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Almost contra-sgp-cleavability

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Highlights

- **We introduce a new kind of cleavability is called Almost Contra-sgp-cleavability using special map and some special topological spaces.**
- **We study different types of almost Contra-sgp- cleavability of some special topological spaces as almost Contra-sgp pointwise cleavability, almost Contra-sgp absolutely cleavability and Contra-sgp- cleavability.**
- **We proved this case: if a space X which is almost Contra-sgp (pointwise or absolutely) cleavable over a class P of some special topological spaces then X does not belong to the class P.**

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ABSTRACT

In this paper we introduce basic definitions and some properties, of special mapping called “almost contra sgp continuous” used to show the concept of cleavability over some special topological spaces as (sgp -T_0, sgp-T_1, sgp -T_2, weakly Hausdorff, Ultra Hausdroff, Ultra normal, sgp –normal, sgp- Ultra –connected ,hyper connected, sgp –compact–sgp- compact, sgp- Lindelöf, S-Lindelöf, S-closed) spaces.

1. Introduction

Skii *et al.* (1985) introduced different types of cleavability (originally called splittability) as following: A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$, if for $A \subset X$ there exists a continuous mapping $f: X \to Y \in \mathcal{P}$, Such that $f^{-1}f(A) = A$, $f(X)=Y$.

A new class of functions called contra and almost contra sgp-continuous functions introduced and studied by (Hanif and Patil, 2016) as a generalization of contra continuity which was introduced and investigated by (Dontchev, 1996). In this paper we studied this case of cleavability as following: A space X is almost contra sgp-pointwise (rsp. absolutely double)-cleavable over a class $\mathcal{P}$ of weakly Hausdroff, Ultra Hausdroff (Ultra normal, sgp.) spaces respectively, if X admits almost contra sgp- continuous bijection onto some space in $\mathcal{P}$ (but X need not be in $\mathcal{P}$). Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) denote Topological spaces which no separation axioms are assumed otherwise mentioned. For a subset B of a space X the closure and interior of B with respect to τ are denoted by Cl(B) and Int(B) respectively.

2. Preliminaries

In this section, we recall some definitions, which we need in this paper.

2.1. Definition

A subset A of a space X is called

(i) A semi-open set (Levine, 1963) if A⊂Cl(Int(A)).

(ii) A semi-closed set (Crossley and Hildebrand, 1972) if Int(Cl(Int(A)))⊆A.

(iii) A regular open set (Stone, 1937) if A=Int(Cl(Int(A))).

2.2. Dentition (Navalagi and Bhat, 2007)

A topological space X is called semi-generalized preclosed (briefly, sgp-closed) set if pCl(A)⊆U whenever A⊆U and U is semi-open in X.

2.3. Definition (Hanif and Patil, 2016)

A topological space X is said to be

(i) SgpTc-space if every sgp-closed set is closed set.

(ii) Sgp-T_0 space if for any two distinct points x, y in X, x ≠ y there exists sgp-open set of X containing x but not y or containing y but not x.

(iii) Sgp-T_1 space if for any pair points x and y there exist sgp-open sets G and H such that x ∈G, y ∉G and x ∉H, y ∈H.

(iv) Sgp-T_2 space if for any two points x, y in X, x ≠ y, there exist sgp- open sets U and V such that U containing x but not y and V containing y but not x, where U ∩V =∅.

2.4. Definition (Hanif and Patil, 2016)

Let X and Y be topological spaces. A function f :X⟶Y is said to be Sgp-continuous if the inverse image of every closed set in Y is sgp-closed set in X.

2.5. Definition (Dlaska, Ergun and Ganster, 1994)

A function *f*: X ⟶Y is said to be sgp-open (resp., sgp -closed) if f (U) is sgp-open (resp., sgp-closed) in Y for every open set (resp., closed) U in X.

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2.6. Definition (Staum, 1974)

(i) A topological space X is called ultra Hausdroff space if every pair of distinct points of x and y in X there exist disjoint clopen sets U and V in X containing x and y respectively.

(ii) Ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

2.7. Definition (Hanif and Patil, 2016)

A topological space X is said to be:

(i) sgp-normal if each pair of non-empty disjoint closed sets can be separated by disjoin sgp -open sets.

(ii) sgp-connected provided that X is not the union of two disjoint nonempty sgp -open sets of X

(iii) sgp-compact if every sgp -open cover of X has a finite subcover.

2.8. Definition

A topological space (X, τ) is said to be

(i) weakly Hausdorff (Stone, 1937) if each element of X is the intersection of regular closed sets of X.

(ii) hyperconnected (Noiri, 1984) if every open set is dense

2.9. Definition (Hanif and Patil, 2016)

A function f: X⟶Y is said to be

(i) Almost contra-semi generalized pre-continuous (briefly, almost contra sgp-continuous) if f^{-1} (V) is sgp-closed in X for each regular open set V in Y.

(ii) Contra sgp-continuous if f^{-1}(F) is said to be sgp-closed set in X for every open set F of Y.

3. Almost contra sgp-cleavability

3.1. Definition

A topological space X is said to be an almost contra sgp -cleavable over a class of spaces $\mathcal{P}$, if for any subset A of X, there exists a contra sgp-continuous mapping $f: X \to Y$, such that $f^{-1}f$(A)=A. and $f(X) = Y$.

3.2. Definition

A topological spaces is said to be an almost contra sgp-point wise cleavable over a class of spaces. If for every point $x \in X$there exists an injective almost contra sgp -continuous mapping $f: X \to Y$, such that $f^{-1}f$ { x}={ x }.

3.3. Remark

By an almost contra sgp-pointwise cleavable, we mean that a contra sgp-continuous function

$\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y} \in \boldsymbol{\mathcal{P}}$ is an injective and a contra sgp-continuous.

3.4. Definition

A topological space X is said to be an almost contra sgp-absolutely cleavable over a class of spaces $\mathcal{P}$, if for any subset A of X, there exists an injective almost contra sgp-continuous mapping f: $X \to Y$, such that $f^{-1}f$(A)=A.

3.5. Definition

A topological space **X** is said to be double almost contra sgp open (closed) cleavable over a class of spaces $\boldsymbol{\mathcal{P}}$, if for any subsets $\mathbf{A} \subset \mathbf{X}$ and $\mathbf{B} \subset \mathbf{X}$, there exists almost contra sgp open (closed) function $\boldsymbol{f}: \boldsymbol{X} \longrightarrow \boldsymbol{Y}$ such that $\boldsymbol{f^{-1}f}(\mathbf{A}) = \mathbf{A}$ and $\boldsymbol{f^{-1}f}(\mathbf{B}) = \mathbf{B}$

3.6. Proposition

Let space X be an almost contra sgp-pointwise cleavable over a class of weakly Hausdorff spaces $\mathcal{P}$, then X is sgp -T_1 – space .Hence $X \notin \boldsymbol{\mathcal{P}}$

Proof:

Let $x \in X$, then there exists a weakly Hausdorff space $Y \in \mathcal{P}$ and an injective almost contra sgp-continuous mapping

$f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$. This implies that for every y ∈ Y with $x \neq y$, we have $f(x) \neq f(y)$. Since Y is a weakly Hausdorff, so there exist two regular closed sets U and V such that $f(x) \in U$, $f(y) \notin U$, $f(y) \in V$, $f(x) \notin V$ and then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1} f(y) \notin U$, $f^{-1} f(y) \in f^{-1}(V)$, $f^{-1}f(x) \notin V$ this implies that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$, $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$ since $f$is almost contra-sgp-continuous, so $f^{-1}(U)$, $f^{-1}(V)$ are sgp -open sets of X, then X is sgp -T_1-space. Hence $X \notin \boldsymbol{\mathcal{P}}$

3.7. Proposition

Let space X be an almost contra sgp-pointwise cleavable over a class of Ultra Hausdorff spaces $\mathcal{P}$, then X is sgp -T_2-space.

Proof:

Let ∈ X x, then there exists an Ultra Hausdorff space Y and an almost contra sgp-continuous mapping

$f: X \to Y \in \mathcal{P}$ such that $f^{-1}f\{x\} = \{x\}$. This implies that for every y ∈ Y with $x \neq y$, we have $f(x) \neq f(y)$.

Since $\boldsymbol{Y}$ is ultra Hausdorff , so there exist two clopen sets U and V such that $f(x) \in U, f(y) \in V$ and $U \bigcap V$ =∅, then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1} f(y) \in f^{-1}(V)$, this implies that $x \in f^{-1}$(U) , $y \in f^{-1}$(V), since f is an almost contra sgp-continuous, so $f^{-1}(U)$), $f^{-1}(V)$ are two sgp-open sets and $f^{-1}(U) \bigcap f^{-1}(V)= f^{-1}(U \bigcap V)= f^{-1}(\emptyset)= \emptyset$. Thus X is sgp-T_2space. Hence $X \notin \boldsymbol{\mathcal{P}}$

3.8. Proposition

Let X be a closed almost contra sgp-absolutely double cleavable spaces over a class of ultra normal space $\mathcal{P}$, then X is sgp–normal space

Proof:

Suppose F_1, F_1 be two disjoint closed sub sets of X, then there exists an injective closed contra sgp-continuous mapping f:X⟶ Y such that $f^{-1}f(F_1)$=F_1, $f^{-1}f(F_2)$=F_2 Since f is a closed almost contra sgp injective, then $f(F_1)$, $f(F_2)$ are two disjoint closed sets of Y ,since Y is ultra normal, so there exist two clopen sets U, V such that $f(F_1) \subset U$, $f(F_2) \subset V$, $U \cap V = \emptyset$, since f is surjective, then $f^{-1}f(F_1) \subset f^{-1}(U)$, $f^{-1}f(F_2) \subset f^{-1}(V)$ since f is contra sgp–continuous ,then$F_1 \subset f^{-1}(U)$. This implies that $F_2 \subset f^{-1}(V)$ and $f^{-1}(U), f^{-1}(V)$, are sgp-open sets of X, $f^{-1}(U) \cap f^{-1}(V)$ =$f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$. Thus X is sgp-normal. Hence $X \notin \boldsymbol{\mathcal{P}}$.

3.9. Proposition

Let X be sgp-connected almost contra sgp-absolutely cleavable space over a class of spaces Y, then Y is connected space.

Proof:

Suppose Y is not connected space, then Y=U∪ V, where U, V are disjoint non empty open sets of Y, so U and V are clopen sets of Y, then there exists an injective almost contra-sgp-continuous mapping f: $X \to Y$, such that $f^{-1}f\{f^{-1}(U)\}=f^{-1}(U)$, $f^{-1}f\{f^{-1}(V)\}=f^{-1}(V)$ since $Y=U \cup V$, then $f^{-1}(Y)=f^{-1}(U \cup V) \Longrightarrow X=f^{-1}(U) \cup f^{-1}(V)$.
Since f is a almost contra sgp-continuous function, then f^{-1}(U),f^{-1}(V) are non-empty disjoint sgp-open sets in X, which contradicts that X

is sgp-connected. Therefore, Y is connected space sgp-open sets in X, which contradicts that X is sgp–connected. Therefore, Y is connected space.

3.10. Definition (Hanif and Patil, 2016)

A topological space X is said to be sgp-ultra-connected if every two non-empty sgp-closed subsets of X intersect.

3.11. Proposition

Let X be sgp-ultra connected almost contra sgp– cleavable space over a class of spaces Y, then Y is hyper connected space.

Proof:

Suppose Y is not hyper connected space, then there exists an open set V such that V is not dense in Y, and there exists an injective almost contra-sgp-continuous mapping f: $X \to Y$, such that:

$f^{-1}f\{f^{-1}(V)\}=f^{-1}(V)$. Therefore, there exist nonempty regular open subsets G = Int(Cl(V)) and H=Y-Cl(V) in Y, since f is a almost contra sgp-continuous function, then f^{-1}(G), f^{-1}(G) are non-empty disjoint sgp -closed sets in X, which contradicts that X is sgp–ultra-connected. Therefore, Y is hyper connected space.

3.12. Definition

A space X is said to be

(i) Countably sgp-compact (Dlaska, Ergun and Ganster, 1994) if every countable cover of X by sgp-open sets has a finite subcover.

(ii) sgp-Lindelöf (Maio, 1984) if every sgp-open cover of X has a countable subcover.

(iii) S-Lindelöf (Maio, 1984) if every cover of X by regular closed sets has a countable subcover.

(iv) Countably S-closed (Dlaska, Ergun and Ganster, 1994) if every countable cover of X by regular closed sets has a finite subcover.

(v) S-closed (Staum, 1974) if every regular closed cover of X has a finite subcover.

3.13. Proposition

Let X be an almost contra countably–sgp-compact and cleavable-closed space over a class of spaces Y, then Y is countably S-closed space.

Proof:

Suppose $\{V_i\}_{i\in I}$ be any countable regular closed cover of Y, since X is Countably sgp- compact an almost contra sgp-closed cleavable, so there exists an almost contra sgp-continuous mapping f:$X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f\{f^{-1}\{\boldsymbol{V_i}\}_{i\in I}\} = f^{-1}\{\boldsymbol{V_i}\}_{i\in I}$, since f is an almost contra sgp-continuous , then $f^{-1}\{\boldsymbol{V_i}\}_{i\in I}$ is countable-sgp-open cover of X. But X is Countably-sgp-compact, so there exists a finite sub cover $\{f^{-1}\{\boldsymbol{V_i}\}_{i\in I}, \ldots, f^{-1}\ \boldsymbol{V_n}\}$ of X, such that:

X $\subset \bigcup_{i=1}^{n}$ $\{f^{-1}(\boldsymbol{V_i})\}$, since $ff^{-1}\boldsymbol{V_i}= \boldsymbol{V_i}$, So $\{\boldsymbol{V_1}, \ldots, \boldsymbol{V_n}\}$ is a finite subcover of Y. Therefore Y is countably S-closed.

3.7 Proposition

Let X be sgp–Lindelöf-space is an almost contra sgp-cleavable space over a class of spaces $\mathcal{P}$, then Y is S-Lindelöf space.

Proof:

The proof is as in the proof of proposition 3-6.

4-Conclusion

In this paper, we have studied and proved these cases:

(1) If $\mathcal{P}$ is a class of weakly Hausdorff or (Ultra Hausdorff) spaces with certain properties and if X is an almost contra sgp-pointwise cleavable over $\mathcal{P}$, then X is sgp -T_1, (sgp-T_2space)–space respectively , also if $\mathcal{P}$ is a class of ultra-normal spaces with certain properties and if X is a closed almost contra sgp-absolutely double cleavable over $\mathcal{P}$, then X is sgp-normal space.

(2) If X be a sgp-connected spaces, and X is an almost contra sgp-absolutely cleavable over a class of spaces Y, then Y is countably S-closed space.

(3) If X be a sgp connected (sgp-ultra connected, countably sgp–compact, sgp–Lindelöf) spaces, and X is almost contra sgp cleavable over a class of spaces Y, then Y is connected (hyper connected, countably S-closed, S-Lindelöf)–space-respectively.

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