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Equilibrium Studies of the magnetic surfaces function ψ of Tokamak system with poloidal divertor

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ABSTRACT

An analytical solution for the magnetic field **B** produced by current filaments and the related magnetic surface function ψ are given. The use of ψ as a coordinate in this system is explored, and its hyperbolic form is given. The simple character of the plasma drift velocity υ_D for this coordinate is derived.

1. Introduction

The magnetic fields in plasma confinement devices approximately describe magnetic surfaces, since this is a condition for good confinement. It is well known that systems possessing closed magnetic surfaces consist topologically of nested tubes of flux (Hazeltine et al., 1992). The exact fields are thus conveniently described in terms of perturbations of an idealized equilibrium field, which possesses magnetic surfaces. Present equilibrium and stability analyses use magnetic coordinates based on the existence of these surfaces (Wesson et al,. 1987; White et al,. 1982).

It is the purpose of this paper to show how the magnetic geometry of a tokamak with poloidal divertor can be approximated by hyperbolic magnetic fields and magnetic surfaces function ψ. The shapes of the magnetic field geometry in the scrap-off layer (SOL) of a tokamak with divertor affect the profiles of electron density $n_e(\psi)$ and electron temperature $T_e(\psi)$. It has been shown that the electron density and temperature decrease smoothly with distance from their maximum values at the separatrix (Alhasi et al., 1992b; White et al., 1982; Taylor, 1974).

To this end the coordinate system is that of toroidal geometry, which is suitable to most promising fusion reactors mainly tokomaks. A simple torus shown in Fig. 1 and Fig. 2 depicts the toroidal geometry of a tokomak reactor and the divertor field lines configuration in the poloidal cross section respectively. Where the toroidal axis vertical by convention; it is encircled by the magnetic axes, a single toroidal field line that generally locates the peak of the plasma current and plasma density profiles as function of ψ the radial distance. The magnetic axis also identified with the toroidal direction parameter (ξ) (Hazeltine et al., 1992). Similarly, closed poloidal curves encircling the magnetic axis, indicate the local poloidal direction (θ) (Alhasi, 2016; Wesson et al., 1987).

In this work, we use an analytical model with cylindrical magnetic geometry to present the argument, noting that it can be readily generalized to toroidal geometry where the magnetic equilibrium is available. The generalization produces no qualitative change in the result (Loarte et al., 1992; Callen, 1991).

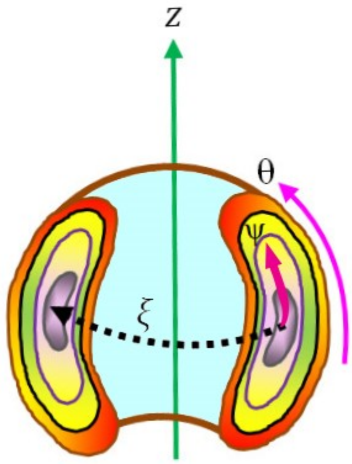

Fig.1. General toroidal coordinates

Firstly, an analytical solution for the magnetic field **B** produced by current filaments and the related magnetic surface function ψ are given. Secondly, the use of ψ as a coordinate in this system is explored, and its hyperbolic form is given. Thirdly, the simple character of the plasma drift velocity υ_D for this coordinate is derived.

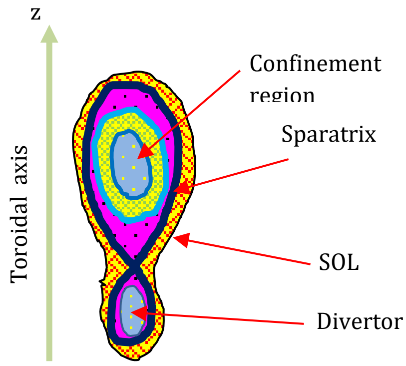

Fig. 2. Divertor field line configuration, seen in poloidal cross-section.

2. Magnetic field structure of poloidal divertor

We consider magnetic field structure of the simplest distribution of currents that presents the characteristics of a poloidal divertor. This is the case of two equal currents (I) along two infinite conductors at a vertical distance (b) from the origin of coordinates, which is at the x-point position. A uniform toroidal field B_T in the direction shown created by external coils is necessary for stability and balance of the effect of the plasma current (I_P) (Shimomura et al., 1983), see Fig. 3.

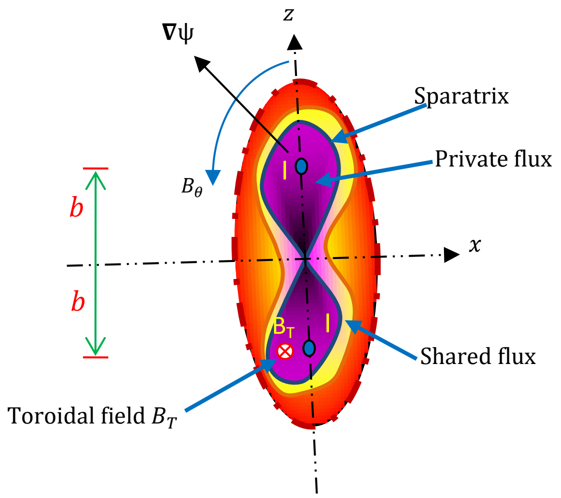

Fig. 3. Layout of the magnetic geometry used in the analytical model for the poloidal divertor.

For magnetic structure distribution of two conductors carrying current (I) at $x = 0$, $z = \pm b$ we obtain the field strength at (x, z), see Fig. 4:

$$B_1 = \frac{\mu_0 I}{2\pi}\frac{1}{\sqrt{x^2+(b+z)^2}}, \text{ and } B_2 = \frac{\mu_0 I}{2\pi}\frac{1}{\sqrt{x^2+(b-z)^2}} \quad (1)$$

The field components are:

$$B_{1z} = B_1 sin\theta_1 = B_1\frac{x}{\sqrt{x^2+(b+z)^2}} = \frac{\mu_0 I}{2\pi}\frac{x}{x^2+(b+z)^2} \quad (2)$$

$$B_{2z} = B_2 sin\theta_2 = B_2\frac{x}{\sqrt{x^2+(b-z)^2}} = \frac{\mu_0 I}{2\pi}\frac{x}{x^2+(b-z)^2} \quad (3)$$

Therefore the total B_z component is;

$$B_z = \frac{\mu_0 I x}{2\pi}\left\{\frac{1}{x^2+(b+z)^2} + \frac{1}{x^2+(b-z)^2}\right\} \quad (4)$$

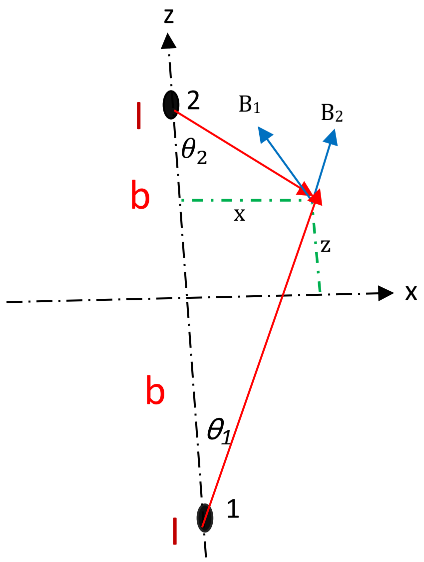

Fig. 4. Layout geometry used to show the structure of the magnetic field components in (x, z) plane.

Similarly the total B_x component is;

$$B_x = \frac{\mu_0 I}{2\pi}\left\{\frac{(b-z)}{x^2+(b-z)^2} - \frac{(b+z)}{x^2+(b+z)^2}\right\} \quad (5)$$

The field components B_z and B_x satisfy the following relations;

$$B_z = \frac{\partial\psi}{\partial x}, \text{ and } B_x = -\frac{\partial\psi}{\partial z} \quad (6)$$

$$\text{where } \psi = \frac{\mu_0 I}{4\pi}\{\ln[x^2 + (b+z)^2] + \ln[x^2 + (b-z)^2]\} \quad (7)$$

Since only the derivatives of ψ are defined, we can add any constant to it. It is convenient to take ψ = 0 at the x-point where $(x = 0, z = 0)$. So, if we add a constant to ψ equal to $-\frac{\mu_0 I}{2\pi}\ln b$, then we can write:

$$\psi = \frac{\mu_0 I}{4\pi}\ln\frac{[x^2+(b+z)^2][x^2+(b-z)^2]}{b^4} \quad (8a)$$

The hyperbolic nature for the magnetic field surfaces near a divertor separatrix can be shown, if we write it as (Alhasi, 2016; Alhasi, 1995):

$$\psi = \frac{\mu_0 I}{4\pi}\ln\Lambda, \text{ where } \Lambda = \frac{1}{b^4}[x^2 + (b+z)^2][x^2 + (b-z)^2] \quad (8b)$$

The magnetic surfaces are given by:

(i)-$\Lambda = 1$ implies $\psi = 0$, the separatrix.

(ii)- $0 < \Lambda < 1$ implies $\psi < 0$, negative inside the separatrix (private flux region).

(iii)- $\Lambda > 1$ implies $\psi > 0$, positive outside the separatrix (shared flux region). Rewriting Eq. (8b) for Λ as

$$x^2 = b\sqrt{4z^2 + b^2\Lambda} - (b^2 + z^2) \quad (9)$$

Let $\Lambda \leq 1$ and $\ll b$, that is in the private region inside the separatix, then we can this as:

$$x^2 \cong b^2\ \Lambda^{\frac{1}{2}}\left(1 + \frac{2z^2}{b^2\Lambda^{\frac{1}{2}}}\right) - (b^2 + z^2) \cong b^2\left(\Lambda^{\frac{1}{2}} - 1\right) + z^2 \quad (10)$$

Thus, roughly we have: $x^2 - z^2 \cong b^2\left(\Lambda^{\frac{1}{2}} - 1\right)$ (11)

using Eq. (8a) we obtain: $\Lambda = exp\frac{4\pi\psi}{\mu_0 I}$ (12)

expanding the exponential in Eq. (12) we get

$$\Lambda \approx 1 + \frac{4\pi\psi}{\mu_0 I} \quad (13)$$

Taking the square root of both sides of Eq. (13) and expand once more to have:

$$\Lambda^{\frac{1}{2}} - 1 \approx \frac{2\pi}{\mu_0 I}\psi \quad (14)$$

Substituting this into Eq. (11) gives that, the magnetic surfaces near the x-point are approximately hyperbolic as:

$$x^2 - z^2 \cong \frac{2\pi b^2}{\mu_0 I}\psi \quad (15)$$

3. Magnetic flux surfaces

Since ψ is positive for $z = 0$, $|x| \geq 0$, that is in the shared flux region, and negative for $x = 0$, $|z| \geq 0$, that is in the private flux region, (Rusbridge, 1992; Alhasi, 2007). The field components B_z and B_x are determined from ψ, this allow the interpretation of ψ as:

a-The ψ satisfies the following identity (Alhasi, 1995; Woods, 1987):

$$(\boldsymbol{B}\cdot\boldsymbol{\nabla})\psi = B_x\frac{\partial\psi}{\partial x} + B_z\frac{\partial\psi}{\partial z} = B_x B_z - B_z B_x = 0 \quad (16)$$

which implies that ψ is constant along field lines and can be used as a coordinate to distinguish them.

b-The vector magnetic potential **A** defined by **B** = **∇** × **A** can be related to ψ. In an axisymmetric system with no variation in the toroidal direction ξ (which incidentally coincide with the axial coordinate of the cylindrical system in this representation). As we have $\frac{\partial}{\partial \xi}=0$, thus;

$$B_x=\frac{\partial A_\xi}{\partial z}-\frac{\partial A_z}{\partial \xi} \text{ and } B_z=\frac{\partial A_x}{\partial \xi}-\frac{\partial A_\xi}{\partial x}\text{, which implies that} \quad (17)$$

$$B_x=\frac{\partial A_\xi}{\partial z}=-\frac{\partial \psi}{\partial z} \text{ and } B_z=-\frac{\partial A_\xi}{\partial x}=\frac{\partial \psi}{\partial x} \quad (18)$$

from which we obtain: $\psi=-A_\xi$ (19)

c- If B_x vanishes, at z = 0, then the magnetic field **B** is normal to x-axis, and the flux contained between the origin and a field line say $\psi=\psi_0$ is $\int_0^{x_0} B_z dx$, x_0 is the x-coordinate where the field line crosses the axis. But, since $B_z \equiv \frac{\partial \psi}{\partial x}$, we get:

$$flux=\int_0^{x_0}\frac{\partial \psi}{\partial x}dx=\Delta\psi \quad (20)$$

Therefore, $\psi(0)$ =0, and $\Delta\psi=\psi_0$. Thus, ψ_0 represents the flux (strictly flux per unit axial length) contained between the field line labeled by ψ_0 and the separatrix. The separatrix $\psi=0$ is the field line, which passes the x-point twice, see Fig. 5.

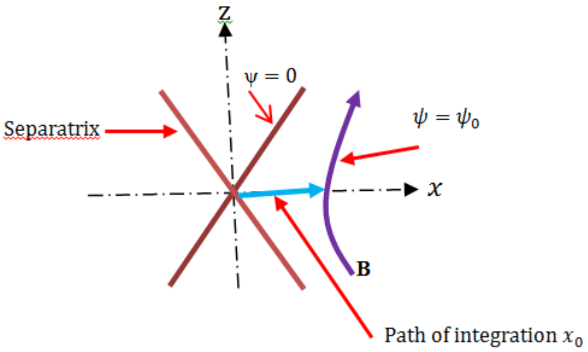

Fig. 5. Sketch depicts the field line path of integration.

d- Field line coordinates

Properly the value of ψ defines a flux surfaces rather than line, since in an ideal conductors (without ends) everything is independent of ξ, and a given field line traces out a surface, cylindrical in the general sense, as ξ is varied. In general, any field line in any magnetic field structure requires two functions of position, say α and β to defined it uniquely (Hazeltine et al., 1992; Woods, 1987). These can always be chosen so that the magnetic field **B** is given by:

$$\boldsymbol{B}=\nabla\alpha\times\nabla\beta \quad (21)$$

In this cylindrical geometry the field lines are intersections of surfaces of constant ψ and surfaces of constant ξ. We choose, $\alpha=\psi$ and $\beta=\xi$, hence $\nabla\beta=\nabla\xi=\hat{k}$, where $\hat{k}$ is a unit vector in the ξ- direction. Thus:

$$\boldsymbol{B}=\nabla\psi\times\nabla\xi=\nabla\psi\times\hat{k} \quad (22)$$

e-The use of ψ as a coordinate**:**

Many expressions which would be very complicated if expressed in terms of special coordinate becomes simple if expressed in terms of ψ, and in many cases the effect is to take account automatically of the variation of field strength along a line of force. To show this effect, consider the drift velocity of charged particle due to an electric field normal to the magnetic field **B** (Alhasi, 2010; Rusbridge, 1992). Assume **B** is constant so that **E** must be derived from scalar potential, $\boldsymbol{E}=-\nabla\phi$. We assume ϕ to be constant on a field line (often true when a plasma is present) so ϕ must be a function only of the coordinates ψ and ξ which are constant on a field line, (Woods, 1987; Wesson et al., 1987).

Consider when ϕ is independent of ξ, and $\phi=\phi(\psi)$. Then at any particular place, we can see that:

$$E=-\frac{d\phi}{dn} \quad (23)$$

Where dn is an element length normal to **B** and ξ. Therefore the drift velocity can be given as (Alhasi, 1992a; Rusbridge, 1992):

$$v_D=\frac{E}{B}=-\frac{1}{B}\frac{d\phi}{dn}=-\frac{d\phi}{d\psi} \quad (24)$$

Hence we can always write $= d\psi$, we see that since $\phi=\phi(\psi)$ where v_D is a function of ψ only and is constant along a field line. Now assume ϕ to be constant in any cross-section, but to vary with ξ, and $\phi=\phi(\xi)$. This gives rise to a drift normal to **B** but in the cross-sectional plane normal to ξ , we call this the ψ-direction. The drift velocity is:

$$v_D=-\frac{1}{B}\frac{d\phi}{d\xi} \quad (25)$$

which is not constant along a field line. We can also write:

$$Bv_D=B\frac{dn}{dt}=\frac{d\psi}{dt} \text{ so that } \frac{d\psi}{dt}=-\frac{d\phi}{d\xi} \quad (26)$$

This quantity is constant along a field line. Thus in a given time particles move through a fixed flux difference irrespective of where they are on the field line, and all particles starting together on one field line finish together on another (Alhasi, 1995; Rusbridge, 1992).

4. Conclusion

In this work, we have derived the hyperbolic structure of the magnetic field lines, and the magnetic surfaces coordinates. This is created by the introduction of the so-called magnetic poloidal divertor due to two current carrying conductors inside the tokamak chamber. The simple expression for vector magnetic potential and the drift velocity in this coordinates is obtained, which will be very complicated otherwise. The study of the multipole filaments magnetic surface function and their solutions by the Green's method, and the method of images, which gave our solution for Eq. (7) as special case is lengthily and deferred to future work (Hazeltine et al., 1992; Taylor, 1974).

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