Libyan Journal of Science & Technology 14:1 (2022) 44-46

Faculty of Science - University of Benghazi

Libyan Journal of Science & Technology

journal home page: www.sc.uob.edu.ly/pages/page/77

Particle Diffusion in Toroidal Fusion Systems (Tokamak).

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Highlights

- **In the cylindrically symmetric configuration, the diffusion process is termed classical transport.**
- **The use of toroidal configuration leads to an enhanced level of transport known as neoclassical.**
- **Diffusion occurring in highly collisional plasma is known as Pfirsch-Schlutre diffusion.**

A R T I C L E I N F O

Article history:

Received 26 November 2021
Revised 22 March 2022
Accepted 24 March 2022

Keywords:

Diffusion, Neoclassical, Trapped particle..

A B S T R A C T

The plasma transports in two different geometrical configurations are studied, wherein in a system having a cylindrically symmetric shape, the diffusion process is termed classical transport. The use of toroidal configuration leads to an enhanced level of transport known as neoclassical. The plasma collisionality produces different forms of diffusion. The fluid-like diffusion occurring in highly collisional plasma is known as Pfirsch-Schlutre diffusion. In low collisionality plasma, the trapped particle leads to the so-called banana diffusion. The ion thermal diffusivity χ_i exceeds the electron thermal diffusivity χ_e by a factor$\sim\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}$, which is a landmark of this collisional diffusion process. In this paper, the classical diffusion coefficient with random walk step is covered, and the neoclassical transport and the trapped particle diffusion are studied.

1. Introduction

The thermonuclear condition in tokamak can be achieved if the plasma energy is confined to a sufficient amount of time. We defined the global energy confinement time as $\tau_E = \frac{3}{2} n \frac{(T_e+T_i)}{P}$, where n is the plasma density, P the total input power, T_e and T_i are the electrons and the ions temperatures respectively. Thermal conduction, convection, and radiation processes may hinder this confinement. For plasma consisting of a set of nested toroidal magnetic surfaces in tokamak, there is an irreducible loss rate resulting from Coulomb collisions (Boozer, 2004). In a system having a cylindrically symmetric configuration, this diffusion process is termed classical transport. The use of toroidal configuration leads to an enhanced level of transport known as neoclassical. The plasma collisionality produces different forms of diffusion. The fluid-like diffusion occurring in highly collisional plasma is known as Pfirsch-Schlutre diffusion. In low collisionality plasma, the trapped particle leads to the so-called banana diffusion. The ion thermal diffusivity χ_i exceeds the electron thermal diffusivity χ_e by a factor $\sim\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}$ is a landmark of this collisional diffusion process. The leading collisional transport theories predict that the energy loss is mainly determined by the ion confinement time $\tau_{Ei}\sim\frac{a^2}{\chi_i}$, where a is the plasma minor radius. Detailed experimental analysis using transport codes shows that ion thermal transport is similar to the neoclassical value; however that of electron thermal losses exceed the neoclassical prediction by up to two orders of magnitude. Line radiation from impurities can cause serious energy loss, and there is an irreducible component arising from bremsstrahlung and electron cyclotron emission also. When particle transport appears to be substantially higher, these enhanced losses are termed anomalous transport. In order to understand the cause of this anomalous transport, there are several potential explanations, generally connected with the presence of instabilities (Woods, 1987).

If the instabilities preserve the topology of the magnetic surfaces, the enhanced loss can arise from $\vec{E} \times \vec{B}$ due to fluctuating electric fields (Hazeltine, 1992). Instabilities involving magnetic perturbations can modify the magnetic field structure. Thus, tearing modes produce magnetic islands and the rapid transport of energy along the distorted magnetic field produces an enhanced radial transport. For magnetic islands coming to vary close to each other the magnetic field lines become ergodic, then rapid transport along the magnetic field lines, which themselves diffuse in space, provides a further loss mechanism. Given these substantial possibilities for the occurrence of anomalous transport and the technical difficulties in calculating their consequences, therefore it goes without saying that no convincing theoretical model has been found. Thus, much reliance has therefore been placed on empirical scaling laws for the confinement time in order to estimate the parameters required for a tokamak reactor. In this work classical diffusion coefficient with random walk step is covered, the neoclassical transport is given and the trapped particle diffusion is also shown.

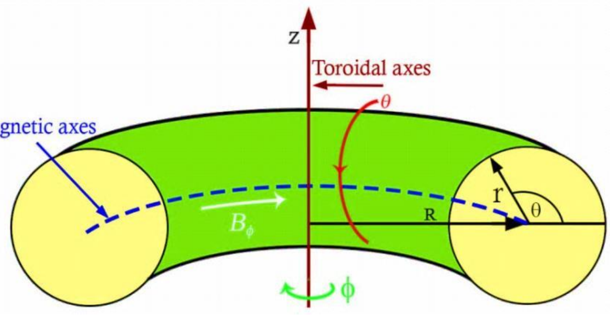

Fig.1. Toroidal tokamak coordinates

 ISSN: Online 2663-1407, Print 2663-1393; National Library of Libya, Legal number: 390/2018

2. Anomalous Transport

Given the substantial possibilities for the occurrence of anomalous transport, they can be classified as;

2.1 Classical transport

Consider plasma with cylindrical configuration and uniform magnetic field $\vec{B}$, if the velocity of the moving particle is υ during the collision time τ and using the random walk step size during this time to be Δx. Then the particle would move a distance $\Delta x = \upsilon\tau$, hence the parallel classical diffusion coefficient is:

$$D_{\parallel}^{C} = \frac{\langle(\Delta x)^2\rangle}{\tau} = \frac{\langle(\upsilon\tau)^2\rangle}{\tau} = \upsilon_e^2\tau \tag{1}$$

where this is a classical coefficient parallel to the magnetic unit vector $\hat{b}$, for the diffusion across the magnetic field the two coefficients are related as:

$$D_{\perp}^{C} \approx \frac{D_{\parallel}^{C}}{(\omega_{ce}\tau)^2} \simeq \frac{\upsilon_e^2}{\omega_{ce}^2} \approx \frac{\upsilon_e^2}{\omega_{ce}^2}\nu_e \tag{2}$$

where υ_e is the electron thermal velocity, ω_{ce} is the electron cyclotron frequency and ν_e is the electron collision frequency. When $\omega_{ce} = \frac{eB}{m_e}$, the diffusion coefficient $D_{\perp}^{C}$ scales as $\frac{1}{B^2}$.

2.2 Neoclassical transport

The use of toroidal configuration leads to an enhanced level of transport known as neoclassical, where the magnetic field $\vec{B}$ is nonuniform, see Fig. 1. The drift velocity in the absence of the electric field $\vec{E}$ and in nonuniform $\vec{B}$ is given by the following contributions (Alhasi, 2021);

$$\upsilon_D^{\text{Neo}} = m_e\frac{(\vec{\upsilon}_{\parallel i} - \vec{\upsilon}_{\parallel e})}{\tau_c} \times \left(\frac{\vec{B}}{eB^2}\right) + \frac{m\upsilon_{\perp}^2}{2eB^3}\vec{B}\times\vec{\nabla}B + \frac{m\upsilon_{\parallel}^2}{eB^4}\vec{B} \times (\vec{B}\cdot\vec{\nabla})\vec{B} \tag{3}$$

Consider the first term of Eq. (3) due to the plasma current to obtain the particle flux as;

$$\Gamma_p = nm_e\frac{(\vec{\upsilon}_{\parallel i} - \vec{\upsilon}_{\parallel e})}{\tau_e} \times \frac{\vec{B}}{eB^2} = -\frac{m_e}{\tau_e}\frac{1}{e^2B^2}\vec{j}\times\vec{B} \tag{4}$$

where n is the plasma density. In the case of isothermal equilibrium plasma the electromagnetic forces balance the gradient thermal forces to have;

$$\vec{j}\times\vec{B} = \vec{\nabla}p = T_e\vec{\nabla}n \tag{5}$$

Eq. (4) can be written in this form;

$$\Gamma_p = \left(-\frac{m_e}{\tau_e}\frac{T_e}{e^2B^2}\right)\vec{\nabla}n = -D_{\perp}^{C}\vec{\nabla}n \tag{6}$$

Eq. (6) is just the cross-field diffusion coefficient obtained in the classical case of Eq. (2). Thus the effect of tokamak geometry is included in the second and third terms of Eq. (3), These two terms when they are combined give the diffusion velocity drift due to the geometry effect which can be written (Bittencourt, 2004) as;

$$\upsilon_D^{geo} = \frac{\upsilon_e^2}{\omega_{ce}R} \tag{7}$$

where R is the machine major radius, $\omega_{ce} = \frac{eB_\phi}{m_e}$, and B_ϕ is the toroidal magnetic field.

2.2.1 Neoclassical transport of transient particles

In tokamak geometry the factor qR is defined as the maximum distance over which the magnetic curvature doe not change sign, $q \equiv \frac{r}{R}\frac{B_\phi}{B_\theta}$ is the safety factor and $\varepsilon \equiv \frac{r}{R}$ is the inverse aspect ratio, see Fig. 1. The diffusion velocity in Eq. (7) has two components, thus the diffusion coefficients are fast $D_{\parallel}$ along $\vec{B}$ and a slower $D_{\perp}$cross $\vec{B}$. Using the random walk $\Delta x = qR$, the time t taken to diffuse a distance qR along $\vec{B}$ is;

$$t \approx \frac{(qR)^2}{D_{\parallel}} = \frac{(qR)^2}{\upsilon_e^2\tau_e} \tag{8}$$

where the definition of $D_{\parallel}$ is taken from Eq. (1). Using Eq. (7) and Eq. (8) to estimate the maximum distance Δx cross $\vec{B}$ moved by the transit particle to have;

$$\Delta x \sim \upsilon_D^{geo}t = \left(\frac{\upsilon_e^2}{\omega_{ce}R}\right)\left(\frac{q^2R^2}{D_{\parallel}}\right) = q^2\frac{R}{\omega_{ce}\tau_e} \tag{9}$$

Again using the random walk technique and the help of Eq. (8) and Eq. (9) to estimate the diffusion coefficient along $\vec{B}$ as;

$$D_{\parallel}^{geo} = \frac{(\Delta x)^2}{t} = \frac{(q^2R/\omega_{ce}\tau_e)^2}{(qR/\upsilon_e^2\tau_e)^2} = q^2\left[\frac{\upsilon_e^2\tau_e}{(\omega_{ce}\tau_e)^2}\right] = q^2D_{\perp}^{C} \tag{10}$$

This is just the cross $\vec{B}$ coefficient enhanced by the geometrical factor q^2. If we now add the contribution of Eq. (2) to Eq. (10) we get;

$$D_{Neo}^{geo} = D_{\perp}^{C} + q^2D_{\perp}^{C} = (1+q^2)D_{\perp}^{C} \tag{11}$$

This is the so-called Pfirsch-Schluter diffusion factor (Woods, 1987).

2.2.2 Neoclassical transport of trapped particles

The resulting mean distance for trapped electrons in a banana tokamak islands (Hazeltine, 1992) is;

$$\delta r = \frac{B_\phi\varepsilon^{\frac{1}{2}}}{B_\theta\omega_{ce}}\upsilon_e = \frac{qr_{Le}}{\varepsilon^{\frac{1}{2}}} \tag{12}$$

where r_{Le} is the electron Larmor radius. The diffusion coefficient with δr as a random walk step is

$$D_{Bana}^{geo} \approx \varepsilon^{\frac{1}{2}}\frac{(\delta r)^2}{\tau_{eff}} = \frac{\varepsilon^{\frac{1}{2}}}{\varepsilon\tau_e}\left(qr_{Le}/\varepsilon^{\frac{1}{2}}\right)^2 = \varepsilon^{-\frac{3}{2}}q^2D_{\perp}^{C} \tag{13}$$

where $\tau_{eff} = \varepsilon\tau_e$. The total neoclassical diffusion coefficient due to all types of particles (tapped and transient) is, see Fig. 2.

$$D_{Neo} = \left(1 + q^2 + \varepsilon^{-\frac{3}{2}}q^2\right)D_{\perp}^{C} \tag{14}$$

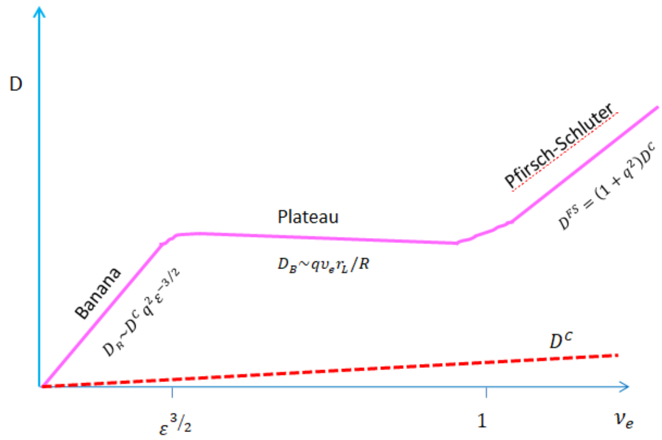

Fig.3. Transport diffusion coefficients

3. Thermal Heat Diffusivity Coefficients

Just as the gradient of the particle density defined the coefficients of diffusion, also the gradient of the particle temperature defined it's thermal diffusivities. The thermal heat diffusivity χ_e is related to the coefficient of diffusion as $\chi_e \sim D$ for electrons. The ion thermal diffusivity coefficient χ_i is much larger than χ_e by factor of $\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}$; implies

$$\chi_i \sim \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \chi_e \tag{15}$$

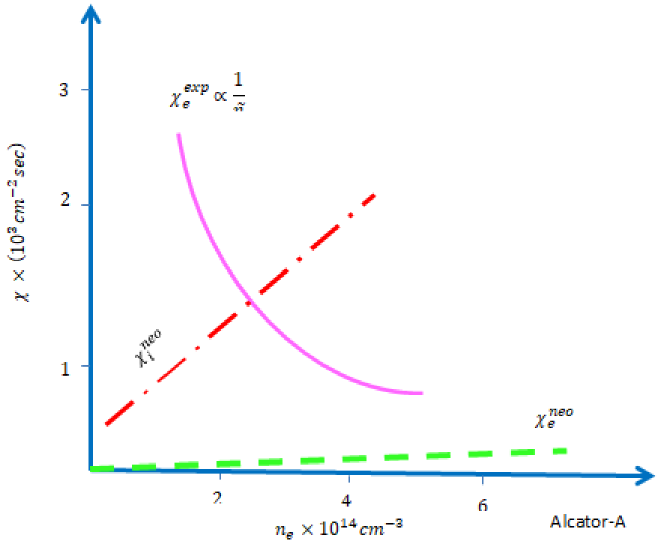

Fig. 3. Alcator (A) experimental results

The experimental measurement of χ_i agrees well with the neoclassical transport calculations. The case for electrons is very different; the disagreement between experiment and neoclassical theory calculations for electron χ_e is huge. There must be some other factors, which might contribute to χ_e , this type of transport is termed anomalous diffusion. Several theories are used to explain this anomaly:

(i) Drift wave transport due to electrostatic fluctuation gives a particle flux as:

$\Gamma_{wave} \sim -\frac{\gamma}{2}\left(\frac{k_\perp \phi}{\omega B}\right)^2 \vec{\nabla} n$, with wave diffusion coefficient, $D_{wave} = \frac{\gamma}{2}\left(\frac{k_\perp \phi}{\omega B}\right)^2$, Where, ϕ the electro-static potential, ω the wave frequency, $k_\perp$ the normal wave vector and γ the wave growth factor.

(ii) Magnetic fluctuation leading to the creation and distraction of magnetic islands such as tearing mode instabilities.

(iii) Magnetic ripples transport. Even this contribution is not enough to bring theory and experiment into an agreement. Therefore, scaling laws are required to explain the experimental results, such as M.I.T. Alcator tokamak scaling law: $\chi_e = 5 \times 10^{17} \frac{1}{n}$ sec cm^{-2}, the Alcator experimental results are shown in Fig. 3

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