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# Basic concepts of neutrosophic crisp sets

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**Highlights**

- **This study was conducted to present new types of Neutrosophic sets, which are a generalization of classic sets, as they take into account a new vision and a logical system that fits our imperfect information in the world around us. We were able to remove privacy, ambiguity and contradiction.**
- **Neutrosophic sets are gaining great interest in solving many real-life problems involving uncertainty and imprecision, ambiguity, incompleteness and inconsistency.**
- **The study of the relationship between Neutrosophic crisp sets and the application of some algebraic operations to them is of great importance in many applications of life.**

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ABSTRACT

The world around us is characterized by specificity, ambiguity, and contradiction, which led to a lack of our full knowledge of its facts and events. Therefore, we became in need of a new vision and a logical system that fits our incomplete information. In 1965, the American scientist F. Smarandache presented neutrosophic logic as a generalization of fuzzy logic and intuitive fuzzy logic. He added a new component to the degrees of a membership function and a non-membership function, which is the degree of indeterminacy. The crispneutrosophic sets presented by Ahmed Salama is considered a generalization of the classical set theory, as he developed and introduced new concepts in the fields of mathematics, statistics, computer science, and classical information systems. The purpose of this paper is to introduce basic concepts of neutrosophic crisp sets and give some types and operations on neutrosophic crisp sets. Also, we discussed the relationship between neutrosophic crisp sets and others. Finally, we investigate some properties of neutrosophic crisp sets supported by many theorems and illustrative examples. We can use the new of neutrosophic notions in the following applications: database, networks robots, compiler, and codes.

## 1-Introduction

In 1965, Zadeh (1965) introduced the concept of fuzzy sets (FS for short). Where each element had a degree of membership. The intuitionistic fuzzy set (IFS for short) introduced by Atanassov (1983) is a generalization of fuzzy set, where he added to the degree of membership defined in the fuzzy sets, the degree of non-membership of each element. The intuitionistic fuzzy sets consider both truth-membership $\mu_A(x)$ and falsity-membership $\nu_A(x)$, with $\mu_A(x),\nu_A(x) \in[0,1]$ and $0\leq\mu_A(x)+\nu_A(x)\leq 1$.

One of the interesting generalizations of theories of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by Smarandache (1995). Itis a powerful tool to deal with incomplete, indeterminate, and inconsistent information, which exists in the real world. The fundamental concepts of neutrosophic set, introduced by Smarandache (1999, 2002, 2005); Salama *et al.* (2014); Salama & Alblowi (2012a, 2012b); Albblowi *et al.* (2014); Salama *et al.* (2014). Neutrosophic sets are characterized by three functions: a membership function, an indeterminacy function, and a non-membership function that are independently related. The great importance of this theory in many fields of application is due to the possibility of explicitly determining the indeterminacy, the function of truth membership, indeterminacy membership function, and falsity membership functions, which are independent. The features that characterized this theory have led to great success in various fields such as medical diagnosis, Ansari (2011), topology, Lupiáñez (2008), image processing, Zhang (2010), and the problem of decision-making, Kharal (2013). Therefore, the neutrophil group is a powerful tool for dealing with unspecified and inconsistent data.

We can assert that Neutrosophic logic has become the basis for most of the new mathematical theories that generalize their classical and fuzzy counterparts (Atanassov, (1988); Salama *et al.* (2014); Zhang (2010)). In addition, rough set theory (Salama & Broumi, 2014). The neutrosophic set provides a generalization of all the above-mentioned sets from a philosophical point of view.

Salama & Smarandache (2015); Salama *et al.* (2014)introduced and studied some possible definitions for basic concepts of an instance of a neutrosophic set called neutrosophic crisp sets (NCSs) and its operations. In this paper, we will introduce the basic concepts of neutrosophic crisp sets and define new types of neutrosophic crisp sets. Moreover, we study some operations and properties on neutrosophic crisp sets.

## 2. Preliminaries

In this section, we present some preliminaries that will be useful to our work in the next sections.

**Fuzzy set:**

In a fuzzy set (FS), each element has a degree of membership that indicates belonging to this set.

**Definition 2.1** (Zadeh, (1965)):

Let $X$ be a non-empty set. A fuzzy set $A$ is an object having the form

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$A = \{(x, \mu_A(x)); \in X, \mu_A(x) \in [0,1]\}$ represent the degree of membership of each $x \in X$ to the set $A$.

If $\mu_A(x) = 1$, then $x \in A$.

If $\mu_A(x) = 0$, then $x \notin A$.

**Intuitionistic fuzzy set:**

The intuitionistic fuzzy set (IFS) on a universe $X$ is a generalization of (FS), where besides the degree of membership $\mu_A(x) \in [0,1]$ of each element $x \in X$ to a set A, there was considered a degree of non-membership $\nu A(x) \in [0,1]$.

**Definition 2.2** (Atanassov (1986)):

Let $X$ be a non-empty set. The set $A \subseteq X$ is said to be intuitionistic fuzzy set (IFS), if it has the form $A = \{( , \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership function and the degree of non-membership respectively of each $x \in X$ to the set $A$ and $x \in X$, $\mu_A(x), \nu_A(x) \in [0,1]$ : $\mu_A: X \to [0,1], \nu_A: X \to [0,1]$ and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$.

**3-Neutrosophic Set and its Generalizations**

This section provides a summary of the basic concepts of neutrosophic sets and some of their generalizations, as introduce some possible definitions of these concepts. We also study some of its properties.

**Definition 3.1 Neutrosophic set** (Smarandache, 2002):

Let $X$ be a non-empty set. A neutrosophic set (NS for short) $A \subseteq X$ is an object having the form $A = \{<\mu_A(x), \sigma_A(x), \nu_A(x))>, x \in X\}$, where $\mu_A(x)$ represents the degree of membership function, $\sigma_A(x)$ represents the degree of indeterminacy and $\nu_A(x)$ represents the degree of non-membership function respectively of each element $x \in X$ to the set $A$.

**Remark 3.1:**

A neutrosophic set $A = \{<\mu_A(x), \sigma_A(x), \nu_A(x)>, x \in X\}$ can be expressed as an ordered triple

$<\mu_A(x), \sigma_A(x), \nu_A(x)>$in$]0^-, 1^+[$in $X$. Where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x)> \le 1^+$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

**Remark 3.2:**

We use the symbol $<\mu_A(x), \sigma_A(x), \nu_A(x)>$for neutrosophic set $A = \{<\mu_A(x), \sigma_A(x), \nu_A(x)>, x \in X\}$ for simplicity.

Since Smarandache provided the neutrosophic logic by the neutrosophic components T, I, and F, which represents the membership, indeterminacy, and non-membership values respectively, where $]0^-, 1^+[$ is a nonstandard unit interval. Thus, the neutrosophic set can be expressed in this context, as in the following definition.

**Definition 3.2** (Smarandache, 2002):

Suppose $X$ is a non-empty set. We express the neutrosophic set $A \subseteq X$ witha truth-membership function $T_A$, an indeterminacy membership function $I_A$ and a falsity-membership function $F_A$. We can be recognized as an ordered triad $<T_A(x), I_A(x), F_A(x)>$.Where $T_A$: $X \to ]0^-, 1^+[$, $I_A: X \to ]0^-, 1^+[$, $F_A: X \to ]0^-, 1^+[$, and $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

**Definition 3.3**

Let $X$ be a non-empty set. A neutrosophic $A \subseteq X$ is said to be a null or empty neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$, $F_A(x) = 1$ for all $x \in X$ .It is denoted by:

$0_N = \{< x, 0, 0, 1 > : x \in X\}$.

**Definition 3.4**

A neutrosophic $A \subseteq X$is said to be an absolute (universe) neutrosophic set if

$T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by:

$1_N = \{< x, 1, 1, 0 > : x \in X\}$.

**Definition3.5** (Smarandache, 2002):

Let $T, I, F$ be subsets of $]0^-, 1^+[$, with $\sup T = t_{sup}$, $\inf T = t_{\inf}$, $\sup I = i_{sup}$, $\inf I = i_{\inf}$, $\sup F = f_{sup}$and $\inf F = f_{\inf}$. So:

$n_{sup} = t_{sup} + i_{sup} + f_{sup} \le 3^+$, $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf} \ge 0^-$.

Therefore $0^- \le \inf(n) \le \sup(n) \le 3^+$.

In the previous definition, T, I, and F are called neutrophilic components, where T represents the value of truth, while I represents the value of indeterminacy and F represents the value of falsehood respectively referring to neutrosophy, neutrosophic logic.

Let X be a non-empty set, A⊆ Xand∈ X.We can analyze the element's belonging $x$to set A by the following method: it is $t$% true in the set, $i$% indeterminate (unknown if it is) in the set, and $f$% false, where $t$ varies in T, $i$ varies in I, $f$ varies in F. For software engineering proposals, the classical unit interval [0, 1] is used.

**Remark 3.3:**

In the single-valued neutrosophic logic (t, i, f), we observe the following:

1- When the three components are independent, the sum of the components have a value ranging between 0 and 3, which meaning that $0 \le t + i + f \le 3$
2- If two of the components are dependent, while the third is independent of them, then $0 \le t + i + f \le 2$
3- When the three components are dependent, we get $0 \le t + i + f \le 1$
4- If two or three components are independent. We have one of the following options:
a- (Sum<1) because one of them made a field for incomplete information.
b- (Sum> 1) because the information is paraconsistent and contradictory.
c- (Sum = 1) that is, the information is complete.

In the neutrosophic philosophy, one might say that any neutrosophic element belongs to any set, due to the percentages of truth, indeterminacy, and falsity involved ranging from 0 to 1, or even less than 0 or greater than 1.

**Example 3.1:**

Let A be a neutrosophic set.

1- $x$ (0.7, 0.1, 0.2) ∈A: means, with a probability of 70% $x$∈ A, with a probability of 20% $x$∉ A.As for the rest, we cannot report it.

2- $y$ (0, 0, 1) ∈A: Means sure $y$∉ A.

3- $z$ (0, 1, 0) ∈A: This means that we cannot determine whether the element $z$ belongs to the set A or not.

**Definition 3.6:**

Let $A$ and $B$be NSs of the form $A = < \mu_A(x), \sigma_A(x), \nu_A(x) : x \in X >$ and

$B = < \mu_B(x), \sigma_B(x), \nu_B(x) : x \in X >$. Then:

(i) $A \subseteq B$ *if and only if* $\mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$.

(ii) $A^C = < \nu_A(x), \sigma_A(x), \mu_A(x) : x \in X >$.

(iii) $A \cap B = < \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x) : x \in X >$.

(iv) $A \cup B = < \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_A(x), \nu_A(x) \wedge \nu_B(x) : x \in X >$.

**Definition 3.7:**

Let $X$ be a non-empty set, a neutrosophic set $A$ with $\mu_A(x)$=1, $\sigma_A(x)$=1 and $\nu_A(x)$=1, is called normal neutrosophic set. In other words, $A$ is called normal if and only if $\max_{x\in X}\mu_A(x)$)= $\max_{x\in X}\sigma_A(x)$=$\max_{x\in X}\nu_A(x)$=1.

**Proposition 3.1:**

Let $A$ = <$\mu_A(x)$,$\sigma_A(x)$,$\nu_A(x)$>be a neutrosophic set on a set $X$. Then the following properties hold:

1) $A\cup 0_N = A$.
2) $A\cup 1_N = 1_N$.
3) $A\cap 0_N = 0_N$.
4) $A\cap 1_N = A$.

**Proof:** since$0_N = \{< x, 0, 0, 1 > : x \in X\}, 1_N = \{< x, 1, 1, 0 > : x \in X\}$.

So we have:

1) $A\cup 0_N = \langle \mu_A(x)\cup 0, \sigma_A(x)\cup 0, \nu_A(x)\cap 1\rangle = \langle \mu_A(x), \sigma_A(x), \nu_A(x) >= A$.

2) $A\cup 1_N$= $\langle \mu_A(x)\cup 1, \sigma_A(x)\cup 1, \nu_A(x)\cap 0\rangle$ = $\langle 1,1,0\rangle = 1_N$.

3) $A\cap 0_N = \langle \mu_A(x)\cap 0, \sigma_A(x)\cap 0, \nu_A(x)\cup 1\rangle$ = $\langle 0,0,1\rangle$=$0_N$.

4) $A\cap 1_N = \langle \mu_A(x)\cap 1, \sigma_A(x)\cap 1, \nu_A(x)\cup 0 >$ =$\langle \mu_A(x), \sigma_A(x), \nu_A(x) >$=$A$.

**Definition 3.8 Generalized neutrosophic set** (Salama & Alblowi, 2012a)**:**

Let X be a non-empty set. A generalized neutrosophic (GNS for short) set is an object having the form $A$ = <x,$\mu_A(x)$, $\sigma_A$(x), $\nu_A$(x)>where$\mu_A(x)$ represents the degree of membership function, $\sigma_A$(x) represents the degree of indeterminacy and$\nu_A$(x) represents the degree of non-membership function respectively of each element x $\in$ X to the set $A$, where$0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$and the functions satisfy the condition:

$\mu_A(x)\wedge\sigma_A(x)\wedge\nu_A(x) \le 0.5$ and $0^- \le \mu_A(x)+\sigma_A(x)+\nu_A(x) \le 3^+$.

**Remark 3.4:**

A generalized neutrosophic $A$ = <$\mu_A(x)$,$\sigma_A$(x),$\nu_A$(x)>can be expressed as an ordered triple

<$\mu_A, \sigma_A, \nu_A$> in$]0^-, 1^+[$ in X, with the requirement to fulfill the three functions of the condition:

$\mu_A(x)\wedge\sigma_A(x)\wedge\nu_A(x) \le 0.5$.

**Definition 3.9 Intuitionistic neutrosophic set** (Bhowmik & Pal, 2010)**:**

Let X be a non-empty set. An intuitionistic neutrosophic set A (INS for short) is an object having the form A = <x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$> where$\mu_A(x)$, $\sigma_A$(x)and$\nu_A$(x)which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely$\sigma_A$(x)) and the degree of non-membership (namely $\nu_A$(x)) respectively of each element x $\in$ X to the set A, where $0.5 \le \mu_A(x)$, $\sigma_A$(x),$\nu_A$(x)and the functions satisfy the condition $\mu_A(x)\wedge\sigma_A(x) \le 0.5$,$\mu_A(x)\wedge\nu_A(x) \le 0.5$, $\sigma_A(x)\wedge\nu_A(x) \le 0.5$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+$.

## 4- Neutrosophic Crisp Set

In this section, we will discuss some possible definitions for the different types of neutrosophic crisp sets with some illustrative examples.

**Definition 4.1:**

Let $X$ be a non-empty set. A neutrosophic set $A$ with $\mu_A(x)$=$A_1$, $\sigma_A(x)$=$A_2$ and $\nu_A(x)$=$A_3$, where$A_1, A_2$, and $A_3$are subsets of $X$is called neutrosophic crisp set (NCS for short). In other words, A neutrosophic crisp set $A$ is an object having the form A= $\langle A_1, A_2, A_3\rangle$ where$A_1, A_2$, and $A_3$are subsets of $X$.

**Types of neutrosophic crisp sets:**

**Definition 4.2** (Salama & Smarandache, 2014)**:**

(i) The neutrosophic crisp set is of Type 1 if satisfying the condition:

$A_1\cap A_2 = \emptyset$, $A_1\cap A_3 = \emptyset$and$A_2\cap A_3 = \emptyset$. We denoted to it as: (NCS-1).

(ii) The neutrosophic crisp set is of Type2 if satisfying the condition:

$A_1\cap A_2 = \emptyset, A_1\cap A_3 = \emptyset, A_2\cap A_3 = \emptyset$ and $A_1\cup A_2\cup A_3$ =$X$.

We denoted to a neutrosophic crisp set as: (NCS-2).

(iii) The neutrosophic crisp set is of Type 3 if satisfying the condition:

$A_1\cap A_2\cap A_3 = \emptyset$ and $A_1\cup A_2\cup A_3$ =$X$. We denoted it as: (NCS-3).

**Example 4.1:**

Let $X$ ={1,2,3,4,5,6 }, $A$ = ⟨{1,2,3,4},{5},{ 6}⟩ ,$B$ = ⟨{1,2},{3,5},{ 4,6}⟩ be a NCS-2,

C= ⟨{1,2,3},{4},{5}⟩ be a NCT-1, $D$= ⟨{1,2},{3,4},{1,5,6}⟩ be a NCS-3.

**Remark 4.1:**

To study and develop the neutrosophic crisp set, it is necessary to know the tools necessary to deal with these sets. Salama et al. constructed the tools for a developed neutrosophic crisp set, to learn more, see Salama & Smarandache (2014).

**Definition 4.3:**

Empty crisp neutrosophic set denoted as$\varphi_N$, we can define it as one of the following four types:

(i) T1: $\varphi_N$ = $\langle\varphi, \varphi, X\rangle$,

(ii) T2: $\varphi_N$ = $\langle\varphi, X, X\rangle$,

(iii) T3: $\varphi_N$ = $\langle\varphi, X, \varphi\rangle$,

(iv) T4: $\varphi_N$ = $\langle\varphi, \varphi, \varphi\rangle$.

**Definition 4.4:**

Absolute crisp neutrosophic set denoted as$X_N$ and defined as one of the following four types:

(i) T1: $X_N$ =$\langle X, \varphi, \varphi\rangle$,

(ii) T2: $X_N$ =$\langle X, X, \varphi\rangle$,

(iii) T3: $X_N$ =$\langle X, \varphi, X\rangle$,

(iv) T4: $X_N$ =$\langle X, X, X\rangle$.

**Definition 4.5:**

A crisp neutrosophic set $\varphi_N$ in definition 4.2.2 (iv) is called normal empty crisp neutrosophic set with $A_1 = \varphi, A_2 = \varphi, A_3 = \varphi$ .In other words, $A$ is called normal empty crisp neutrosophic set if and only if $\max_{x\in X}\mu_A(x)$)= $\max_{x\in X}\sigma_A(x)$=$\max_{x\in X}\nu_A(x)$ = 0. In addition, a crisp neutrosophic set $X_N$ in definition 4.2.3 (iv) is called normal absolute crisp neutrosophic set with $A_1 = X, A_2 = X, A_3 = X$ .In other words, $A$ is called normal crisp neutrosophic set if and only if $\max_{x\in X}\mu_A(x)$)= $\max_{x\in X}\sigma_A(x)$=$\max_{x\in X}\nu_A(x)$ = 1.

## 5- Some Operations on Neutrosophic Crisp Sets

Neutrosophic crisp sets are involved in many applications of life, so it is necessary to study the relationship between these sets and apply some algebraic operations to them.

### 5.1 The complement of the NCS

**Definition 5.1**

Let $A = \langle A_1, A_2, A_3 \rangle$ be an NCS in $X$, then the complement of the set $A$ ($A^C$ for short) maybe defined as three kinds of complements:

(i) $A^C = \langle {A_1}^C, {A_2}^C, {A_3}^C \rangle$ or

(ii) $A^C = \langle A_3, A_2, A_1 \rangle$or

**(iii)**$\boldsymbol{A^C = \langle A_3, {A_2}^C, A_1 \rangle}$.

The complement of the NCS $A$ can be defined as$A^C = 1_N - A$ or $A^C = X_N - A$, note that $1_N$ equivalent to $X_N$of type 2.

**Example 5.1:**

In Example 4.2.1, $A$ = ⟨{1,2,3,4},{5},{ 6 }⟩is a NCS-2,

$A^C$ = ⟨{5,6}, {1,2,3,4,6}, {1,2,3,4,5}⟩or$A^C$= ⟨{ 6 }, {5}, {1,2,3,4}⟩ or

$A^C$= ⟨{ 6 }, {1,2,3,4,6}, {1,2,3,4}⟩.

### 5.2 Containment

We can present that a neutrosophic set $A = \langle A_1, A_2, A_3 \rangle$ is contained in the other neutrosophic set $B = \langle B_1, B_2, B_3 \rangle$, write ($A \subseteq B$ ) as in the following definition.

**Definition 5.2:**

Let $X$ be a non-empty set, and NCSs $A$ and $B$ be in the form
$A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$.

$A \subseteq B$ may be defined as two types:

Type 1: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$and $A_3 \supseteq B_3$ .

Type 2: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$and $A_3 \supseteq B_3$ .

**Remark 5.1:**

The equality relation on neutrosophic sets $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$is defined as:

$$A = B \ iff \ A \subseteq B \ and \ B \subseteq A.$$

**Proposition 5.1:**

For any neutrosophic crisp set $A$. Then

(i)$\varphi_N \subseteq A$, $\varphi_N \subseteq \varphi_N$.

(ii) $A \subseteq X_N$, $X_N \subseteq X_N$.

**Example 5.2:**

In Example 4.2.1, C= ⟨{1,2,3},{4},{5}⟩, D= ⟨{1,2},{3,4},{1,5,6}⟩, E= ⟨{1,2},{3},{6}⟩, we have $D \subseteq C$ of Type-2 and $E \subseteq B$ of Type-1.

### 5.3 Intersection and union of the NCSs:

The intersection and union of two neutrosophic crisp sets $A$ and $B$ in $X$ can be presented in the following definition.

**Definition 5.3:**

Let $X$ be a non-empty set, and the NCSs $A$ and $B$ be of the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$. Then:

(i) $A \cap B$ may be defined as two types:

Type1: $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$.

Type 2: $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$.

(ii) $A \cup B$ may be defined as two types:

Type1: $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$.

Type 2: $\cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$.

**Proposition 5.2:**

For NCSs $A$ and $B$ in $X$, De Morgan's laws on NCSs are true:

$(A \cap B)^c = A^c \cup B^c$ , $(A \cup B)^c = A^c \cap B^c$.

It is easy to generalize the operations of intersection and union above to an arbitrary family of neutrosophic crisp subsets as it follows.

**Proposition 5.3** (Salama & Smarandache, 2015):

Let $\{A_j : j \in J\}$ be an arbitrary family of neutrosophic crisp subsets in $X$, then:

1) $\cap Aj$ may be defined as the following two types:

(i) Type1: $\cap Aj = \langle \cap Aj_1, \cap Aj_2, \cup Aj_3 \rangle$.

(ii) Type2: $\cap Aj = \langle \cap Aj_1, \cup Aj_2, \cup Aj_3 \rangle$.

2) $\cup Aj$ may be defined as the following types:

(a) Type 1: $\cup Aj = \langle \cup Aj_1, \cup Aj_2, \cap Aj_3 \rangle$

(b) Type 2: $\cup Aj = \langle \cup Aj_1, \cap Aj_2, \cap Aj_3 \rangle$.

**Remark 5.2:**

Because $A \backslash B = A \cap B^c$, the difference can be defined using intersection and negation operators.

**Example 5.3:**

Let $X$ ={a,b, c, d}, $A$ =⟨{a,b},{c},{d}⟩,$B$ =⟨{a },{c},{d,b}⟩, be NCSs-1 in $X$ and

C = ⟨{a,b},{c, d},{a,d}⟩, D= ⟨{a,b, c},{c },{d}⟩ be NCSs-3 in $X$, then we have:

$A \cap B$; Type1= Type2 =⟨{a },{c},{ d,b }⟩,

$A \cup B$; Type1= Type2 =⟨{a ,b},{c},{ d }⟩.

$A \cap C$; Type1: $A \cap C$=⟨{a ,b},{c},{ d }⟩,

Type2: $A \cap C$=⟨{a,b },{c,d},{ a,d }⟩.

$C \cup D$; Type1: $C \cup D$ = ⟨{a ,b,c},{c},{ a,d }⟩,

Type2: $C \cup D$=⟨{a ,b,c},{c},{ d }⟩.

$A^c$; Type1:$A^c$ = ⟨{c,d},{a,b,d},{a,b,c}⟩ NCS -3

Type2:$A^c$ = ⟨{ d},{a,b,d},{ a,b }⟩ NCS -3

Type3:$A^c$ = ⟨{ d},{c},{ a,b }⟩ NCS -3.

$B^c$; Type1:$B^c$ = ⟨{b,c,d},{a,b,d},{a,b}⟩NCS-3

Type2:$B^c$ = ⟨{b ,d},{c},{ a }⟩ NCS-1

Type3:$B^c$ = ⟨{b,d},{a,b,d},{a}⟩ NCS-3.

$A \backslash B = A \cap B^c$: Type1= Type 2= ⟨{b},{ c},{a, d}⟩, where $B^c$is Type-1.

From definition 4.2.2 and definition 4.2.3, we can classification $\varphi_N, X_N$ as follows

**Definition 5.4:**

An NCSofType-1 $\varphi_{N_1}, X_{N_1}$in $X$can be classified as follows:

1. $\varphi_{N_1}$can be defined as one of the following types:

(i) Type1: $\varphi_{N_1} = \langle \varphi, \varphi, X \rangle$,

(ii) Type2: $\varphi_{N_1} = \langle \varphi, X, \varphi \rangle$,

(iii) Type3: $\varphi_{N_1} = \langle \varphi, \varphi, \varphi \rangle$.

2. $X_{N_1}$can be defined as one type: (i) Type1:$X_{N_1}$ =$\langle X, \varphi, \varphi \rangle$.

**Definition 5.5:**

An NCSofType-2, $\varphi_{N_2}, X_{N_2}$in $X$can be classified as follows:

(1) $\varphi_{N_2}$can be defined as two types:

(i) Type1: $\varphi_{N_2} = \langle \varphi, \varphi, X \rangle$,

(ii) Type2: $\varphi_{N_2} = \langle \varphi, X, \varphi \rangle$.

(2) $X_{N_2}$ can be defined as one type:

(i) Type1: $X_{N_2} = \langle X, \varphi, \varphi \rangle$.

**Definition 5.6:**

An NCS of Type-3, $\varphi_{N_3}$, $X_{N_3}$ in $X$ can be classified as follows

1) $\varphi_{N_3}$ can be defined as one of the following three types:

(i) Type1: $\varphi_{N_3} = \langle \varphi, \varphi, X \rangle$,

(ii) Type2: $\varphi_{N_3} = \langle \varphi, X, \varphi \rangle$,

(iii) Type3: $\varphi_{N_3} = \langle \varphi, X, X \rangle$.

2) $X_{N_3}$ can be defined as one of the following three types:

(i) Type1: $X_{N_3} = \langle X, \varphi, \varphi \rangle$,

(ii) Type2: $X_{N_3} = \langle X, X, \varphi \rangle$,

(iii) Type3: $X_{N_3} = \langle X, \varphi, X \rangle$.

**Corollary 5.1:**

From Definition 4.2, we have:

(i) Every NCS of Type1, Type 2 and Type 3 are NCS.

(ii) Every NCS of Type1 is not NCS of Type2 nor Type3.

(iii) Every NCS of Type 2 is not NCS of Type1 nor Type 3.

(iv) Every NCS of Type3 is not NCS of Type2 nor Type 1.

(v) Every crisp set is NCS.

**Example 5.4:**

Let $X=\{1,2,3,4,5,6\}$, $A = \langle\{1,2,3,4\},\{5\},\{6\}\rangle$, $B=\langle\{1,2\},\{3,5\},\{4,6\}\rangle$ be a NCS-2, but not NCS-1,3, C= $\langle\{1,2,3\},\{4\},\{5\}\rangle$ be a NCT-1, but not NCS-2,3, $D=\langle\{1,2\},\{3,4\},\{1,5,6\}\rangle$ be a NCS- 3, but not NCS-1,2.

**Remark 5.3:**

Let $X$ be a non-empty set, $A = \langle A_1, A_2, A_3 \rangle$.

1- If $A$ is an NCS-1 in $X$, then $(A^C)$ may be defined as one kind of complement Type1: $A^C = \langle A_3, A_2, A_1 \rangle$.

2- If $A$ is an NCS-2 in $X$, then $(A^C)$ may be defined as one kind of complement $A^C = \langle A_3, A_2, A_1 \rangle$.

3- If $A$ is NCS-3 in $X$, then $(A^C)$ can be defined as one type of the three types:

(i)Type1: $A^C = \langle A^C{}_1, A^C{}_2, A^C{}_3 \rangle$.

(ii)Type2: $A^C = \langle A_3, A_2, A_1 \rangle$.

(iii)Type3: $A^C = \langle A_3, A^C{}_2, A_1 \rangle$.

**Example 5.5:**

Let $X=\{a,b,c,d,e,f\}$, $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$ is a NCS- 2,

$B=\langle\{a,b,c\},\{\varphi\},\{d,e\}\rangle$ is a NCS-1, $C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ is a NCS-3, then

(i) $A^C = \langle\{f\},\{e\},\{a,b,c,d\}\rangle$ NCS-2;

(ii) $B^C = \langle\{d,e\},\{\varphi\},\{a,b,c\}\rangle$ NCS-1;

(iii) $C^C$ may be defined as three types:

Type 1: $C^C = \langle\{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$, Type 2: $C^C = \langle\{e,f\},\{a,b,e,f\},\{a,b\}\rangle$,

Type 3: $C^C = \langle\{e,f\},\{c,d\},\{a,b\}\rangle$.

**Remark 5.4:**

The set of processes previously defined on types of neutrosophic crisp sets is not closed in general. The following example illustrates this observation.

**Example 5.6:**

Let $X=\{1,2,3,4,5\}$, $A=\langle\{1,2\},\{4\},\{3,5\}\rangle$, $B=\langle\{1,3\},\{2,5\},\{4\}\rangle$ are NCS-3, but

$A \cap B=\langle\{1\},\{\varphi\},\{3,4,5\}\rangle$ is not NCS-3. Also, $A \cup B=\langle\{1,2,3\},\{\varphi\},\{3,4,5\}\rangle$ is not NCS-3.

## 6. Basic Properties of Neutrosophic Crisp Sets:

We present some theorems to clarify the basic characteristics of neutrosophic crisp sets with notice that $(0_N)^C = 1_N$, $(1_N)^C = 0_N$

**Proposition 6.1:**

1- $(\varphi_N)^C = X_N$.

2- $(X_N)^C = \varphi_N$.

**Proof:** It is straightforward from the definition of $\varphi_N$, $X_N$ and the complement of the NCS.

**Proposition 6.2:**

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be neutrosophic crisp sets on a set $X$, then

$$A \subseteq B \Leftrightarrow B^C \subseteq A^C.$$

**Proof:** Case 1: Since $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$.

$\Leftrightarrow 1 - B_1 \subseteq 1 - A_1, 1 - B_2 \subseteq 1 - A_2, 1 - B_3 \supseteq 1 - A_3$

$\Leftrightarrow B_1{}^C \subseteq A_1{}^C, B_2{}^C \subseteq A_2{}^C, B_3{}^C \supseteq A_3{}^C$.

$\Leftrightarrow B^C \subseteq A^C$.

Case 2: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$.

$\Leftrightarrow 1 - B_1 \subseteq 1 - A_1, 1 - B_2 \supseteq 1 - A_2, 1 - B_3 \supseteq 1 - A_3$

$\Leftrightarrow B_1{}^C \subseteq A_1{}^C, B_2{}^C \supseteq A_2{}^C, B_3{}^C \supseteq A_3{}^C$.

$\Leftrightarrow B^C \subseteq A^C$.

**Proposition 6.3:**

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set on a set $X$, then $(A^C)^C = A$.

**Proof:** Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set on a set $X$, then $A^C$ have three cases.

case1: $A^C = \langle A_1{}^C, A_2{}^C, A_3{}^C \rangle = \langle 1 - A_1, 1 - A_2, 1 - A_3 \rangle$. So

$(A^C)^C = \langle (A_1{}^C)^C, (A_2{}^C)^C, (A_3{}^C)^C \rangle = \langle (1 - A_1)^C, (1 - A_2)^C, (1 - A_3)^C \rangle = \langle 1 - (1 - A_1), 1 - (1 - A_2), 1 - (1 - A_3) \rangle = \langle A_1, A_2, A_3 \rangle = \text{A}$.

case 2: $A^C = \langle A_3, A_2, A_1 \rangle$, so $(A^C)^C = \langle A_1, A_2, A_3 \rangle = \text{A}$.

case 3: $A^C = \langle A_3, A_2{}^C, A_1 \rangle$, so, $(A^C)^C = \langle A_1, (A_2{}^C)^C, A_3 \rangle = \langle A_1, A_2, A_3 \rangle = \text{A}$.

Hence $(A^C)^C = \text{A}$.

**Proposition 6.4:**

Let $A_i = \langle A_{i_1}, A_{i_2}, A_{i_3} \rangle$ and $B = \langle B_1, B_2, B_3, \ i \in J$ be neutrosophic crisp sets on a set $X$.

If $B \subseteq A_i$ for each $i \in J$, then $B \subseteq \cap A_i$.

**Proof:** Let $B \subseteq A_i$, it means that $B \subseteq A_1$, $B \subseteq A_2$, $B \subseteq A_3 \ldots B \subseteq A_n$.

We distinguish two cases,

Case 1: $B \subseteq A_i \Longrightarrow B_1 \subseteq A_{i_1}, B_2 \subseteq A_{i_2}, B_3 \supseteq A_{i_3}$

$\Rightarrow B_1 \subseteq (A_{1_1}, A_{2_1}, A_{3_1}, \dots \dots . . A_{n_1})$,

$\Rightarrow B_2 \subseteq (A_{1_2}, A_{2_2}, A_{3_2}, \dots \dots . . A_{n_2})$,

$$\Rightarrow B_3 \supseteq (A_{1_3}, A_{2_3}, A_{3_3}, \dots \dots . . A_{n_3}).$$

$So B_1 \subseteq \cap A_{i_1}, B_2 \subseteq \cap A_{i_2}, B_3 \supseteq \cap A_{i_3}$.Then $B \subseteq \cap A_i$ .

Case 2:$B \subseteq A_i \Rightarrow B_1 \subseteq A_{i_1}, B_2 \supseteq A_{i_2}, B_3 \supseteq A_{i_3}$

$$\Rightarrow B_1 \subseteq (A_{1_1}, A_{2_1}, A_{3_1}, \dots \dots . . A_{n_1}),$$

$$\Rightarrow B_2 \supseteq (A_{1_2}, A_{2_2}, A_{3_2}, \dots \dots . . A_{n_2}),$$

$$\Rightarrow B_3 \supseteq (A_{1_3}, A_{2_3}, A_{3_3}, \dots \dots . . A_{n_3}).$$

$So B_1 \subseteq \cap A_{i_1}$, $B_2 \supseteq \cap A_{i_2}$, $B_3 \supseteq \cap A_{i_3}$. Then $B \subseteq \cap A_i$ .

**Proposition 6.5:**

Let $A_i = \langle A_{i_1}, A_{i_2}, A_{i_3}\rangle$ and $B= \langle B_1, B_2, B_3$, $i \in J$ be a neutrosophic crisp set on a set $X$.

If $A_i \subseteq B$ for each $i \in J$, then $\cup A_i \subseteq B$.

**Proof:** Let$A_i \subseteq B$, it means that $A_1 \subseteq B, A_2 \subseteq B, A_3 \subseteq B., A_n \subseteq B$

$$\Rightarrow A_{1_1} \subseteq B_1 \ , A_{1_2} \subseteq B_2, A_{1_3} \supseteq B_3$$

$$\Rightarrow A_{2_1} \subseteq B_1 \ , A_{2_2} \subseteq B_2, A_{2_3} \supseteq B_3$$

$$\Rightarrow A_{3_1} \subseteq B_1 \ , A_{3_2} \subseteq B_2, A_{3_3} \supseteq B_3$$

$$A_{n_1} \subseteq B_1 \ , A_{n_2} \subseteq B_2, A_{n_3} \supseteq B_3$$

$$(A_{1_1} \cup A_{2_1} \cup A_{3_1} \cup \dots \cup A_{n_1}) \subseteq B_1$$

$$(A_{1_2} \cup A_{2_2} \cup A_{3_2} \cup \dots \cup A_{n_2}) \subseteq B_2$$

$$(A_{1_3} \cup A_{2_3} \cup A_{3_3} \cup \dots \cup A_{n_3}) \supseteq B_3$$

So $\cup A_i = \langle \cup A_{i_1}, \cup A_{i_2}, \cup A_{i_3}\rangle \subseteq B$. Then $\cup A_i \subseteq B$.

**Proposition 6.6:**

Let $A$ be a neutrosophic crisp set on a set $X$. Then the following properties hold:

1)$A \cup 0_N = A$.

2)$A \cup 1_N = 1_N$.

3)$A \cap 0_N = 0_N$.

4) $A \cap 1_N = A$.

Proof: Let $A = \langle A_1, A_2, A_3\rangle$ , $0_N = \{< x, 0, 0, 1 >: x \in X\}, 1_N = \{< x, 1, 1, 0 >: x \in X\}$

1) $A \cup 0_N = \langle A_1 \cup 0, A_2 \cup 0, A_3 \cap 1\rangle = \langle A_1, A_2, A_3\rangle = A$ .

2)$A \cup 1_N = \langle A_1 \cup 1, A_2 \cup 1, A_3 \cap 0\rangle = \langle 1,1,0\rangle = 1_N$.

3) $A \cap 0_N = \langle A_1 \cap 0, A_2 \cap 0, A_3 \cup 1 \rangle = \langle 0,0,1\rangle = 0_N$.

4) $A \cap 1_N = \langle A_1 \cap 1, A_2 \cap 1, A_3 \cup 0 \rangle = \langle A_1, A_2, A_3\rangle = A$.

**Proposition 6.7:**

Let $A$ be a neutrosophic crisp set on a set $X$. Then the following properties hold:

1) $A \cup \varphi_N = A$.

2) $A \cup X_N = X_N$.

3) $A \cap \varphi_N = \varphi_N$.

4) $A \cap X_N = A$.

**Proof:** It is similar to Proposition 6.6 with regard to definitions of $\varphi_N$and$X_N$.

**Proposition 6.8:**

Let $A = \langle A_1, A_2, A_3\rangle$ and $B= \langle B_1, B_2, B_3\rangle$be two neutrosophic crisp sets on a set $X$. Then

$A \cup B = A$if and only if $B \subseteq$ A.

**Proof:** Let $A = \langle A_1, A_2, A_3\rangle$ and $B= \langle B_1, B_2, B_3\rangle$ be two neutrosophic crisp sets such that

$$A \cup B = A$$

$\Leftrightarrow A_1 \cup B_1 = A_1, A_2 \cup B_2 = A_2, A_3 \cap B_3 = A_3$

$$\Leftrightarrow B_1 \subseteq A_1, B_2 \subseteq A_2, B_3 \supseteq A_3$$

$\Leftrightarrow B \subseteq A$.

**Proposition 6.9:**

Let $A = \langle A_1, A_2, A_3\rangle$ and $B= \langle B_1, B_2, B_3\rangle$ be two neutrosophic crisp sets on a set $X$. Then

$$A \backslash B = B^c \backslash A^c$$

**Proof:** It is direct from the difference operator and the complement of the NCS.

**Proposition 6.10:**

$A \cup B$is the smallest neutrosophic set containing both $A$ and $B$.

**Proof:** It results directly from the definition of the union operator.

**Proposition 6.11:**

$A \cap B$is the largest neutrosophic set contained in both $A$ and $B$.

**Proof:** It results directly from the definition of the intersection operator.

**Proposition 6.12:**

Let $A = \langle A_1, A_2, A_3\rangle$, $B= \langle B_1, B_2, B_3\rangle$ and $C = \langle C_1, C_2, C_3\rangle$ be neutrosophic crisp sets on a set $X$ .Then

1) $A \cap B = B \cap A$ .

2) $A \cup B = B \cup A$.

3) $A \cap (B \cap C) = (A \cap B) \cap C$.

4) $A \cup (B \cup C) = (A \cup B) \cup C$.

5)$A \cap (B \cup C)) = (A \cap B) \cup (A \cap C)$.

6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

7) $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.

8) $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$.

**Proof:** It is straightforward from the definition of difference, intersection, and union operators.

## 6. Conclusion

Neutrosophic sets are well equipped to deal with missing data. They are characterized by truth, indeterminacy, and falsity membership functions, which are independent in nature. In this paper, we have studied an instance of neutrosophic sets called neutrosophic crisp sets. The neutrosophic crisp set is a generalization of a classic set. The notion of inclusion, complement, union, intersection, and difference have been defined on neutrosophic crisp sets. We investigate their properties. We also present a comparison between different types of neutrosophic sets and some of their basic operations. We present some examples and theorems to clarify the basic characteristics of neutrosophic crisp sets. There are some ideas that we would have liked to try during the description and the development of the neutrosophic crisp sets in future work.

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