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Using Markov Chains to Predicate Pandemic Trend: A Case Study in Libya for COVID–19.

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Highlights

- **This research, discusses using mathematical models to predict the long-term trends of pandemics.**
- **In this study, the stationary Markov chain is applied to predict the status of the COVID-19 pandemic in Libya.**
- **The data used (from WHO) and the results showed that the chain was convergent (the limiting probability being very close to the initial distribution).**
- **The study results show that the probability of staying in a good situation is 70.9%, and the probability of becoming worse is 29.1%.**

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ABSTRACT

Many predictive models have been developed by various academic institutions to support health systems in strategic decision-making, planning, and policies that help in the challenge against COVID-19. These models are useful in determining, the expected number of cases and deaths due to COVID–19, as well as the required resources such as hospital beds for isolation period and ICU, and necessary supplies such as protective equipment. In this article, the stationary Markov Chain is applied to the Libyan population to predict the status of the pandemic in Libya after more than four years of its spread. The data used was collected by WHO, and the results showed that the chain had converged due to the large sample size taken, resulting in the limiting probability being very close to the initial distribution. Additionally, the probability of staying in a good situation is 70.9% and to become worse is 29.1%. Finally, due to the convergence of the chain, these results will remain the same regardless of the initial state of the chain.

1. Introduction

One of the most serious problems resulting from changes in weather patterns is disease spread. According to the WHO climate change has exacerbated over 200 infectious diseases such as COVID-19, Lyme disease and various fungal afflictions. Despite the severity of climate change's impact on life on Earth, the steps taken by the world, especially the industrialized countries, are very modest and do not meet the level of imminent danger. On the other hand, scientists in various fields have begun working on preparing ways to confront the dangers resulting from these rapid changes.

One of these efforts, in the health field, aims to understand the behaviour of viruses that cause epidemics after they begin to spread (not to mention working on finding vaccines resistant to them). In this research, the Markov chains will be used to identify the behaviour of the spread of the epidemic, specifically the Corona epidemic that swept the world at the end of 2019 and is still among us now.

The methods used up to now can be classified into three categories: statistics-based method, deep learning method, and machine learning method (Ma *et al*., 2021). However, researchers in this field tend to prefer mathematical models due to their simplicity and ability to predict the long-term trends of a pandemic.

Aldila *et al*., (2018) studied the MERS – CoV and introduced a SIR model, while Wang *et al*., (2016) used a stationary Markov chain for optimizing combinatory drugs, and their algorithm showed better performance compared to two other stochastic algorithms in terms of reliability and efficiency. Another application to the Markov chain was by Szczepanik and Mrozek (2013) to estimate the electron conditions in atomic orbitals. LSTM-Markov was the model introduced by Ma, *et al*., (2021), where they used the Markov model to minimise the prediction error of LSTM model. Depending on confirmed records in Russia, Brazil, the US and Britain, they determined the training errors of LSTM and created the PTM of the Markov model based on these errors. Then, by combining the results of LSTM with the errors in the prediction of the Markov Model, the results were obtained. In this article, we aim to predict the long-term spread of COVID-19 in Libya by using a stationary Markov chain, with its probability transition matrix (PTM) associated with data collected by WHO from the first case in March 2019 up to the 23rd of November 2023.

2. Stochastic Process

Definition: A stochastic process is a collection of random variables that is indexed by some mathematical set $\{X_t\}$. The state space $\{X_t\}$ will be denoted by S. If S is countable then it is called a discrete state process.

2.1 Types of Stochastic Processes

The following are the most important types of stochastic processes:

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1. Independent stochastic sequence.
2. Renewal process.
3. Independent increment process.
4. Markov process.
5. Martingale process.
6. Stationary process.
7. Point processes.

2.2 Discrete Time Markov Chain

Definition: as stated by Cassady and Nachlas, (2009, p.137), "If {$X(t)$, t = 0, 1, ... } is a discrete-valued stochastic process having a countable state space K, then {$X(t)$, t = 0, 1, ... } is said to be a discrete-time Markov chain if and only if for all {i_0, i_1, ... , i_{t-1}, i, j} ⊆ K and for all t = 0, 1, ...,

$$Pr\left(X(t+1)=j\,\middle|\,X(t)=i, X(t-1)=i_{t-1},\dots,X(1)=i_1,X(0)=i_0\right)$$

$$=Pr\left(X(t+1)=j\,\middle|\,X(t)=i\right)=P_{ij} \quad (1)$$

This is known as the Markov property. Also, P_{ij} is referred to as the probability value of transition from state i to state j. Colloquially, the property (Markov property) can be stated as Given the present, the future is independent of the past". Let us suppose that the Markov chain (discrete-time) has three state spaces {1, 2, 3} then, the PTM is

$$P=\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix} \quad (2)$$

Where,

$$P_{ij}\geq 0 \ and \sum P_{ij}=1$$

Assume {X(t), t = 0, 1, ... } be a discrete-time Markov chain with S state space and PTM P. A class of states is a subset of S if and only if:

(1) All states in the subset communicate with one another and;

(2) No state not in the subset communicates with any state in the subset.

Also, the discrete-time Markov chain is said to be ***irreducible*** if it has only one class. Otherwise, the chain is ***reducible***.

2.3 Limiting Behavior

If a discrete-time Markov chain is irreducible and has a finite state space, then as the number of transitions increases, the initial state of the process becomes irrelevant. This can be summarized by saying that the process approaches steady-state (stationary) behavior (Cassady and Nachlas, 2009, p.149).

Let {X(t), t = 0, 1, ... } be a Markov chain with discrete-time state space S and TPM P. Let π_j denote the stationary (or limiting) probability of state j where:

$$\pi_j=\lim_{n\to\infty}P_{ij}^{(n)} \quad (3)$$

For all $i\in S, j\in S$, if

$$\pi=[\pi_0 \quad \pi_1 \quad \dots]$$

Then, the vector π is referred to as the limiting probability. Equivalently, the elements of vector π can be defined as follows:

$$\pi_j=\lim_{n\to\infty}v_j^{(n)} \quad (4)$$

As implied by the definition, we use the TPM to compute the limiting probabilities.

Let {X(t), t = 0, 1, ... } be a Markov chain with discrete-time state space S and TPM P. The vector π is the unique non-negative solution to the set of linear equations

$$\pi_j=\sum_{i\in S}\pi_i P_{ij} \quad (5)$$

For all $j\in S$ and

$$\sum_{j\in S}\pi_i=1 \quad (6)$$

The previous set of equations (Eq.(3, 4, 5 & 6)) can be representing by the following matrix:

$$\pi=\pi P \quad (7)$$

3. The case study

In this study, based on the data from WHO regarding the infection status in Libya starting from the first detection of Covid-19 in March 2019 up to November 23th , 2023, we will analyze the status of the pandemic after more than four years. A sample of the data used is shown in Table 1.

3.1 Markov Model

To design the Markov chain and define its states, we found that there are only three possible cases for the daily infection status:

1. The number of cases today is greater than the number of cases yesterday (increased).

2. The number of cases today is less than the number of cases yesterday (decreased).

3. The number of cases today is equal to the number of cases yesterday (stable).

Therefore, the possible state space for the Markov model (Fig. 1) includes the following:

- *the daily cases of the pandemic have decreased from the previous day (denoted by N_0),*
- *the daily cases of pandemic are the same as the previous day (denoted by N_1),*
- *the daily cases of pandemic have increased from the previous day (denoted by N_2).*

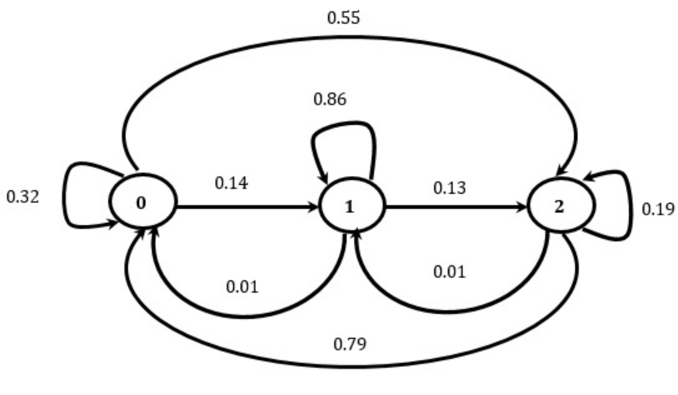

Fig. 1. Markov chain for the considered case

From analyzing the available data (containing 1399 daily figures), the following totals are observed for each status:

N_0: the number of occurrences of state 0 = 477

N_1: the number of occurrences of state 1 = 515

N_2: the number of occurrences of state 2 = 407

Based on the data provided in Table 1 and the observed daily fluctuation, Table 2 shows the number of transitions between the different states in the chain at the front of each row of the data sample in Table 1.

Table 1

Sample of the used data (Daily cases of Cov-19 in Libya from Dec. 23rd, 2021 to Jun 20th, 2022)

Date	12/23/2021		12/24/2021	12/25/2021	12/26/2021	12/27/2021	12/28/2021	12/29/2021	12/30/2021	12/31/2021	1/1/2022	1/2/2022	1/3/2022	1/4/2022	1/5/2022	1/6/2022
New Cases	**561**		**560**	**658**	**0**	**735**	**881**	**599**	**665**	**640**	**551**	**0**	**916**	**634**	**651**	**698**
Date	1/7/2022		1/8/2022	1/9/2022	1/10/2022	1/11/2022	1/12/2022	1/13/2022	1/14/2022	1/15/2022	1/16/2022	1/17/2022	1/18/2022	1/19/2022	1/20/2022	1/21/2022
New Cases	**643**		**592**	**0**	**579**	**536**	**487**	**599**	**618**	**765**	**0**	**867**	**736**	**885**	**1173**	**1331**
Date	1/22/2022		1/23/2022	1/24/2022	1/25/2022	1/26/2022	1/27/2022	1/28/2022	1/29/2022	1/30/2022	1/31/2022	2/1/2022	2/2/2022	2/3/2022	2/4/2022	2/5/2022
New Cases	**1700**		**0**	**2281**	**2333**	**3063**	**2245**	**3157**	**3320**	**0**	**5694**	**4429**	**4266**	**4371**	**3656**	**3917**
Date	2/6/2022		2/7/2022	2/8/2022	2/9/2022	2/10/2022	2/11/2022	2/12/2022	2/13/2022	2/14/2022	2/15/2022	2/16/2022	2/17/2022	2/18/2022	2/19/2022	2/20/2022
New Cases	**0**		**4242**	**2832**	**3326**	**3272**	**3773**	**3345**	**0**	**3648**	**2800**	**2490**	**2884**	**2457**	**1208**	**0**
Date	2/21/2022		2/22/2022	2/23/2022	2/24/2022	2/25/2022	2/26/2022	2/27/2022	2/28/2022	3/1/2022	3/2/2022	3/3/2022	3/4/2022	3/5/2022	3/6/2022	3/7/2022
New Cases	**2292**		**2307**	**1815**	**1373**	**1276**	**938**	**0**	**1394**	**898**	**669**	**857**	**806**	**501**	**0**	**679**
Date	3/8/2022		3/9/2022	3/10/2022	3/11/2022	3/12/2022	3/13/2022	3/14/2022	3/15/2022	3/16/2022	3/17/2022	3/18/2022	3/19/2022	3/20/2022	3/21/2022	3/22/2022
New Cases	**495**		**386**	**293**	**333**	**302**	**0**	**233**	**304**	**193**	**129**	**134**	**104**	**0**	**151**	**120**
Date	3/23/2022		3/24/2022	3/25/2022	3/26/2022	3/27/2022	3/28/2022	3/29/2022	3/30/2022	3/31/2022	4/1/2022	4/2/2022	4/3/2022	4/4/2022	4/5/2022	4/6/2022
New Cases	**76**		**85**	**83**	**41**	**0**	**75**	**64**	**51**	**47**	**48**	**33**	**0**	**0**	**0**	**0**
Date	4/7/2022		4/8/2022	4/9/2022	4/10/2022	4/11/2022	4/12/2022	4/13/2022	4/14/2022	4/15/2022	4/16/2022	4/17/2022	4/18/2022	4/19/2022	4/20/2022	4/21/2022
New Cases	**0**		**0**	**96**	**0**	**0**	**0**	**0**	**0**	**0**	**28**	**0**	**0**	**0**	**0**	**0**
Date	4/22/2022		4/23/2022	4/24/2022	4/25/2022	4/26/2022	4/27/2022	4/28/2022	4/29/2022	4/30/2022	5/1/2022	5/2/2022	5/3/2022	5/4/2022	5/5/2022	5/6/2022
New Cases	**0**		**42**	**0**	**0**	**0**	**0**	**0**	**0**	**12**	**0**	**0**	**0**	**0**	**0**	**0**
Date	5/7/2022		5/8/2022	5/9/2022	5/10/2022	5/11/2022	5/12/2022	5/13/2022	5/14/2022	5/15/2022	5/16/2022	5/17/2022	5/18/2022	5/19/2022	5/20/2022	5/21/2022
New Cases	**3**		**0**	**0**	**0**	**0**	**0**	**0**	**35**	**0**	**0**	**0**	**0**	**0**	**0**	**33**
Date	5/22/2022		5/23/2022	5/24/2022	5/25/2022	5/26/2022	5/27/2022	5/28/2022	5/29/2022	5/30/2022	5/31/2022	6/1/2022	6/2/2022	6/3/2022	6/4/2022	6/5/2022
New Cases	**0**		**0**	**0**	**0**	**0**	**0**	**29**	**0**	**0**	**0**	**0**	**0**	**0**	**24**	**0**
Date	6/6/2022		6/7/2022	6/8/2022	6/9/2022	6/10/2022	6/11/2022	6/12/2022	6/13/2022	6/14/2022	6/15/2022	6/16/2022	6/17/2022	6/18/2022	6/19/2022	6/20/2022
New Cases	**0**		**0**	**0**	**0**	**0**	**36**	**0**	**0**	**0**	**0**	**0**	**0**	**34**	**0**	**0**

Table 2

The state of transitions in front of each row of sample dates in Table 1

From → to	0 → 0	0 → 1	0 → 2	1 → 0	1 → 1	1 → 2	2 → 0	2 → 1	2 → 2
No. of transitions	2		6				7		
	4		5				5		1
	2		6				5		2
	3		4				4		4
	1		5				5		4
	4		5				6		
	7		4				3		1
	5		4				5		1
	6	1	3		2		3		
		2			9	2	2		
		2			9	2	2		
		2			8	3	2		
		2			8	2	3		
		3			8	2	2		
		2			9	2	2		

The overall transition status for all the data is shown in Table 3.

Table 3

Total number of transitions between the different states

From → to	0 → 0	0 → 1	0 → 2	1 → 0	1 → 1	1 → 2	2 → 0	2 → 1	2 → 2
# of transitions	151	66	260	5	443	67	322	6	79
Total transitions from each state	477			515			407		

Based on the results in Table 3, the transition probability p_{ij} and the probability transition matrix (Eq.(2)) are as follows:

$$P = \begin{bmatrix} 0.32 & 0.14 & 0.55 \\ 0.01 & 0.86 & 0.13 \\ 0.79 & 0.01 & 0.19 \end{bmatrix}$$

From Fig. 1 and the PTM P shown above, it is clear that the considered chain is irreducible because it has only one class. Therefore, the chain is stationary, and the steady state probability exists. However, because the stationarity of the chain is a condition that must be met to lay on the study results, the reader can prove the irreducibility of the chain (Medhi J., 2009, p.80)

3.2 Steady state probability (π_0, π_1, π_2)

The stationary distribution Markov chain for daily cases of Covid-19 (Eq.(7)) is:

$$(\pi_0\ \pi_1\ \pi_2) = (\pi_0\ \pi_1\ \pi_2)\begin{bmatrix} 0.32 & 0.14 & 0.55 \\ 0.01 & 0.86 & 0.13 \\ 0.79 & 0.01 & 0.19 \end{bmatrix}$$

Then,

$$\pi_0 = 0.32\pi_0 + 0.01\pi_1 + 0.79\pi_2$$

$$\pi_1 = 0.14\pi_0 + 0.86\pi_1 + 0.01\pi_2$$

$$\pi_2 = 0.55\pi_0 + 0.13\pi_1 + 0.19\pi_2$$

and from Eq.(6);

$$\pi_0 + \pi_1 + \pi_2 = 1$$

Mathematically, it is clear that any one of the first three equations is redundant. By solving any two of them with equation three, we obtain:

$$(\pi_0\ \pi_1\ \pi_2) = (0.344\ , 0.365\ , 0.291)$$

So, from the previous calculations, we found that the probability of increasing the infliction rate, denoted as π_2 is 0.291 (29.1%), the probability of decreasing π_0 is 0.344 (34.4%), and the probability of remaining stable, denoted as π_1 is 0.365 (36.5%).

3.3 Initial Probability of the Chain states

The probability that the chain will now be at any one of the given statuses (initial probability) can be determined depending on the data in Table 4 as follows:

Table 4

Probability of transition from state ***i*** to status ***j*** in the chain

$0 \to 0$	$0 \to 1$	$0 \to 2$	$1 \to 0$	$1 \to 1$	$1 \to 2$	$2 \to 0$	$2 \to 1$	$2 \to 2$	Total
0.1080	0.0472	0.1860	0.0029	0.3169	0.0479	0.2303	0.0043	0.0565	1

Then, the probability that the states are now at status 0, 1, or 2 is as in Table 5:

Table 5

Initial Probability

In "0" (0→0 + 1→0 + 2→0)	In "1" (0→1 + 1→1 + 2→1)	In "2" (0→2 + 1→2 + 2→2)
0.341	0.368	0.290

The reader could note the convergence in the values between the limited probability and the initial probability. As we mentioned earlier, a regular Markov chain with transition matrix P has a unique stationary distribution vector π such that πP = π. In addition, it is proven that starting from any initial distribution q, if the iteration q, qP, qP^2 . . . converges, then it must converge to this unique stationary distribution. This can happen (but not necessarily) when the experiment has a large sample of data, and this is actually what has happened in the studied case. However, in this instance, the Markov Chain is denoted as "***Convergence Markov Chain***" and it must be a stationary chain.

4. Conclusion

In this article, a stationary Markov chain was introduced as a method to predict the behaviour of the COVID-19 pandemic in the long term. The proposed method was applied to the Libyan population in the period from March 2019 up to November 2023. The analyzed data showed that the probability that the virus will reactivate and increase the number of infected cases is about 29.1%. While the probability will remain in its current state (weak) or decrease further (semi-dormant) is 70.9% (stable + decreases). Additionally, the study showed that the Markov chain converged due to the long period of recorded data, making the results more confident and reliable. Moreover, the limited probability will remain the same regardless of the starting state of the chain. Finally, it is hoped that nowadays with the increases in the epidemic emergence rates, the methods of prediction and tracing their behaviour will receive more attention from researchers in different fields of sciences.

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