mathcal{P}}} = p\bar{\bar{\delta}} \tag{5}$$

The magnitude of the gyroscopic approximately is $\left|\sigma \vec{j} \times \frac{\vec{B}}{en_e}\right| \sim \omega_{ce}\tau_e |\vec{j}|$ shows that this term can be dropped if $\omega_{ce} t_e \ll 1$, where $\omega_{ce} = \frac{eB}{m_e}$ is the electron cyclotron frequency, the plasma conductivity $\bar{\bar{\sigma}}$, becomes isotropic and leads to the following form of Ohm's law:

$$\eta \vec{j} = \vec{E} + \vec{v} \times \vec{B} - \frac{1}{en_e}\vec{j} \times \vec{B} \tag{6}$$

Where, η is the plasma resistivity equal to; ($\frac{1}{\sigma} = \frac{m_e}{2e^2 n_e \tau_e}$). On the limit of $\sigma \to \infty$

Eq.(6) becomes:

$$\vec{E} + \vec{v} \times \vec{B} - \frac{1}{en_e}\vec{j} \times \vec{B} = 0 \tag{7}$$

3. Unbound Plasma MHD Waves Propagation

We ignore wave damping by basing this work on Eq. (1) to Eq. (5), using Eq.(7) into Eq.(1a-1b) the electromagnetic equations become:

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$, and $\vec{\nabla} \cdot \vec{B} = 0$

(8a)

$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B} - \frac{1}{e n_e} \vec{j} \times \vec{B} \right)$

(8b)

The ideal MHD equations are:

$\mathcal{D}\rho + \rho \vec{\nabla} \cdot \vec{v} = 0$ and $\rho \mathcal{D} \vec{v} + \vec{\nabla} p - \vec{j} \times \vec{B} = 0$

(9a)

$\mathcal{D}s = 0$ and $p \propto \rho^{\frac{5}{3}}$

(9b)

The ideal constitutive relations are:

$\bar{\bar{\mathcal{P}}} = p \bar{\bar{\delta}}$ and $\bar{\bar{q}} = 0$

(10)

Apart of Eq.(8a) the rest of the equations below it are nonlinear. To find the modes of propagation of small-amplitude waves, we linearize them as follows. Using a typical dependent variable such as $\psi(\vec{r}, t)$ which is then separated into a uniform part ψ_0 and a perturbation, which is Fourier analyzed into components such as $\psi_1(\vec{r}, t) = \hat{\psi}\, exp^{i(\vec{k} \cdot \vec{r} - \omega t)}$, representing plane waves propagating in the $\vec{k}$-direction. The propagation vector $\vec{k}$ and frequency ω satisfy a dispersion relation function;

$$\mathcal{g}(\omega, \vec{k}) = 0$$

(11)

This function is a principle object of our analysis. For a given mode;

$$\psi(\vec{r}, t) = \psi_0 + \hat{\psi}\, exp^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

(12)

Where the wave amplitude $\hat{\psi}$ depends on the initial conditions: complete solutions are obtained by summing over modes. In linearization we omitted all terms that are not linear in the perturbations. For this reason we show that; the term $\vec{v} \times \vec{B}$ in Eq.(8b) can be linearized as

$\vec{v} x \vec{B} = \vec{v}_0 \times \vec{B}_0 + \vec{v}_0 \times \vec{B}_1 + \vec{v}_1 \times \vec{B}_0 + \vec{v}_1 \times \vec{B}_1 \sim \vec{v}_1 \times \vec{B}_0$

(13)

Where we have chosen a steady state in which the change $\mathcal{D}s_0 = \mathcal{D}\rho_0 = 0$, $\vec{v}_0$ and $\vec{j}_0$ are zeros and omitted the second- order terms such as $\vec{v}_1 \times \vec{B}_1$ in Eq.(13). Also in the linearization we adopt the replacements of:

$\mathcal{D} \longrightarrow -i\omega t$ and $\vec{\nabla} \longrightarrow i\vec{k}$

(14)

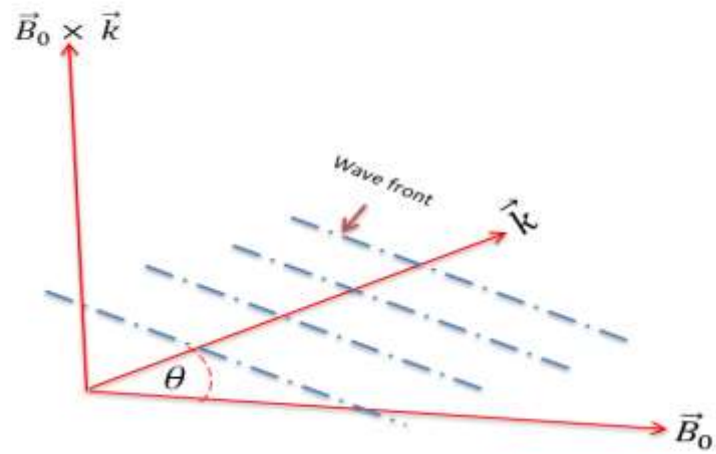

Figure.(2): Propagation vector and wave front.

The scalar pressure in Eq.(9b) is linearized to obtain the definition of the sound speed;

$$p_0 + p_1 \propto (\rho_0 + \rho_1)^{\frac{5}{3}} \sim \rho_0^{\frac{5}{3}} + \frac{5}{3}\rho_0^{\frac{2}{3}}\rho_1 \quad (15)$$

By comparing terms of Eq.(15) we get;

$$p_0 = \rho_0^{\frac{5}{3}} \quad \text{and} \quad p_1 = \frac{5}{3}\rho_0^{\frac{2}{3}}\rho_1 = c_s^2\rho_1 \Longrightarrow c_s^2 = \frac{5}{3}\rho_0^{\frac{2}{3}} = \frac{5}{3}\frac{p_0}{\rho_0} \quad (16)$$

We defined $c_s = \left(\gamma\frac{p_0}{\rho_0}\right)^{\frac{1}{2}}$ and $c_a = \left(\frac{B_0^2}{\mu_0\rho_0}\right)^{\frac{1}{2}}$ as the speed of sound and the Alfven speed in the plasma fluid respectively.

Linearizing Eq.(8) – Eq.(9) using Eq.(14) and Eq.(16) , after reorganizing the equations order for the entropy the first part of Eq.(9b) we obtain:

$$\mathcal{D}s = \mathcal{D}s_0 + \mathcal{D}s_1 = \mathcal{D}s_1 = \mathcal{D}\hat{s}\, exp^{i(\vec{k}\cdot\vec{r}-\omega t)} = -i\omega\hat{s}exp^{i(\vec{k}\cdot\vec{r}-\omega t)} = 0 \Longrightarrow \omega\hat{s} = 0 \quad (17)$$

For mass continuity equation first part of Eq.(9a) to have

$$\mathcal{D}\rho + \rho\vec{\nabla}\cdot\vec{v} = \mathcal{D}\rho_1 + \rho_0\vec{\nabla}\cdot\vec{v}_1 = -i\omega\rho_1 + i\vec{k}\cdot\vec{v}_1 = 0 \Longrightarrow \omega\hat{\rho} - \rho_0\vec{k}\cdot\hat{v} = 0 \quad (18)$$

For momentum balance equation second part of Eq.(9a) to get

$$\rho\mathcal{D}\vec{v} + \vec{\nabla}p + \vec{B}\times\vec{j} = 0 \Longrightarrow \omega\rho_0\hat{v} - c_s^2\vec{k}\hat{\rho} + \frac{1}{\mu_0}kB_0cos\theta\hat{B} - \frac{1}{\mu_0}\vec{k}\hat{B}_{\parallel} = 0 \quad (19)$$

Where θ *is the angle between* $\vec{B}_0$ and $\vec{k}$, see Fig. 2, ∥ denotes a component parallel to B_0 and Eq.(8a) *is used in* Eq.(19) in the following form

$$\Longrightarrow \hat{j} = \frac{i}{\mu_0}\vec{k}\times\hat{B} \Longrightarrow \vec{B}_0\cdot\hat{j} = \hat{j}_{\parallel} = \frac{i}{\mu_0}\vec{B}_0\times\vec{k}\cdot\hat{B} \quad \text{and} \quad \Longrightarrow \vec{k}\cdot\hat{B} = 0 \quad (20)$$

And for the electromagnetic Eq.(8b) we obtain

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times \left(\vec{v} \times \vec{B} - \frac{1}{en_e}\vec{j} \times \vec{B}\right) = 0 \Longrightarrow \omega\hat{B} + kB_0 cos\theta\hat{v} - \left(\vec{k}\cdot\hat{v}\right)\vec{B}_0 - \frac{kB_0}{en_0}cos\theta\hat{j} = 0$$
(21)

Where we set $n_{oe} = n_0$, another variable of considerable importance is the fluid vorticity $\vec{\xi} = \vec{\nabla} \times \vec{v}$ relate to the angular velocity $\vec{\xi} = 2\vec{\Omega}$. A convenient dependent variable is the amplitude of the perturbation in the vorticity:

$$\vec{\xi} = \vec{\nabla} \times \vec{v} \qquad \Longrightarrow \hat{\xi} = i\vec{k} \times \hat{v} \Longrightarrow \vec{B}_0 \cdot \hat{\xi} = \hat{\xi}_{\parallel} = i\vec{B}_0 \times \vec{k} \cdot \hat{v}$$
(22)

It is natural to project Eq.(19) and Eq.(21) in the directions of the vectors $\vec{k}$, $\vec{B}_0$ and $\vec{B}_0 \times \vec{k}$, where Eq.(20) and Eq.(22) are used . The projections of Eq.(19) along these vectors respectively are:

$$-k^2c_s^2\hat{\rho} + \omega\rho_0\vec{k}\cdot\hat{v} - k^2c_a^2\frac{\rho_0}{B_0^2}\hat{B}_{\parallel} = 0$$
(23a)

$$-kc_s^2 cos\theta\,\hat{\rho} + \omega\frac{\rho_0}{B_0}\hat{v}_{\parallel} = 0$$
(23b)

$$\frac{1}{\mu_0}kB_0\cos\theta\,\frac{\mu_0}{B_0}\hat{j}_{\parallel} + \omega\frac{\rho_0}{B_0}\hat{\xi}_{\parallel} = 0$$
(23c)

Eq.(21) has no component along $\vec{k}$ when Eq.(20) is used. Its projections along $\vec{B}_0$ and $\vec{B}_0 \times \vec{k}$ respectively are:

$$-\rho_0\,\vec{k}\cdot\hat{v} + kcos\theta\frac{\rho_0}{B_0}\,\hat{v}_{\parallel} + \omega\frac{\rho_0}{B_0^2}\hat{B}_{\parallel} - \frac{k}{en_0}\frac{\rho_0}{\mu_0}\,cos\theta\frac{\mu_0}{B_0}\hat{j}_{\parallel} = 0$$
(24a)

$$-k^3\frac{c_a^2}{en_0}cos\theta\frac{\rho_0}{B_0^2}\hat{B}_{\parallel} + \omega\frac{\mu_0}{B_0}\hat{j}_{\parallel} + \frac{kB_0}{\rho_0}\,cos\theta\frac{\rho_0}{B_0}\hat{\xi}_{\parallel} = 0$$
(24b)

Closer look to the Eq.(17)-Eq.(18) and Eq.(23)-Eq.(24) shows that there are seven eigenvectors: $\hat{s}$, $\hat{\rho}$, $\rho_0\vec{k}\cdot\hat{v}$, $\frac{\rho_0}{B_0}\hat{v}_{\parallel}$, $\frac{\rho_0}{B_0^2}\hat{B}_{\parallel}$, $\frac{\mu_0}{B_0}\hat{j}_{\parallel}$ and $\frac{\rho_0}{B_0}\hat{\xi}_{\parallel}$. The coefficients of these eigenvectors comprise the following matrix whose determinant, must vanish for a solution of the $g\left(\omega,\vec{k}\right) = 0$, Eq.(11).

$$\begin{bmatrix} \omega & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega & -1 & 0 & 0 & 0 & 0 \\ 0 & -k^2c_s^2 & \omega & 0 & k^2c_a^2 & 0 & 0 \\ 0 & -\mathrm{k}c_s^2 cos\theta & 0 & \omega & 0 & 0 & 0 \\ 0 & 0 & -1 & \mathrm{k}cos\theta & \omega & -\frac{\mathrm{k}}{en_0}\frac{\rho_0}{\mu_0}\,cos\theta & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{en_0}k^3c_a^2cos\theta & \omega & \frac{kB_0}{\rho_0}\,cos\theta \\ 0 & 0 & 0 & 0 & 0 & -\frac{kB_0}{\mu_0}\cos\theta & \omega \end{bmatrix} \begin{bmatrix} \hat{s} \\ \hat{\rho} \\ \rho_0\,\vec{k}\cdot\hat{v} \\ \frac{\rho_0}{B_0}\,\hat{v}_\parallel \\ \frac{\rho_0}{B_0^2}\hat{B}_\parallel \\ \frac{\mu_0}{B_0}\hat{j}_\parallel \\ \frac{\rho_0}{B_0}\,\hat{\xi}_\parallel \end{bmatrix} = [0]$$

(25)

Where two special velocities have emerged: $c_s = \sqrt{\gamma\frac{p_0}{\rho_0}}$, and $c_a = \frac{B_0}{\sqrt{4\pi\rho_0}}$, the sound speed and the Alfve'n speed, respectively. The former is familiar from fluid dynamics, while the latter is another speed, arising in MHD, at which perturbations can travel.

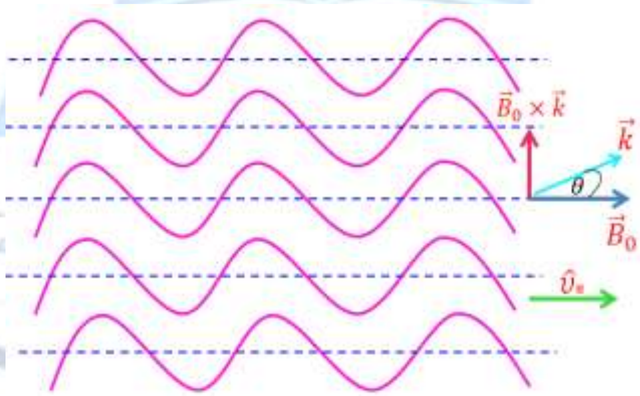

Figure.(3): Distortion in the magnetic field lines due to torsional Alfven waves

As implied by the second part of Eq.(20) if the perturbation magnetic field $\hat{B}$ is perpendicular to $\vec{B}_0$, the field lines are distorted due to torsional Alfve'n waves, see Fig.(3). However if the perturbation magnetic field parallel $\vec{B}_0$ as $\hat{B}_\parallel$, compression of the magnetic field lines under the effect of the compressional Alfve'n waves take place, shown in Fig.(4).

3. Conclusion

The above matrix splits into four submatrices, corresponding to: 1) the top row 'marked blue' give the non-propagating entropy waves, it represents the convective transport of a constant pressure and temperature fluctuation and always decoupled, 2) the classical sound (acoustic) waves, top left submatrix 'marked black', 3) the magnetoacoustic waves consisting of the submatrix for the acoustic waves plus the column 'marked red', 4) the Alfve'n waves, lower for the acoustic waves plus the column 'marked red', 4) the Alfve'n waves, lower right submatrix 'marked green'.

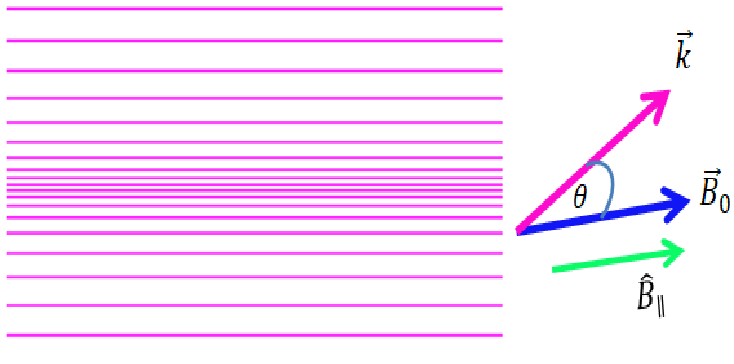

Figure.(4): Compression of the magnetic field lines under the effect of the compressional Alfve'n wave.

By Eq.(25) the magnetosound waves and the Alfve'n waves are decoupled if the 'gyroscopic' term is omitted from Eq.(7), [4]. The ratio of the square of the characteristic speeds, the sound c_s to the Alfve'n c_a , is a typical plasma parameter measuring the kinetic plasma pressure to the confining magnetic pressure as:

$$\beta = \left(\gamma \frac{p_0}{\rho_0}\right) / \left(\frac{B_0^2}{\mu_0 \rho_0}\right) = \frac{\gamma p_0}{B_0/\mu_0}$$

(26)

Thinking in terms of displacements makes sense in MHD but not so much in (homogeneous) hydrodynamics because in the latter case, just displacing a fluid element produces no back reaction, whereas in MHD, since magnetic fields are frozen into the fluid and are elastic, displacing fluid elements causes magnetic restoring forces to switch on. In other words, an (ideal) MHD fluid "remembers" the state from which it has been displaced, whereas neutral (Newtonian) fluids only "know" about velocities at which they flow.

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