*ISSN 2522-6967*

# Using Least Square for Develop a New Formation Resistivity Factor Correlation and Relationship between the Salinity and the Corrosion

Neima Mazeb and Nassr AlFadil

*Petroleum Engineering Dept., Benghazi University, Benghazi, Libya*
*Corresponding author. Tel.: +218-91-813-8563; e-mail: nassr.mussa@uob.edu.ly & fatima_n1967 @yahoo.com*

**ABSTRACT**

Formation resistivity factor (Fr),introduced by Archie , is an important parameter for electric log interpretation .It is defined as the resistivity of rock fully saturated with brine (Ro) divided by the resistivity of brine (Rw). This factor is function of porosity, resistivity, tortuosity and cementation factor, which are consequently affected also by particle size and degree of compaction .Formation resistivity factor is usually determined experimentally or by using empirical approach. During the present paper, we try to develop a new formation resistivity factor (Fr) correlation based on statistical methods, Least square, by taking into account different parameter such as brine salinity, particle size and degree of compaction. In addition, this study aims to determine the relationship between salinity and corrosion.

***Keywords:*** *Formation Resistivity factor, Porosity, Tortuosity, Particle size, compaction, Salinity, Corrosion*

## 1. Introduction

Sedimentary formations are capable of transmitting an electric current only by means of the interstitial and adsorbed water they contain. They would be non-conductive if they were entirely dry.

The interstitial or connate water containing dissolved salts constitutes an electrolyte capable of conducting current, as these salts dissociate into positively charged cations, such as Na+ and Ca++ , and negatively charged anions, such as Cl- and SO4 - .

These ions move under the influence of an electrical field and carry an electrical current through the solution. The greater the salt concentration, the greater the conductivity of the connate water. The electrical resistivity (reciprocal of conductivity) of a fluid-saturated rock is its ability to impede the flow of electric current through that rock. Dry rocks exhibit infinite resistivity. The resistivity of reservoir rocks is a function of salinity of formation water, effective porosity, and quantity of hydrocarbons trapped in the pore space [1]. Relationships among these quantities indicate that the resistivity decreases with increasing porosity and increases with increasing petroleum content. Resistivity measurements are also dependent upon pore geometry, formation stress, and composition of rock, interstitial fluids, and temperature. Resistivity is, therefore, a valuable tool for evaluating the producibility of a formation. A rock that contains oil and/or gas will have a higher resistivity than the same rock completely saturated with formation water and the greater the connate water saturation, the lower the formation resistivity.

Archie defined the formation resistivity factor Fr as [2]:

$$Fr = Ro/Rw \quad (1.1)$$

Where Ro is the resistivity of a formation that is fully saturated with water, Rw is the resistivity of the water .Ro will be greater than Rw and Fr will always be greater than unity.

Figure (1.1) shows the qualitative effect of brine resistivity (assuming allother factors, such as porosity, cementation, and amount of shale remain constant) on (Fr) for limestone and clean sand, and shaly (dirty) sand. The formation factor is essentially constant for clean sand and limestone. For dirty or shaly sand, (Fr) decreases as brine resistivity, Rw, increases; and although Ro increases, it does not increase proportionately because the clay in the water acts as a conductor.

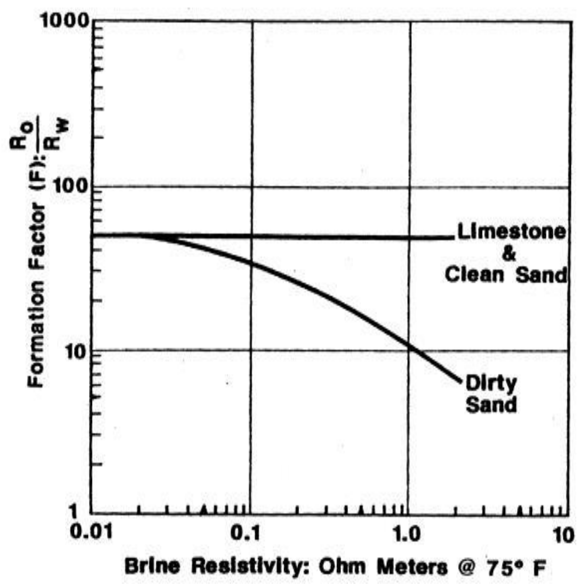


**Figure 1.1:** General relationship between formation factor Fr and brine resistivity Rw factor (Courtesy of Core Laboratories). [2]

ISSN 2522-6967

This effect is dependent upon the type, amount, and manner of distribution of the clay in the rock Equation (1.1) is an important relation in well-log interpretation for locating potential zones of hydrocarbons. Several methods for determining the reservoir water resistivity have been developed, including: chemical analysis of produced water sample, direct measurement in resistivity cell, water catalogs, spontaneous potential (SP) curve, resistivity-porosity logs, and various empirical methods.

## 2.0 Literature Review

The value of Fr is one of the most important parameters in water saturation calculations. The presence of Fr or equivalent parameters in all different formulas of water saturation calculation such as Archie, Indonesia, Popoun, etc… indicate the important role of this parameter in original oil in place estimation of a field.

### 2.1. Theoretical Formula for $F_r$

The formation resistivity factor,

$$Fr = a\, Ø^{-m} \tag{2.1}$$

has theoretical derivation in some of the early literature and textbooks on well log analysis and core analysis. Most all published derivations start with the fundamental definition of formation resistivity factor.

$$Fr = Ro / Rw \tag{2.2}$$

Where $R_o$ is the resistivity of the porous media 100% saturated with a conductive fluid and $R_w$ is the resistivity of the conductive fluid.

Each derivation requires a simplified model of the porous media using geometric shapes of pores, pore throats, and bulk volume that are easily described in terms of length and cross-sectional area for the conduction of ions through the model.

A general derivation similar to Amyx et al [3]is shown here. The definition of resistivity (R) of many materials is

$$R = rA/L \tag{2.3}$$

where, r = resistance of the material.
A = the cross-sectional area perpendicular to ionic flow.
L = Length of the ionic flow path.

Using a cube of salt water, the resistance of the cube could be defined as:

$$r_w = (R_w L)/A \tag{2.4}$$

where L and A describe the dimensions of the cube of water. A cube of porous media of the same dimension of the cube of water would have a lesser volume available for water. The matrix is assumed to be an insulator as such the portion of the cube. That can conduct ionic flow is only the pore space.

Therefore, an apparent cross-sectional area (Aa) and apparent flow path (La) are used. The resistance of the cube is

$$r_2 = R_w L_a/Aa \tag{2.5}$$

By definition the resistivity of the cube of core saturated with water is

$$R_o = r_2\, A/L \tag{2.6}$$

Substituting the last two equations yields:

$$R_o = R_w L_a\, A/AaL \tag{2.7}$$

Using this definition of $R_o$ in the $F_r$ equation result in:

$$F_r = \frac{L_a/L}{A_a/LA} \tag{2.8}$$

which is the ratio of the apparent flow path to the length of the cube compared to the ratio of the apparent cross-sectional area to the cross-sectional area of the cube. The ratio of the lengths is proportional to tortuosity and is given the symbol a, the tortuosity factor. The apparent cross-sectional area is assumed to be equal to the product of the actual area and the porosity of the porous media (ØA). Using this definition yields

$$\boldsymbol{F_r} = \frac{\boldsymbol{a}}{\boldsymbol{\emptyset}} \tag{2.9}$$

Porosity has no power as such m can be seen as one. Several attempts have been made to obtain a universal formula relating porosity, formation resistivity, and cementation factor. If an electric current is passed through a block of non-conducting porous rock saturated with a conducting fluid, only a portion of the pore space participates in the flow of electric current. Consequently, total porosity Ø can be divided into two components such that [4]:

$$\Phi = \Phi_{ch} + \Phi_{tr} \tag{2.10}$$

Where Øch and Øtr are, the flowing porosity associated with the channels and the porosity associated with the regions of stagnation (traps) in a porous rock, respectively. It seems that, Øch is corresponding to the ' effective porosity ' used by Chilingarian and Øtr is corresponding to the irreducible fluid saturation [5].Figures (2.1)and (2.2) show that the electrical current can flow only through the channel indicated by C, while no current can flow through the trap indicated by T. In Figure 2.1, the traps are of the dead-end type. The trap in figure 2.2 is called an open or symmetry trap. A universal relationship between Fr and $Ø_f$ may be written as [6]:

$$F_r = 1 + f_G\left[\frac{1}{\emptyset_{ch}} - 1\right] \tag{2,11}$$

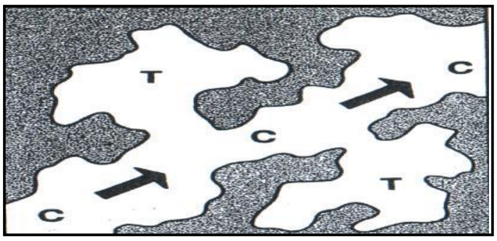


**Figure 2.1:** Portion of porous rock showing dead end trap [4]

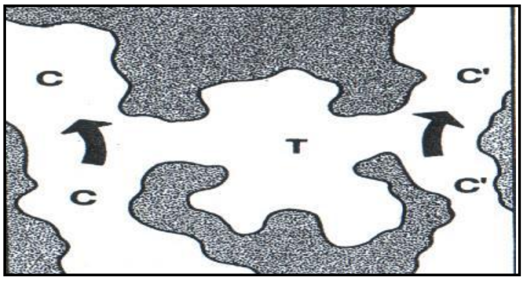


**Figure 2.2:** Portion of porous rock showing an open or symmetry trap [4]

ISSN 2522-6967

$$Ø_{ch} = Ø^{m} \quad (2.12)$$

Where $f_G$ is defined as the interior geometry parameter of the porous rock, and m ≥ 1. Combining Equations 2.11 and 2.12 gives:

$$F_r = 1 + f_G\left[\frac{1}{Ø^m} - 1\right] \quad (2,13)$$

This is the Rosales relationship between formation resistivity, porosity, and cementation factor. If $f_G$ = 1, Equation 2.13 gives Archie's formula. Equation 2.13 can be expressed as:

$$F_r = \frac{f_G}{Ø^m} + (1 - f_G) \quad (2,14)$$

The value of $f_G$ for most porous rocks is close to unity. Hence, fG/Øm>>(1-$f_G$) and Equation 2.14 can be approximated by :

$$F_r = \frac{f_G}{Ø^m} \quad (2,15)$$

This expression is the Humble formula where $\mathbf{f_G}$ = a. Thus, Archie's formula and Humble's formula are special cases of Rosales general formula. Rosales showed experimentally that, for sandstones, Equation 2.15 can be written as follows [6]:

$$F_r = 1 + 1.03\left[\frac{1}{Ø^{1.73}} - 1\right] \quad (2,16)$$

This expression was compared graphically with the Humble formula, Equation 2.17, and Timur et al. formula [7]:

$$R_{wc} = \frac{E}{I_{wc}}\frac{ØA(L/L_a)}{L} \quad (2,17)$$

$$F_r = \frac{1.13}{Ø^{1.73}} \quad (2,18)$$

Figure 2.3 is a log-log plot of Humble Equation (line A), 2.11 (line B), and 2.18 (line C). The three formulas give approximate results within the region of practical interest, i.e., 10 ≤ Ø ≤ 40. As Ø approaches unity, however, Equation (2.11) gives a curved line that satisfies the condition Fr =1 when Ø =1, whereas Humbler's formula and Timur's et al. formula (Equation 2.18) are straight lines for all values of Ø, which does not satisfy that condition.

The tortuosity is:

$$\tau = ØF_r \quad (2.19)$$

Substituting Equation 2.13 into Equation 2.19 yields a general expression for calculating tortuosity:

$$\tau = Ø\left[1 + f_G\left(\frac{1}{Ø^m} - 1\right)\right] \quad (2.20)$$

Inasmuch as the value of $f_G$ is approximately equal to unity for most porous rocks, Equation 2.20 can be written as follows:

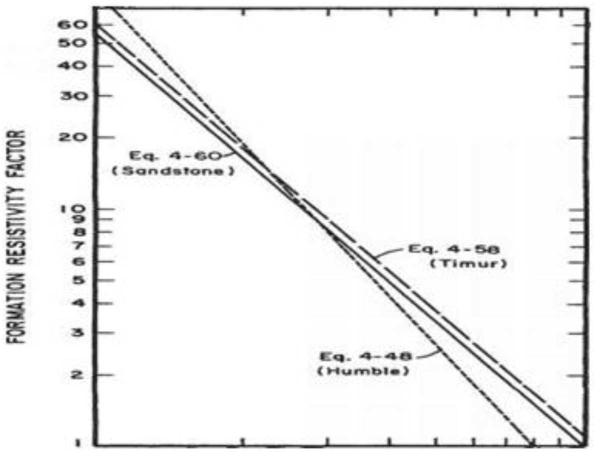


**Figure 2.3:** Graphical comparisons of Humble Equation (A), Rosales Equation (B), and Timur Equation (C) [4].

**Table 3.1:**Natural core samples (sandstone) (Attia 2005) [12]

| Core # | Compaction pressure(psi) | Porosity (fraction) | Fr at 0.3% NaCl | Fr at 1% NaCl | Fr at 5% NaCl |
|---|---|---|---|---|---|
| 1 | 3000 | 0.30 | 3.15 | 5.80 | 5.64 |
| 2 | 4000 | 0.29 | 3.91 | 5.96 | 7.11 |
| 3 | 5000 | 0.29 | 3.62 | 6.09 | 7.52 |
| 4 | 3000 | 0.28 | 2.66 | 6.85 | 6.78 |
| 5 | 4000 | 0.27 | 3.65 | 7.58 | 9.79 |
| 6 | 5000 | 0.25 | 3.57 | 7.75 | 10.20 |
| 7 | 3000 | 0.24 | 4.12 | 9.76 | 11.13 |
| 8 | 4000 | 0.24 | 4.57 | 9.38 | 13.75 |
| 9 | 5000 | 0.22 | 3.87 | 10.13 | 15.18 |
| 10 | 3000 | 0.21 | 6.42 | 13.19 | 14.08 |
| 11 | 4000 | 0.20 | 7.56 | 13.47 | 21.14 |
| 12 | 5000 | 0.19 | 6.51 | 13.06 | 18.55 |

$$\tau = \frac{1}{Ø^{m-1}} \quad (2.21)$$

Combining Equations 2.10, 2.12, and 2.21 gives:

$$\tau = 1 + \frac{Ø_{tr}}{Ø_{ch}} \quad (2.22)$$

This expression indicates the physical significance of tortuosity in terms of stagnant and flowing porosities. Equation 2.22 is approximation valid only for consolidated porous rocks. For unconsolidated sands, the general expression (Equation 2.20) should be used, where $f_G$= 1.49 and m =1.09.

## 2.2 Formation Resistivity Factor, Fr and Porosity

As clean sedimentary rocks conduct electricity by virtue of the salinity of water contained in their pores, it is natural that the porosity is an important factor in controlling the flow of electric current.

ISSN 2522-6967

As a first approximation, one would expect that the current conductance would be no more than that represented by the fractional porosity, e.g., a formation with 20% connate water saturation and 80% oil saturation would be expected to transmit no more than 20% of the current that would be transmitted if the entire bulk volume conducted to the same degree as the water [8].

**Table (3-2)** clean sandstone 0.3%NaCl

| Fr | Ø | Log Fr = Yi | Log Ø = Xi |
|---|---|---|---|
| 5.7971 | 0.3038 | 0.7632 | -0.5174 |
| 5.9613 | 0.2926 | 0.7753 | -0.5337 |
| 6.0851 | 0.2922 | 0.7843 | -0.5343 |
| 6.8483 | 0.2792 | 0.8355 | -0.5540 |
| 7.5760 | 0.2654 | 0.8794 | -0.5761 |
| 7.7475 | 0.2525 | 0.8892 | -0.5977 |
| 9.7597 | 0.2410 | 0.9894 | -0.6179 |
| 9.3788 | 0.2387 | 0.9721 | -0.6221 |
| 10.1348 | 0.2217 | 1.0058 | -0.6542 |
| 13.1945 | 0.2085 | 1.1203 | -0.6808 |
| 13.4740 | 0.2015 | 1.1295 | -0.6957 |
| 13.0598 | 0.1870 | 1.1159 | -0.7281 |
| | | ? 11.2599 | ? -7.312 |

$F_r = (1/Ø)$ (2-23)

- Shlumberege [9], well surving corporations stated that the relation between (Fr) and porosity is reported as:

  $F_r = (0.81/Ø^2)$ (2-24)

- Zaafran et al, found that the formation resistivity factor may well correlated to the porosity and introduce this equation based on the better sandstone samples.

  $F_r = (1.48/Ø^{1.66})$ (2-25)

- Perez Rosales, et al. [11], Reported a new formulation for formation resistivity factor of fracture porous media .

They tried to fit an Archie type equation to the investigated data. They showed that it is impossible to get a good match . They established a new model based on the physical consideration and according to that they obtained the following formula:

$F_r = (1/1-(1-Ø)^{0.78})$ (2-26)

## 3.0 For Natural Core Sample

To predict a correlation represents the relation between the formation resistivity factor and each of rock porosity and cementation factor , we used the laboratory measurements of the formation resistivity factor and the porosity for 12 natural core samples .The measurements were taken from a previous study and are shown in Table (3.1) [12].

By measuring the electric resistivity of the fully saturated samples with (Ro) and water resistivity (Rw), the formation resistivity factor (Fr) using the known formula Fr=Ro/Rw for each core was determined. Resistivity factor and core porosity was obtained.

**Table (3-3)** clean sandstone, Fr at 1% NaCl.

| Fr at 0.3% NaCl | Ø | Fr=Yi | Log Ø = Xi |
|---|---|---|---|
| 3.1526 | 0.3038 | 0.4986 | -0.5174 |
| 3.9148 | 0.2926 | 0.5927 | -0.5337 |
| 3.6180 | 0.2922 | 0.5584 | -0.5343 |
| 2.6633 | 0.2792 | 0.4254 | -0.5540 |
| 3.6520 | 0.2654 | 0.5625 | -0.5761 |
| 3.5735 | 0.2525 | 0.5530 | -0.5977 |
| 4.1154 | 0.2510 | 0.6144 | -0.6179 |
| 4.5727 | 0.2387 | 0.6601 | -0.6221 |
| 3.8690 | 0.2217 | 0.5875 | -0.6542 |
| 6.4150 | 0.2085 | 0.8071 | -0.6808 |
| 7.5588 | 0.2015 | 0.8784 | -0.6957 |
| 6.5118 | 0.1870 | 0.8137 | -0.7281 |
| | | ? 7.55187 | ? -7.312 |

**Table (3-4):** clean sandstone, Fr at 5% NaCl

| Fr | Ø | Log Fr = Yi | Log Ø = Xi |
|---|---|---|---|
| 05.6410 | 0.3038 | 0.7513 | -0.5174 |
| 07.1090 | 0.2926 | 0.8518 | -0.5337 |
| 07.5190 | 0.2922 | 0.8761 | -0.5343 |
| 06.7760 | 0.2792 | 0.8309 | -0.5540 |
| 09.7878 | 0.2654 | 0.9906 | -0.5761 |
| 10.1968 | 0.2525 | 1.0084 | -0.5977 |
| 11.1310 | 0.2410 | 1.0465 | -0.6179 |
| 13.7517 | 0.2387 | 1.1383 | -0.6221 |
| 15.1780 | 0.2217 | 1.1812 | -0.6542 |
| 14.0810 | 0.2085 | 1.1486 | -0.6808 |
| 21.1420 | 0.2015 | 1.3251 | -0.6957 |
| 18.5500 | 0.1870 | 1.2683 | -0.7281 |
| | | ? 12.4171 | ? -7.312 |

Tables (3-2) to table (3-4) Show the calculations of using the least square method. The best correlation obtained for these different values of water salinities were obtained as follows:

1. Fr at 0.3% NaCl

   Fr = $(a / Ø_m) = (1/Ø^{1.057})$ least square (3-1)

2. Fr at 1% NaCl

   Fr = $(a/Ø_m) = (1/Ø^{1.427})$ least square (3-2)

3. Fr at 5% NaCl

   Fr = $(a/Ø_m) = (1/Ø^{1.698})$ least square (3-3)

A general correlation is obtained by using each of least square method determine an average value for the cementation factor m= 1.427 and constant a=1 the general correlation is given as:

Fr = $(a/Ø_m) = (1/Ø^{1.427})$ general equation (3-4)

*ISSN 2522-6967*

**Validity of the obtained resistivity factor correlations:**

To check the validity of the obtained correlations and possibility of its applications. We used the new correlations obtained for the natural core samples equation (3-1) through equation (3-4) and compared it with the other different correlation in literature. Table (3-5) shows this comparison for a core samples have different values of porosity (Ø) and cementation factor (m). The difference between the data obtained by equation (3-4) and the other correlations is due to neglecting the effect of water salinity and confined pressures.

**Table (3-5)** comparison of equation Humble equation. Perez equation. Zaafran equation Shlumberege equation and present study.

| Porosity (Ø) | Fr Eq.(3-4) | Fr Eq.(Humble) | Fr Eq.(Perez) | Fr Eq.(Zaafran) | Fr (Shlumberege) |
|---|---|---|---|---|---|
| 0.10 | 26.73 | 87.58 | 12.67 | 67.65 | 81 |
| 0.15 | 14.98 | 36.63 | 8.399 | 34.51 | 36 |
| 0.20 | 9.94 | 19.73 | 6.26 | 21.41 | 20.25 |
| 0.25 | 7.23 | 12.21 | 4.97 | 14.78 | 12.96 |
| 0.30 | 5.57 | 8.25 | 4.12 | 10.92 | 9 |
| 0.35 | 4.47 | 5.92 | 3.50 | 8.45 | 6.61 |

## 4.0 Conclusions

**Based on the results obtained from this study, we can conclude the followings:**

1. The formation resistivity factor which is the ratio of the electrical resistivity of porous medium fully saturated with water to the water resistivity is very important factor in electric log interpretation.
2. Using some of the natural core samples specifications measurements; porosity water salinity and formation resistivity factor a new correlation for the formation resistivity factor was obtained as Fr =Ø-1.427 .
3. To check the validity of the obtained correlations and its applications. A comparison between the results of using these corrections and those obtained by the different correlations present in literature. This comparison showed a reasonable agreement between them.
4. The formation resistivity factor increases by increasing the confined pressure (compaction pressure). This is as mentioned before, due to the decrease of the electric current path by increasing the confined pressure that cause an increase in the resistivity of formation and respectively increase the formation resistivity factor. It is clear that the water salinity affect the rate of increasing of resistivity factor.

The formation resistivity factor increases with relatively high rate at high degree of salinity.

### References

[1] Pirson, S.J. Handbook of well log analysis. Prentice-Hall, 1963, 326pp.

[2] Archie, G.E.” The electrical resistivity log as an aid in determining some reservoir characteristics, ”Trans., AIME, VOL.146, 1942.

[3] Amyx, J.W., Bass, Jr., D. M., and Whiting, R.L.,: petroleum reservoir engineering, McGraw-Hill, New York 1960.

[4] Rosales “C.P.” on the relationship between formation resistivity factor and porosity.” Soc. Petrol. Engr. J. Aug.1982.

[5] Chilingarian, G.V. “Relationship between porosity, permeability and grain size distribution of sands and sandstones.” in: L.M.J.U. van straitens (Ed), Deltaic and Shallow Marin Deposits, I. Elsevier Sci. Pub., Amsterdam’1964, pp, 71-75.

[6] Rosales, C. P., ”generalization of the Maxwell equation formation resistivity factors .” Soc. petrol.engr.J.petr.tech., July 1976.

[7] Timur, A., Hemkins, W.B.and Worthington , A.E.” porosity and pressure dependence of formation resistivity factor for sandstones . “Proc. Canadian well logging soc., 4th formation Eval. Symp. Calgary, May 9-10, 1972.

[8] Calhoun, J.C., Jr. Fundamentals of reservoir engineering,fourthed. University of Oklahoma press, Norman, 1960,426 pp.

[9] Shlumberger, Inc. Log interpretation-principles Schlumerger Educational Services, Houston, 1972.

[10] Zaafran, Z.M., Ph.D. Thesis, Leeds University England. 1975.

[11] Perez-Rosales, C., Cruz Hernandez ,J., IsIas-Juarez, R., and Mercade –Diaz, A:"New formulation for formation resistivity of fracture porous media , "SPE 58991 presented at the 2000 Int. Pet. Conf. and Exhib. In Mexico, Villahermosa, 1-3 feb.2000.

[12] Attia M. Attia 2005. £ Effects of petrophsical rock properties on tortuosity factor “. Paper PETROL 1289 Journal of petroleum science and engineering, volume 48, Lssues 3-4, ages 185-198.