"HARD" AND "SOFT" LINES IN ECONOMIC DEVELOPMENT

*By M. Bronfenbrenner**

Introduction

It is flattering to be asked for an essay relevant to the economic problems of North Africa, when one has never visited that area and knows almost nothing about its languages, history, and cultures. If the ideas or "insights" which follow do not fit North African realities, let the reader remember their derivation — partly from the international Ivory Tower, and partly from conditions in developing and reconstructing countries on the Asian side of the Pacific Ocean.

What I hope to accomplish in this essay, expanding on an earlier effort,[1] is to polarize into alternatives I call "hard" and "soft" the menu of policy choices open to a country with developmental ambitions, somewhat as an impresario might divide between "classical" and "popular" the menu of plays, operas, or concerts he presents to his audiences. I hope this division can prove useful, and not just another game of the type the Japanese call "playing with concepts" *(gainen no yugi)* .

Three Income Equations

To distinguish "hardness" from "softness," we begin on an indirect route, which involves three income equations, or rather identities, from elementary macrostatic economics. Our symbols are quite conventional,

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1. "'Hard' versus 'Soft' Economic Development," **Economic Weekly** (Bombay, India), Feb., 1963, This paper arose in turn from lectures at the Economic Development Institute of the International Bank for Reconstruction and Development (Washington, 1961-62). In its expansion and updating I owe especial thanks to my colleagues Michael Lovell and Robert Lucas, and to Simon Rottenberg (University of New York at Buffalo), before whose seminar I presented an intermediate draft in January, 1965.

and redolent of the controversy surrounding Lord Keynes' *General Theory* in the late 1930's :

Y – National income (more strictly, net national product, excluding depreciation on capital, and also "transfer" payments not in return for current services. All items are evaluated at market prices, including all taxes and subsidies).

C – Personal consumption expenditures (on *domestic* goods only).

I – Private net domestic investment expenditures (on *domestic* goods only).

G – Central and local government expenditures for goods and services (not further classified into consumption and investment categories).

T – Central and local government receipts, primarily taxes (net of "transfer" payments).

S – Net private saving (less depreciation of capital).

X – Exports of goods and services (including both consumption and capital goods, but excluding exports under international assistance programs).

M – Imports of goods and services (including both consumption and capital goods, but excluding imports under international assistance programs).

A – Foreign assistance received (>0) or granted (<0).

All values[2] should be considered in deflated or "real" terms. In the absence of special notation, all quantities are actual *(ex post)*. A superscript a denotes an anticipated or planned *(ex ante)* quantity. For example, Y^a is *ex ante* income.

Our first two income equations are standard Keynesian identities relating to the sources and uses, respectively, of national income *ex post.*

2. This breakdown of consumption, investment, and international accounts does not gibe perfectly with the published statistics of any country or international organization known to the writer. It simplifies matters conceptually, but seems less than "operational" statistically.

The third equation relates to the uses of income *ex ante*. A source equation for income *ex ante* is statistically legitimate, but I doubt its economic significance[3] and we do not heed it here.

The sources of any individual's, firm's, or public body's income are the expenditures of others and so we have, aggregating for all receivers and eliminating duplication :

$$Y = C + I + G + X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The *uses* of any individual's, firm's, or public body's income are its own expenditure and saving for all classes of goods and services. Aggregating again, we have :

$$Y = C + S + T + M + A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Aggregating receivers' vague plans to use their *anticipated* income gives the third income equation, with *ex ante* saving S^a as a residual or balancing item :

$$Y^a = C^a + S^a + M^a + A^a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We shall assume that equations (1 - 3) refer to the same time period, so that, for example, income received in period t is disposed of in that period, and *vice versa*.[4] Under this assumption, we can subtract (2) from (1) to derive an important constraint;

$$(I–S) + (G–T) + (X–M) = A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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3. It is hardly likely that many individuals consider, even vaguely, the sources of their income, **ex ante,** by economic sectors.

4. We are following Keynes here rather than D.H. Robertson. In the Robertsonian system, income received in one "day" is spent in the next. We must use time subscripts :

$$Y_1 = C_1 + I_1 + G_1 + X_1$$

$$Y_0 = C_1 + S_1 + T_1 + M_1 + A_1$$

The Robertsonian equipment of (4) is :

$$(I\text{-}S)_1 + (G\text{-}T)_1 + (X\text{-}M)_1 = A_1 + (Y_1 - Y_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4\text{-a})$$

The shorter the time period with which one is concerned, the more meaningful is this analysis. Robertson, and most writers who follow in his footsteps, work in terms of "days" or "weeks." The Keynesian analysis is more helpful when the period is longer, for example three months, a year, a quinquennium, or a decade.

In words, equation (4) says that no economy, whatever its social system or stage of development, can simultaneously invest more than it saves in its private sector, spend more than it receives in its public sector, avoid balance of payments difficulties (by avoiding $X < M$), and remain independent of foreign assistance (by avoiding $A > 0$). (Other interpretations of this equation are equally true and tautologous.) I concentrate on this particular interpretation because so many bright students and development practitioners think of it as a "law" of bourgeois, developed, or stable-price economies only, i.e., as pertaining to a free-enterprise economy in equilibrium rather than as part of a universal algebra. Actually, it is as true for a developing as for an advanced country, for a Socialist as for a capitalist country, for a hyper-inflating country as for a hyper-deflating one.[5] We can restate the principle in different words: Any country which attempts simultaneously to invest privately more than it saves privately, and to spend publicly more than it receives fiscally, will face a balance of payments problem or receive foreign assistance.

If we instead subtract (3) from (1), we obtain an approximation to the "Stockholm" outlook on income analysis. More important, we obtain instead of (4), an equation with different policy implications :

$$Y - Y^a = (C - C^a) + (I - S^a) + (G - T^a) + (X - M^a) - A^a$$

$$A^a + (Y - Y^a) + (C^a - C) = (I - S^a) + (G - T^a) + (X - M^a) \quad .. (5)$$

Equation (5) indicates three ways of combining a surplus of private investment over *anticipated* (not actual) private saving, a deficit of public expenditures over *anticipated* tax receipts, and of exports over *anticipated* imports. One way is the receipt of aid, as before in (4); this time the "aid" variable turns out to be the *ex ante* amount. A second way is what we shall call an income windfall (realized greater than anticipated income, $Y > Y^a$) as by remarkably good crops or remarkably rapid growth in industrial productivity.[6] The third way is the disappointment of consumer expectations (actual consumption less than anticipated, $C < C^a$).

5. Over the short Robertsonian day or week, of course, (4) is not true when income is rising ($Y_1 > Y_0$); compare equation (4-a). It may be that my "bright students and practitioners" had in view only periods as short as this, but I believe otherwise.
6. Assuming the relevant demand elasticities greater than unity.

With the disappointment of consumer expectations, should this method be employed purposefully and systematically, "hard" development appears for the first time in this essay. Such disappointment can be brought about by tax increases, unanticipated (by the public). It can also result from inflation, from rationing, from more or less compulsory purchases of public securities, or from similar ingenious forms of "forced frugality" or "forced saving." Here we have the principal germ of truth underlying my students' refusal to accept the implications of (4). While the algebra is correct for actual, realized *ex post* quantities, it is irrelevant to the planned, anticipated, *ex ante* quantities these students and practitioners had consciously or unconsciously in mind.

We may carry the argument of (5) a step further, making use of (4) and of three simple identities :

$$I - S^a = (I-S) + (S-S^a)$$
$$G - T^a = (G-T) + (T-T^a)$$
$$X - M^a = (X-M) + (M-M^a)$$

Substituting these results in (5) and rearranging terms :

$$A^a + (Y-Y^a) + (C^a-C) = [(I-S) + (G-T) + (X-M)] + [(S-S^a) + (T-T^a) + (M-M^a)]$$

But by (4), the expression in the first square brackets on the right hand side is simply A. Making this substitution, and shifting $(M-M^a)$ to the left hand side :

$$(Y-Y^a) + (C^a-C) + (M^a-M) + (A^a-A) = (S-S^a) + (T-T^a) \quad .. (6)$$

Equation (6) is an alternative to (5) in explaning how a surplus of private investment over planned saving may be combined with a deficit in the public sector and with balanced international payments. Here additional explanations become apparent, making use of *ex ante* concepts. For example : There may be windfalls of income. Anticipations of domestic consumption, receipts of imports, or receipts of aid goods may be disappointed. These will be balanced by unanticipated volumes of saving, taxes, or both, as two sides of a single coin.

If there is no windfall income, and if anticipated foreign aid is received ($Y = Y^a$, $A = A^a$), (6) simplifies to :

$$(C^a - C) + (M^a - M) = (S - S^a) + (T - T^a) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This result we could have reached directly by subtracting (3) from (2), with ($Y = Y^a$, $A = A^a$).[7] Disappointments on the side of cansumption of domestic or imported goods are identical with unexpected (forced?) increases of savings and taxes.

The Harrod Equation

The last section was static, and quite remote from growth theory. A connection can be made, employing an equation (identity) devised by Sir Roy Harrod on the eve of the Second World War :[8]

$$\frac{dY}{Y} = \frac{S}{Y}\frac{dY}{I} \qquad \text{or} \qquad \frac{dY}{Y}\frac{I}{dY} = \frac{S}{Y}$$

In these expressions, dY is the change in real national income ($Y_1 - Y_0$), so that (dY/Y) is the growth rate $(Y_1 - Y_0)/Y_0$. S/Y is the proportion of income saved, usually called the saving ratio, net of capital depreciation. I/dY is the net investment required for a unit increase in income; it has a number of names, the most common being the marginal capital-output ratio or marginal capital coefficient.[9]

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7. The algebra is, substracting (3) from (2) :

$Y - Y^a = (C - C^a) + (S - S^a) + (T - T^a) + (M - M^a) + (A - A^a) = 0$ (by hypothesis)

If ($A = A^a$), we need only transfer the consumption and import terms to the opposite side of the equation to obtain (7).

A special case of (6-7) for a country which neither receives nor grants aid, would have both A and A^a equal to zero.

8. "An Essay in Dynamic Theory," **Economic Journal** (March, 1939).

9. An occasional source of confusion in the literature is the confusion between marginal and average capital-output ratios (capital coefficients). The marginal ratio is as defined in the text. The average one, which may or may not be numerically equal to the marginal, is K/Y, where K is the total capital stock. ($I = dK$). The algebraic relation between marginal and average concepts is :

$I/dY = (K/Y)\ (I/K)/(dY/Y)$

In deriving these expressions, Harrod treats saving and investment (S and I) as equal. In an economy where explicit account is taken of the public sector and of international relations, the most useful equality is not between S and I (as we have defined them), but between two other quantities we may call S* and I*.

These may be defined with the aid of equation (4) :

$$(I-S) + (G-T) + (X-M) = A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$I^* = I + G = S + T + (M-X) + A = S^* \quad . \quad . \quad . \quad . \quad (8)$$

and we may understand the Harrod equation to refer to the "expanded" variables (S*, I*). These expanded forms (and likewise alternative ones we might have used) lose their pristine clarity of meaning, as when purely "consumptive" public expenditure (for military pay and equipment) enters on the same footing as investment in machinery, or when private expenditure for imported luxuries and aid goods is treated as a form of saving. Numerous alternative definitions of the expansions S* and I* are readily conceivable.[10] The most I can say for this one is that it seems to preserve the Harrod equality at minimal cost in realism, and without making ridiculous the notion of an approximately stable capital-output ratio.

The Harrod equation is often written using G as the income growth rate,[11] s as the saving ratio, and v as the capital coefficient, thus:

$$G = s/v \qquad \text{or} \qquad Gv = s \quad . \quad . \quad . \quad (9\ a\text{-}b)$$

A further adjustment, not made by Harrod, involves separating out foreign aid, so that :

$$s = \frac{S + T + (M-X)}{Y} \qquad \text{and} \qquad a = \frac{A}{Y}$$

10. In my Indian article (op. cit., p. 213), I treated all public spending as consumption, so that I and I* were identical. I also had

$$S^* = S + (T\text{-}G) + (M\text{-}X)$$

with A included in M, and all public expenditures treated as "consumption."

11. We have hitherto used G to refer to public expenditures. The context usually makes clear which meaning is intended.

The symbol a represents the relative importance of foreign aid in the national income. It may be positive (receiving country) or negative (donating country). The change modifies (9) to :

$$G = \frac{s + a}{v} \qquad \text{or} \qquad Gv = s + a \,. \quad (10\ a\text{-}b)$$

Hardness and Softness

By "hard" economic growth I intend to mean any process, centrally-planned or not, which operates in either or both of the following ways : (1) to raise the economy's saving ratio (I^*/Y, S^*/Y, or s); (2) to disappoint consumer and importer anticipations ($C < C^a$, $M < M^a$), with a view toward maintenance of s at or near some prior value. By "soft" economic growth I refer to a growth process, centrally-planned or not, which pays little attention to the saving ratio, sometimes letting it fall by reason of "demonstration effects" or "welfare state" measures, and which seldom or never disappoints consumer or importer anticipations.

The theme of the essay is the sort that the *London Economist* has called, in a different connection, an "uneasy triangle."[12] (In this case, it is a quadrangle.) I shall maintain that no country can combine soft development, rapid development, heavy industrialization, and the independence resulting from low foreign aid. A country can have none of these, or any one, any two, or any three of them, but not all four.

If this is correct, developing countries should "harden" their development programs, lower their ambitions for rapid improvements in mass living standards, lessen the heavy-industry emphasis in their hopes and plans, reconcile themselves to a somewhat mendicant dependence upon aid (with strings attached, *ex post* if not *ex ante*)[13] or settle for disappointingly low growth rates by comparison with China, Japan, or the Soviet Union. In terms of equation (10), a higher G requires a higher s, a higher a, or a lower v. Or conversely, a lower s, a lower a, and a higher v can lead only to a decline in G.

12. Anonymous, "The Uneasy Triangle," **op. cit.** (August, 1952).

13. Attachment of strings to aid **ex ante** will be familiar to African readers, being at the foundation of United States aid policy. Attachment of strings **ex post** is a wilier policy. It involves suspension or cessation of ostensibly unconditional aid, as a punishment for the receiving country's ideological or economic sins. Soviet policy toward China and Albania in 1960-61 can serve as an example.

In connection with industrialization, this conclusion assumes a conclusion plausible intuitively and usually true empirically, namely, that heavy industry requires higher capital coefficients v than light industry, agriculture, or handicrafts. This is to say, heavy industry is capital-using (capital-intensive). This is especially true when, in addition to "industrial hardware", heavy industry requires indirect investment to provide power, transport, and allied "social overhead capital," to house newly-urbanized labor forces in some degree of sanitation, to train skilled workers, and to compensate for the inevitable wastes from green hands in process of training.

Capitalist and Socialist Versions.

Can the terms "hard" and "soft" be given any more concrete meaning in practice than we have given them thus far? Can one identify "hard" and "soft" development patterns in history? How does one "harden" or "soften" a development pattern? Is "hardness" synonymous with Socialism or with planning, and "softness" with capitalism and the open market? Let me attempt to answer these questions with four examples, illustrating (in that order) hard and soft capitalism and hard and soft Socialism. These examples will be followed by some comments on contemporary American aid policy.

1. "Hard" capitalistic development is illustrated, in varying degrees, by the British Industrial Revolution, by the France of the Second Empire, by the "Gilded Age" of the United States (1870-1914), by Hohenzollern Germany, and by Meiji Japan. It is a system which fostered saving and investment at the expense of income equality, as under Napoleon III's slogan, "Messieurs les bourgeois, enrichissez-vous!" It kept labor organization weak, delayed social legislation, winked at labor monopsony. It taxed regressively and praised frugality. It delayed as long as possible what we now call "demonstration effects" of "foreign" luxury, at least among the masses. Saving was encouraged under the justifiable assumptions that the great bulk would be invested in productive facilities (not hoards of cash, not land, not residential buildings), that these

facilities would be predominantly domestic rather than foreign, and that foreign investments would pay for themselves in interest, profits, or cheap raw materials.

Capital was generally scarce, and its productivity high. Capital coefficients were correspondingly low.[14] Foreign aid was insignificant. The combination of high s, low v, and infinitesimal a led, by Harrod's equation, to values of G which, while seldom spectacular from year to year, maintained themselves over the long period.

2. "Soft" capitalistic development is illustrated, on the other hand, by the less Socialistic wing of the British Labor Party as well as by the American New Deal and the successors thereto. Its attitude toward saving is different. It tends to fear unemployment and stagnation from any excess of saving over *ex ante* investment, and encourages the high-consumption pattern associated with the American way of life. It fosters economic equality, for the sake of equity as well as of increased consumption. It accordingly encourages trade unionism and collective bargaining, while its tax systems tend toward progression.

Capital coefficients run high, partly because of research and development expenditure, partly because much equipment is operated below its rated capacity, and partly because of trade-union feather-bedding and make-work practices. Of recent years, foreign aid has produced a significant negative a term. With lower s, higher v, and negative a terms, it is not surprising that G should be difficult to mantain. One may even

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14. The relation between the productivity of capital and the capital coefficient is not strictly reciprocal, but can be treated as reciprocal to a first approximation. Suppose a production function :

$Y = f(K, L)$

The total and partial derivatives of income with respect to capital are related by the expression :

$$\frac{dY}{dK} = \frac{\delta Y}{\delta K} + \frac{\delta Y}{\delta L}\,\frac{L}{K}$$

The capital coefficient is the reciprocal of the left hand side, while the marginal productivity of capital is the term $(\delta Y/\delta K)$ on the right hand side. The two are reciprocal if labor is unproductive $(\delta Y/\delta L = 0)$, or if additional capital leaves employment unchanged $(dL/dK = 0)$. It is an unsettled question in development planning whether private investment, concerned with $(\delta Y/\delta K)$ alone, will provide the "right" answers for (dY/dK) and its reciprocal, or whether centralized planning can produce significant improvements.

think of a "new" stagnation thesis or dilemma for such economies. If the saving ratio s is low, then G is disappointingly low as well. If s is high, there is unemployment and facilities are used below capacity. When facilities are used below capacity, the capital coefficient v is high, and G again approaches a stagnation level.

3. "Hard" socialism means Russian and Eastern European Stalinism, not to mention Chinese Maoism. "The expropriators are expropriated" in standard Marxian fashion, but their property income (surplus value) is saved by the State for investment purposes, and is never distributed to the proletariat. Indeed, considerable labor income is itself diverted to the State by taxation before reaching the workers. High real wages and living standards are postponed repeatedly with each new N-Year Plan. Meanwhile, consumer amenities are reserved for the elite — Djilas' "New Class" — and consumer expectations often disappointed by rationing, blockage of currency, "voluntary" contributions, or simple waiting in line. In the symbols of equations like our (6-7), the term $(C-C^a)$ is consistently negative and the term $(S-S^a)$ is consistently positive. Taxes are proportional or regressive, and are supplemented by the voluntary contributions mentioned above; if these are treated as taxes, the term $(T-T^a)$ in (6-7) is also consistently positive.

Both to enforce austerity and to reduce capital coefficients, "hard" Socialist regimes suppress independent trade unions and their principal weapons, such as strikes and slow-downs. Labor is allocated as between regions and occupations. Voluntary extra labor, in the form of overtime or second jobs, is common and reduces capital coefficients. Apparently "blue ants" can be surprisingly effective capital-substitutes over a wide range of industries, even modern heavy industries. We should not be deceived by spectacular failures like the Chinese back-yard steel foundries of 1958-59. Neither should we regard the suppression of independent unionism as an insignificant feature of "hard" Socialism. It may be a major and necessary element in keeping capital coefficients down and growth rates up, just as the demands and restrictions of an organized "labor aristocracy" keeps capital coefficients high and growth rates low under some other regimes.

4. India's "Socialist Pattern of Society" is a prototype of "soft" Socialism. Where private wealth, such as agricultural land, is taken over, the beneficiaries are selected workers and peasants rather than the public treasury. Nationalized industries are operated to provide comfortable and prestigious employment or "modern" goods and services rather than to generate surplus for investment. Land taxes and rural income taxes are low, on humanitarian grounds, in a predominantly rural economy, so that central and local governments run deficits behind progressive-tax facades. In Harrodian terms, "soft" Socialism (which may be inseparable from Democratic Socialism) keeps saving ratios lower than their planned values.

In the same way, "soft" Socialism suffers from unnecessarily high capital coefficients. Heavy industries are introduced, at the same time that trade unions are encouraged and strikes rampant in the public as well as the private sector of the economy. Rich-country wages, working rules, and fringe benefits are avidly sought and sometimes approximated, with little regard for the economy's capacity to pay for them. Factories and offices, particularly publicly-operated ones, are over-staffed to provide "employment" of a sort at the price of inefficiency. The educational system turns out urban lawyers, journalists, and white-collar engineers in unusable numbers, while the economy cries out for skilled workers, foremen, technicians, and rural school teachers. Small wonder growth rates (augmented by aid) are disappointing, even without the pressures of rising population .

5. American economic influence, as exerted through official advisers and aid programs, has consistently been in a "soft" direction — more consistently even than a capitalistic one. The purpose has been to build up "showcases" defined by consumption standards.

Consider such American economic dependencies as South Korea, Taiwan, and South Viet Nam.[15] In each of these lands, and likewise in

15. We do not mention Puerto Rico. Puerto Rico is a more successful showcase than any of the countries cited, but it has special tax, trade, and immigration privileges as an offshore territory of the United States, in which no other developing country shares.

other developing countries where American influence has been weaker (Iran, Pakistan, Thailand, Turkey) it has followed a common pattern. However hard (undemocratic, dictatorial) the governments of these countries have shown themselves in political matters, their development patterns have all been soft. Living standards and growth rates have been raised by foreign aid, while domestic saving and investment ratios have remained low.

American influence, on the other hand, has been hostile to capital-intensive heavy industry. In Korea, for example, the American Aid Mission "line" was for many years a two-to-one division of the aid budget, two-thirds consumers goods and one-third capital equipment, while the Republic of Korea was proposing precisely the opposite two-to-one division. In terms of our analysis in this paper, there are obvious "good" reasons for the American stand; for example, the inability of the Korean economy to "absorb" capital without high capital coefficients, and the greater short-term growth potential of consumer goods imports. (This is not to deny possible "real" reasons, such as the fear of competition in both American and foreign markets.) At any rate, American aid policy has kept capital coefficients low and to some extent encouraged growth in the short run. These coefficients would often have been lower still, but for the effects of widespread waste and corruption, by Americans and "host country" citizens alike.[16]

The measured growth rates resulting from the American-sponsored combination of high a and low v have sometimes been impressive, as in Taiwan and Thailand. They have been less impressive in Korea and Indo-China. Even in the favorable cases, however, the record is marred by the receiving developing country's continued need for aid. This is needed to balance the country's international accounts. More important, it is needed to compensate for the softness of the domestic economy, and the failure of domestic saving and investment to support on its own, over the long run, the "take-off" which aid has counterfeited in

16. Where there has been guerilla fighting or civil war, as in Laos and Viet Nam, capital coefficients are naturally raised substantially.

the short period. What we are saying of growth applies, in greater measure, to "showcase" consumer living standards as well.

Keynesian Objections

Our argument seems innocuous enough in its theoretical aspects. It rests, however, on one important proposition which has been attacked from both Keynesian and neo-classical camps. This proposition states that increased saving increases the rate of growth, rather than having its effects dissipated by unemployment, or by higher capital coefficients. In this section, we shall sample the Keynesian objection to this proposition; the neo-classical objections will come later. For the remainder of this essay, therefore, we return to the abstract theorizing of the second and third sections.

Our sample of the Keynesian position is a 1938 paper by the Polish economist Oskar Lange, "The Rate of Interest and the Optimum Propensity to Consume."[17] This is a peculiar choice, since the essay is understandably overlooked by most historians of growth theory. (Its date is early, and its content is never related specifically to problems of developing countries.) Lange's argument is however clear and simple, and shows in an easily understandable way how a saving ratio may be above as well as below the optimum. This optimum is defined by Lange as one which maximizes net investment — and therefore economic growth, although he does not say so explicitly.

Lange's Keynesian model includes neither a public nor a foreign sector, and we consider only its simpler form, where m (the money supply, however defined) is held constant in real terms. In addition to m, the other "new" symbol is i, the rate of interest.

Three of Lange's functions are a liquidity preference L (indicating demand for cash), an investment function F, and an income identity —

17. **Economica** (Feb., 1938). Reprinted in American Economic Association, **Readings in Business Cycle Theory** (Philadelphia : Blakiston, 1946).

a truncated form of our equation (1). They are drawn as Fig. I a-c, in the forms Lange envisaged for them :

$m = L(Y, i)$ with $\delta L/\delta Y > 0$ and $\delta L/\delta i < 0$

$I = F(C, i)$ with $\delta F/\delta C > 0$ and $\delta F/\delta i < 0$

$Y = C + I$

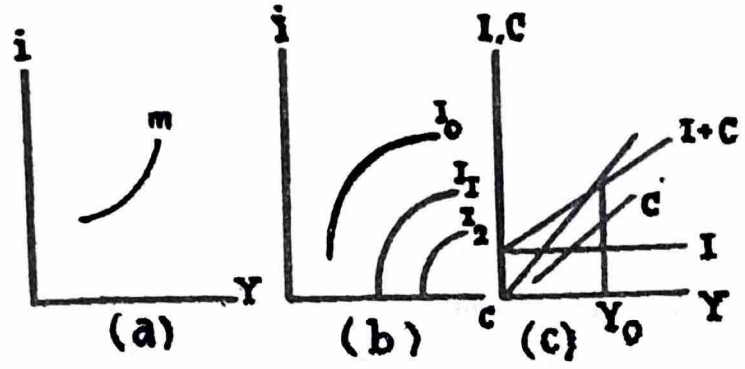


Figure I

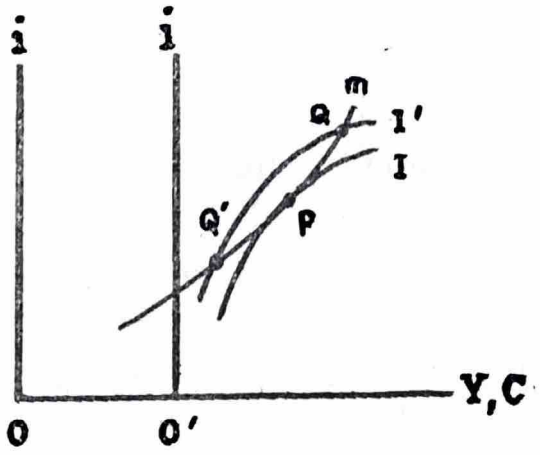


Figure II

Differentiating the three functions, we obtain, with m constant and dm zero :

$$\left.\begin{aligned} \frac{\delta L}{\delta Y}\ dY + \frac{\delta L}{\delta i}\ di = 0 \\ \frac{\delta F}{\delta C}\ dC + \frac{\delta F}{\delta i}\ di = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$dC + dI = dY$$

If investment I is at a maximum, dI is also zero, and dY becomes equal to dC. The differential equations (11) reduce to a two-equation system :

$$\left.\begin{aligned} \frac{\delta L}{\delta Y}\ dY + \frac{\delta L}{\delta i}\ di = 0 \\ \frac{\delta F}{\delta C}\ dC + \frac{\delta F}{\delta i}\ di = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (12)$$

Equations (12) yield an optimum solution, which Lange presents graphically as a tangency between his I and m curves (at P in our Figure II),[18] assuming that I = O − O'.

$$\frac{\delta L/\delta Y}{\delta L/\delta i} = \frac{\delta F/\delta C}{\delta F/\delta i} = -\frac{di}{dY} \quad \ldots\ldots\ldots\ldots \quad (13)$$

But this tangency is only an optimal outcome, not a requirement for economic equilibrium. There is no reason why the interval O − O' (Figure II) should not represent some smaller volume of investment than I, such as I' in the diagram. (A *larger* volume would involve a contradiction!) If the volume of investment is in fact I', there are two intersections with the m curve, indicated by Q and Q' on Figures II. The intersection at Q conforms to the classical interpretation ; I could be increased by lowering consumption (raising the saving ratio s). The rate of interest would fall, but the fall in income suggested on the diagram would be illusory.[19] On the other hand, the intersection at Q' conforms

18. Figure II represents a superimposition of Figures I a-b, which have the same vertical axis and whose horizontal axes are realated by the equation, Y = C + I.

19. Income would not fall because the interval between 0 and 0' would increase, and the fall in consumption balanced by the rise in saving and investment, as per Say's Law.

to an under-consumptionist interpretation, where I could be increased by *raising* consumption (lowering s) and increasing employment. Here the rate of interest, and also current income, would presumably rise.

In any actual situation, there is no *a priori* way of saying whether we are at a point like P or not. If not, there is no *a priori* way of deciding whether a situation corresponds to Q or to Q'. Elaborate econometric study is required to answer these questions. The point is that hardening economic policy and raising the saving ratio may operate to lower investment and growth, not to raise them. For advanced industrial countries in the late 1930's, Lange clearly believed existing equilibria to be unemployment situations like Q'. It is less clear how he would have diagnosed the situation in North African countries in the same period.

The contemporary reader will notice in Lange's model the absence of anything like a production function. This means that investment and growth are held back from the demand side alone. Supply restrictions are ineffective; there is substantial unemployment. Also , as we have seen, there is no public sector, and the money supply is constant in real terms.

For the present paper, I am supposing both an active public sector and an active monetary policy. This means that taxation, expenditure, monetary, and credit measures can be enlisted to maintain employment at any level of saving. This makes the Keynesian objection somewhat irrelevant, at least in the form presented here.

Neo-Classical Objections

Neo-classical growth models assume the relevant obstacle to progress to be on the supply side. That is to say, they take full employment for granted. To put it differently, they hand over the problems of its achievement and maintenance to monetary policy, fiscal policy, and Say's

Law. A production function for the growing economy — or in some cases, a multiplicity of such functions for different economic sectors — is (are) fundamental to their view of the world.

Somewhat surprisingly at first blush, aggregative neo-classical models also deny that the savings ratio (the hardness of development) influences the growth rate. Instead, it influences the capital coefficient inversely. Our sample to illustrate this point will be Robert Solow's "Contribution to the Theory of Economic Growth."[20] Solow uses an aggregate production function for the entire economy :

$$Y = f(K, L) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where both first derivatives are positive and both second derivatives negative.[21] (The mixed derivative is taken positive.) This production function is presumed linear homogenous, meaning that if capital and labor each increased by a multiple h, Y would rise to h times its original value :

$$f(hK, hL) = hY$$

If we set h equal to 1/L, (14) Becomes :

$$\frac{Y}{L} = f\left(\frac{K}{L}, 1\right)$$

Writing y for product per employed worker (Y/L) and x for the capital-labor ratio (K/L, allied to the Marxian "organic composition of capital") we have simply :

$$y = f(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

with a positive first derivative and a negative second derivative.

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20. **Quarterly Journal of Economics** (Feb., 1956) Instead of (14), many neo-classical model-builders prefer the expanded form $Y = e^{gt} f(K, L)$, where g is an index of technical progress and e is the base of natural logarithms.

21. Here L represents the employed labor force, not liquidity or demand for cash as in the Lange model just considered.

The quantity x, (or K/L) may be differentiated with respect to time, to determine its rate of change. Using dot superscripts for time derivatives ($dx/dt = \dot{x}$, etc.) :

$$\frac{d}{dt}\frac{(K)}{(L)} = \frac{L\dot{K} - K\dot{L}}{L^2} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L} \quad \ldots\ldots (16)$$

But $\dot{K}$ equals net investment I*, which in turn equals net savings S*. Expressed as a percentage of income, $\dot{K}$ therefore equals sY. Also, Solow assumes that the growth rate of employment ($\dot{L}/L$) may be treated as an exogenous constant n. (Actually, n need not be completely exogenous to the economic system ; all that is required is the absence of any relation to the saving ratio s.) Letting $\dot{K}$ be sY and $\dot{L}/L$ be n, (16) is:

$$\dot{x} = \frac{sY}{L} - n\frac{K}{L} = sy - nx = s\,f(x) - nx$$

using (15).

If there exists a stable path of balanced growth, the capital-labor ratio x will tend toward constancy, and its rate of change $\dot{x}$ will tend toward zero. If the capital-labor ratio is constant and the level of employment does not change, the capital stock will grow at the same rate n as the labor force. Furthermore, if the aggregate production function (14) is linear and homogenous, income will also grow at the same rate as the labor force and the capital stock.[22] The equilibrium growth path will be simply an exponential function, for however long the labor force growth rate n remains constant. Its formula is :

$$Y = Y_0\, e^{nt} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

where Y_0 is an initial value of Y, and e is the base of natural logarithms. The value of (17) at any time t is *independent* of the saving ratio s, meaning that changes in s reflect themselves entirely in values of v, the capital coefficient.

22. If the national income and the labor force grow at the same rate, per capita income y must be a constant and will not grow at all, in the absence of technological change. For this reason among others, the neo-classical "equilibrium growth path" is increasingly compared to the "stationary state" of the English classical economists.

It is however unclear how much importance need be attached to these formal results. Both economists and practitioners disagree on the point. I shall merely list, without extended comment, six reasons for regarding them as little beyond theoretical *curiosa*. (These reasons are not presented as original.)

1. A single aggregate production function like (14) must be assumed to exist for the entire economy, and to be invariant with the composition of the national output, for these results to hold. In most economies, a multiplicity of sectoral production functions is more realistic. With a multiplicity of sectoral production functions, and with substitution between the outputs of more and less capital-intensive sectors, saving and investment can still raise the over-all growth rate by increasing the capital supply.

2. This aggregate production function must also be assumed linear homogeneous, as per (15). In practice, there may be important scale effects. In particular, an h-fold increase in the effective labor supply[23] and the capital stock of an under-developed country may yield a more than h-fold increase in output.

3. A constant level of employment must also be assumed in these neo-classical models. Full employment is usually chosen, with wages approximately equal to the marginal product of labour. In most under-developed countries the marginal productivity of labor, at least in agriculture, is well below its income level, and there is substantial unemployment, primarily disguised.[24]

4. No account is taken of the probable positive relation of the rate of technical progress (the g term of the expanded production function) to the rate of investment in both material and human capital.

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23. The "effective" labor supply is measured in what Keynes called "efficiency units," which reflect increases in skill and investments in "human capital," as well as mere numbers of unskilled hands.

24. This point has been developed by Arthur Lewis, "Development with Unlimited Supplies of Labour," **Manchester School** (May, 1954), and more fully by John C.H. Fei and Gustav Ranis, **Development of the Labor Surplus Economy** (Irwin, 1964).

5. Independence of the growth rate and the saving ratio holds only along an equilibrium growth path. It has not been shown that such a path actually exists, or that it is stable.[25] (Certainly it is far from Utopian, in that it permits of no rise in per capita income, in the absence of technical progress.)

6. To move from one stable equilibrium growth path to another such path with the same growth rate, following for example the adoption of a "harder" development line, requires approximately a century, as Sato has shown on plausible assumptions.[26] For a long inter-equilibrium short run, at any rate, changes in saving propensities may, therefore influence growth rates.

An Optimal Saving Ratio?

We have considered already, in connection with the Lange model of 1938, the questions of an optimal saving ratio and an optimal degree of "hardness" in development. Equation (13) gives Lange's result in abstract form. It is difficult to translate into ordinary language, and to the best of my knowledge, it has not thus far been rendered operational.

Our treatment of "neo-classical objections" would also be incomplete without some attention to the neo-classical approach to Lange's problem. This has led to the theorem which has been called "The Golden Rule of Accumulation," or more simply, "The Neo-Classical Growth Theorem."[27]

25. In the Harrod-Domar family of growth models, which ruled the field for the decade after World War II, the equilibrium growth path was shown to be unstable. It was the merit of the neo-classical models, including Solow's, to show that the path **might** be stable if capital-labor substitution were introduced into the growth model. They have not shown that the path is in fact stable.

26. Ryuzo Sato."Fiscal Policy in a Neo-Classical Growth Model : An **Analysis** of Time Required for Equilibrating Adjustments," **Review of Economic Studies** (Feb., 1963) and "The Harrod-Domar Growth Model vs. the Neo-Classical Growth Model," **Economic Journal** (June, 1964). The "century" figure is a rough average of the time required for 90 per cent adjustment to new equilibrium in the five cases considered in Table II (p. 385) of the later article.

Research under way by Dale Mortensen at Carnegie Institute of Technology, however, suggests that if consumption and saving behavior is related to wealth (or long-run income) rather than to current income, adjustment periods may be only one-third or one-half the corresponding Sato periods.

27. E.S. Phelps, "The Golden Rule of Accumulation : A Fable for Growthmen," **American Economic Review** (Sept., 1961). A geometric derivation (marred by poor draftsmanship) is found in Alvin Marty, "The Neoclassical Theorem," **Ibid.** (Dec., 1964).

It seems anomalous to speak of optimal saving ratios in a world where equilibrium growth, at least, is independent of their size. It is anomalous in fact, by the Lange definition of optimality. Neo-classical concern with the optimal saving ratio involves, however, a quite different definition and criterion. The neo-classical "optimum saving ratio" has the property of maximizing consumption, both total and per capita, along an equilibrium path whose growth rate is equal to that of population. Since, ignoring the complications of equations (4) and (8) :

$$C = (1-s)\,Y$$

it follows by differentiation that :

$$\frac{dC}{ds} = (1-s)\frac{dY}{ds} - Y \quad \ldots\ldots\ldots\ldots \quad (18)$$

Contrary to a common intuitive guess, consumption is not a maximum, and dC/ds is not zero, for ultra "soft" development (s approching zero). The reason is that, while the equilibrium growth rate of income $\dot{Y}/Y$ may be independent of s, the income level itself is not. It will in fact be shown, paradoxically perhaps, that under standard neo-classical assumptions, most existing development lines are considerably too "soft" for maximum consumption under the Golden Rule.

Before indicating this paradox, let us state and derive the Golden Rule itself. The Golden Rule states that the optimal saving ratio is equal to the property (non-labor) share of the national income, or from 15 to 30 per cent in most countries. The derivation is, as usual, more complex than the statement.

We begin with a neo-classical result, discussed in the last section, that the equilibrium capital stock and income level grow at the same rate G :

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = G$$

We also rember that $\dot{K} = sY$. Equating the capital growth rate to G, and making the substitution of sY for $\dot{K}$, we can solve for K.

$$K = \frac{sY}{G} \quad \ldots\ldots\ldots\ldots \quad (19)$$

The equilibrium value of K from (19) can be substituted in the production function (14). If we specify a particular sort of linear homogeneous production function linear in the logarithms of Y, K, and L – the so-called Cobb-Douglas function :

$$Y = L^a K^{1-a} = L^a (sY/G)^{1-a}$$

(The symbol a is here a statistical parameter, and is not related to foreign aid. Later we shall show it to be an estimate of the labor share of the national income under competitive conditions.)

Algebraic manipulation of the last equation yields :[28]

$$Y = L\,(s/G)^{(1-a)/a}$$

$$Y/s = L\,s^{(1-2a)/a}\,(1/G)^{(1-a)/a}$$

$$\frac{dY}{ds} = \frac{1-a}{a}\,\frac{Y}{s} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

Now substitute (20) in (18), which evaluates the derivative dC/ds. If consumption is to be maximized in equilibrium development, dC/ds must be zero in (18)[29], so that :

$$\frac{dC}{ds} = (1-s)\,\frac{(1-a)}{a}\,\frac{Y}{s} - Y = 0$$

$$\frac{(1-s)}{s}\,\frac{(1-a)}{a} = 1$$

$$s = 1-a \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

28. An intermediate step in deriving the first expression below is to move all Y terms to one side :

$$Y^a = L^a\,(s/G)^{1-a}$$

and take the a-th root.

To obtain the third expression below, take the derivative of the first expression with respect to s, and compare it with the second expression.

29. This is a necessary but not a sufficient condition for a maximum. For the sufficient condition, differentiate (18) with respect to s :

$$\frac{d^2C}{ds^2} = (1-s)\,\frac{d^2Y}{ds^2} - 2\,\frac{dY}{ds}$$

Since d^2Y/ds^2 is non-positive (probably close to zero), this equation is negative. Its sign implies that (18) defines a maximum, when dC/ds vanishes.

To complete the derivation of the Golden Rule, that the optimum saving ratio equals the competitive non-labor share in the national income, we return to the production function (14) in its Cobb-Douglas form. We can then compute the marginal product of capital $\delta Y/\delta K$, and the competitive non-labor share $(K\ \delta Y/\delta K)/Y$:

$$\frac{\delta Y}{\delta K} = (1-a)\ L^a\ K^{-a} = (1-a)\frac{Y}{K}$$

$$\frac{K}{Y}\ \frac{\delta Y}{\delta K} = 1-a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Equations (21-22), taken together, demonstrate the Golden Rule of Accumulation and define an optimum saving ratio in the special neo-classical sense. The actual saving ratio seldom equals as much as half the non-labor share in any under-developed capitalist or soft Socialist country. Thus, even under the restrictive assumptions of the neo-classical development model, the neo-classical Golden Rule of Accumulation has pro-"hardness" implications.

An important variant of the neo-classical model is the dualistic one which Fei and Ranis[30] apply to economies with separate production functions in their industrial and agricultural sectors, and with important amounts of disguised unemployment in the latter. They conceive an optimal development policy as maximizing not the growth rate of output but the absorption rate of the disguised unemployed into productive employment.[31] Fei and Ranis are austere in hoping that what they call the CIW and CPCS (constant institutional wage and constant per capita consumption standard) can in fact be held constant until the redundant agricultural labor has been absorbed.

So "hard" a development line, however, may be only a counsel of perfection. As a precept for action, our menu of choices may be as indicated on a diagram such as Figure III.[32] In this diagram, the horizontal

30. **Op. cit.** For an earlier and brief version, see Ranis and Fei, "A Theory of Economic Development," **American Economic Review** (June, 1961), and for special attention to consumption, Fei, "Per Capita Consumption and Growth," **Quarterly Journal of Economics** (Feb., 1965).

31. To show that the criteria can diverge, see Fei and Ranis **Development of the Labor Surplus Economy**, pp. 136-141, and Diagram 14.

32. Compare this diagram with Fei's Figure III (**op. cit.**, p. 66)

axis measures the labor absorption rate. The vertical axis measures the growth rate of per capita consumption, a useful proxy for the "softness" of the development pattern. A family of community indifference curves (I_1, I_2, I_3) is drawn, with the usual properties of such functions. A transformation curve TT indicates the terms against which the two objectives can be traded off against each other. (The shape of TT reflects the common presumption that slight rises in C will reduce labor absorption less than proportionately to larger ones.) The points S and H indicate practicable limits of "softness" and "hardness" respectively, although further extremes are conceivable (rising labor redundancy in the one case, cuts in mass consumption in the other). An optimum of a subjective, non-operational sort is defined by the intermediate point P, at which TT is tangent to the highest indifference curve, namely I_3, which it meets. Labor absorption is at OX per cent per year, and the consumption standard is rising at OY per cent per year.

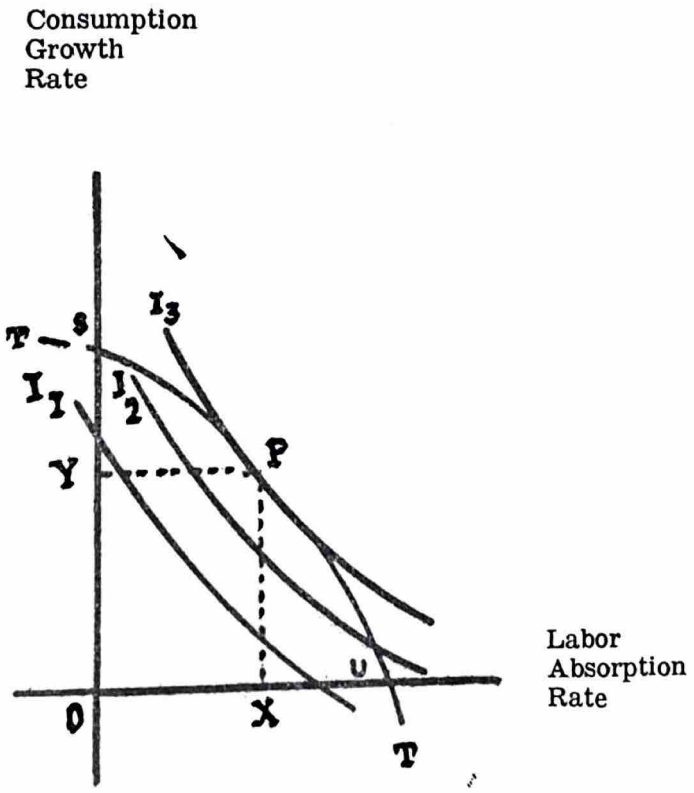


Figure III

This diagram makes, although with different variables, the same point as the present essay, which is itself a special case of the main point of economics, namely, the need for choice among alternatives. At the outset, I put the most relevant issue as involving a quartet of such variables (growth rate, industrialization, independence from aid, and "softness" in development). Following Fei and Ranis, Figure III reduces the variables to two, "softness" and labor absorption, but the conclusion is the same. All objectives cannot be maximized without recognition of their inter-relations. "Softness," in particular, must be sacrificed to some degree for others to be secured.

Premier George Papandreou of Greece once criticized his country's development under his predecessors : "The figures prosper, while the people suffer." This paper suggests that it can hardly be otherwise. For the numbers to prosper, the people must continue to suffer, by postponing higher consumption for the short term. In Greece, Papandreou's predecessors must share the blame with the principles of both economics and mathematics.