Mechanism of Change in Demographic Ageing

*DR. M.G. EL-ROUBY**

INTRODUCTION

Of all contemporary phenomena, Demographic Ageing is, as Sauvy has put it, "the least doubted, the best measured, the most regular in its effects and the easiest to forecast well ahead as well as the most influential. Yet it is probably the least known of all".[1]

Demographic ageing or ageing of human populations differs significantly from ageing of the individuals forming these populations both in the causes underlying the process of ageing and in the consequences occurring after the process has developed. In individuals'ageing, individuals are treated as separate units with the incidence of ageing recorded for each of them as his age increases with time. At advanced age, contribution of the individual to the luxury of his life gradually diminishes up to a stage in which the individual's basic needs can hardly be met as a result of his limited physical and intellectual abilities. Whereas in population ageing, all individuals constituting a particular population are dealt with as a one single unit, the aggregate of the individuals, with an average age computed for the whole unit at varying points of time and the changes in the structure of the aggregate and its age are recorded. The age of the population may not necessarily increase with time as it is the case with individual's ageing and therefore, the population may proceed in either directions: towards ageing or rejuvenation. Contrasted with individual's ageing, population ageing may be advantgeous to the population in various economic and social spheres.

In measuring the ageing of a particular population, the proportional age distribution of the population is observed at consecutive time points. If there is an increase in the proportion of people at old ages with a simultaneous decrease in the proportion of people at younger ages, an ageing process is recorded in which case the population is known as getting older. On the other hand, if there is an increase in the proportion of people at the young ages with a simultaneous decrease in the proportion of people at older ages, a rejuvenation process is alternatively recorded in which case the population is known as getting younger.

In practice, the population age distribution is represented in only three or four broad age classes: the 0-14 age class which embraces children under fifteen years of age, the 15-64 age class which contains people at working ages, and the 65 + age class constituted of old or aged people. Such age grouping is by no means unique for alternative grouping may be used depending on legislative and/or social circumstances of the population in question.

An alternative way of measuring demographic ageing is to calculate an average age of some form for the total population. The median age is often more appropriate. Indeed this is a simple way of expressing the process of ageing where the relevant facts are summarized in one single figure. However, it is sometimes regarded as an elementary measure of ageing owing to its lack of sensitivity. An auxiliary aid of measuring demographic ageing would be the analysis of the population age pyramid at various stages. The age pyramid is merely a graphic represantation that provides an overall picture of the age-sex distribution of the population. The shape of the pyramid and whether it shows regularities or irregularities reflects

---

* Lecturer in the Department of Statistics, University of Garyounis, Benghazi, Libya, Ph. D. in Demography from Glasgow University, U.K.

(1) Sauvy, Alfred "General Theory of Population" (Mathuen & Co. Ltd, 1969), p. 303.

with other competitors in terms of standard deviation, but the underestimation of the final score y seems to its vital characteristic and this grows worse as $p_3 \uparrow$.

REMARKS

(i) For the procedure $\hat{Z}_2$ the above considered aspects of comparison depend intrinsically upon w and Y_4. So, its relative performance cannot be juduciously generalized.

(ii) From Eqs. (3.1), (3.6), (4.2), (5.1) it is easy to see that as w $\downarrow$, the performance of a procedure with respect to bias improves whether it is $\hat{Z}_1$, $\hat{Z}_3$, $\hat{Z}_{1a_0}$, or $\hat{Z}_{1a_0}$. Theoretically it is also not difficult to show that the variance due to each of these procedures $\downarrow$ as w $\downarrow$.

CONCLUSIONS

From Table 1 and above remarks it appears that the use of the proposed procedure $\hat{Z}_{1a_0}$ offers a reasonable estimate of the final score when the probability of missing a student's best score is known not to exceed half. Otherwise, whatever this probability may be, the choice of $\hat{Z}_{1R}$ is another satisfactory alternative.

ACKNOWLEDGEMENT

The assistance rendered by Mr. Tanweer Rathore of AGECO, Benghazi in computational work for Table 1 is acknowledged gratefully.

REFERENCE

Mood, A.M., and Graybill, F.A. Introduction to the Theory of Statistics, 2nd ed. McGraw-Hill Book Co., Inc., 1963.

dents. So, this point does not invoke much support for this procedure. But, later, it will be discovered that $\hat{Z}_{1R}$ cannot be discarded just for this limitation. Its use however may not be made in case a class is of small size.

COMPARISON OF PROCEDURES

To compare the above procedures we use actual scores of thirty eight students in four tests held at the Garyounis University in a course of Statistics. We also include in this study a procedure $\hat{Z}_{1a}$ which gives an estimate of y as a simple mean of the estimates based on $\hat{Z}_1$ and $\hat{Z}_{ao}$. Eight different probability situations of p_1, p_2, p_3 are considered. Corresponding to a specified probability distribution which we assume as applicable to the whole class of thirty eight students, we can compute expected score and mean-squared error pertaining to each student from his scores Y_1, Y_2, Y_3, Y_4 when the use of a given estimating procedure is made. From this information the bias and standard deviation in respect of each student can be easily determined. For comparison of aforementioned procedures we focus our interest on two important aspects, that is, bias and standard deviation averaged over the whole class. Taking the test-weightage $w = 1/4$, such computations are given in Table 1, where the figures in the first row and within parentheses in the second row indicate averages for bias and standard deviation respectively.

Table 1.

p_1, p_2	p_3	$\hat{Z}_1$	$\hat{Z}_2$	$\hat{Z}_3$	$\hat{Z}_{1R}$	$\hat{Z}_{a_0}$	$\hat{Z}_{1a_0}$
0.450	0.100	2.40 (2.16)	-0.49 (1.44)	-2.31 2.61)	-0.14 (2.16)	-1.79 (2.47)	0.31 (2.10)
0.350	0.300	1.20 (3.31)	-1.29 (2.20)	-3.28 (3.05)	-0.07 (3.31)	-1.36 (2.97)	-0.08 (3.08)
0.333	0.333	1.00 (3.34)	-1.42 (2.22)	-3.35 (3.10)	-0.06 (3.34)	-1.17 (3.01)	-0.09 (3.11)
0.300	0.400	0.60 (3.53)	-1.69 (2.36)	-3.76 (3.11)	-0.04 (3.53)	-0.85 (3.25)	-0.13 (3.37)
0.250	0.500	0 (3.61)	-2.09 (2.38)	-4.24 (3.16)	0 (3.61)	0 (3.61)	0 (3.61)
0.200	0.600	-0.60 (3.53)	-2.49 (2.36)	-4.37 (2.96)	0.04 (3.53)	— —	— —
0.150	0.700	-1.20 (3.31)	-2.89 (2.20)	-5.21 (2.72)	0.07 (3.31)	— —	— —
0.005	0.900	-2.40 (2.16)	-3.69 (1.44)	-6.18 (2.26)	0.14 (2.16)	— —	— —

For example, when $p_1 = p_2 = 0.350$, $p_3 = 0.300$, the average bias and average standard deviation due to the use of the procedure $\hat{Z}_{a_0}$ are - 1.36 and 2.97 respectively. A study of Table 1 reveals following features of these procedures.

For the same value of p_3, the procedures $\hat{Z}_1$ and $\hat{Z}_{1R}$ display equal standard deviation but $\hat{Z}_{1R}$ produces an almost neglegible amount of bias. When $p_3 \leqq 1/2$, the absolute bias as well as standard deviation for $\hat{Z}_{1a}$ remain smaller than those for $\hat{Z}_{a_0}$, while the procedure $\hat{Z}_{a_0}$ is, in this regard, much better than Z_3 but not worse than $\hat{Z}_1$. However $\hat{Z}_{1a_0}$ and $\hat{Z}_{1R}$ do not behave much differently from each other. At each level of p_3 the procedure $\hat{Z}_2$ on the other hand works attractively when compared

$$\ddot{Z}_1(\ddot{p}_3) > \ddot{Z}_1(p_3) > 0 \quad \text{if} \quad \ddot{p}_3 < p_3 < 1/2 \tag{3.2}$$

$$\ddot{Z}_1(\ddot{p}_3) < \ddot{Z}_1(p_3) < 0 \quad \text{if} \quad \ddot{p}_3 > p_3 > 1/2 \tag{3.3}$$

$$\ddot{Z}_1(\ddot{p}_3) = -\ddot{Z}_1(p_3) \quad \text{if} \quad \ddot{p}_3 = 1 - p_3 \tag{3.4}$$

So, the procedure $\hat{Z}_1$ underestimates y when $p_3 > 1/2$, and overestimates y when $p_3 < 1/2$. The use of Eq. (3.4) can be helpful in preparing Table 1.

Bias in $\hat{Z}_2$: It can be shown that the bias due to $\hat{Z}_2$ is

$$\ddot{Z}_2 = \frac{w}{1-w}\,[Y_2(w + p_3 - 1) + Y_3(w - p_3) + (1 - 2w)\,y_4]. \tag{3.5}$$

Bias in $\hat{Z}_3$ This is obtained as

$$\ddot{Z}_3 = -w\,[(Y_2 - Y_1)\,p_2 + (Y_3 - Y_1)\,p_3]. \tag{3.6}$$

so that the procedure $\hat{Z}_3$ always underestimates y. However as the probability p_3 increases, this bias increases in general.

THE PROCEDURE $\hat{Z}_a$

From Eqs. (3.1) and (3.6), since

$$\ddot{Z}_3 - \ddot{Z}_1 = w\,[Y_1(1 - p_1) + Y_2(1 - p_2 - 2p_3) - Y_3(1 - p_3)] < w\,(Y_2 - Y_3)(1 - p_3),$$

$$\text{so } \ddot{Z}_3 < \ddot{Z}_1. \tag{4.1}$$

For $p_3 \leqq 1/2$, let us define

$$\ddot{Z}_a = a\ddot{Z}_1 + (1 - a)\,\ddot{Z}_3, \tag{4.2}$$

where $0 \leqq a \leqq 1$. Obviously, $\ddot{Z}_3 \leqq \ddot{Z}_a \leqq \ddot{Z}_1$. From (3.1), (3.6) and (4.2) we can show that

$$\ddot{Z}_a < w\,(Y_3 - Y_2)\,(a - p_3 - ap_3),$$

that is,

$$\ddot{Z}_a < 0 \text{ when } a \leqq p_3 / (1 - p_3). \tag{4.3}$$

Thus, by choosing the value 'a' that does not exceed $p_3 / (1 - p_3)$, we note that the procedure $\hat{Z}_a$, which has the corresponding bias as in (4.2), always underestimates the final score y. Taking a_1, a_2 such that $0 < a_1 < a_2 < a_0$ where $a_0 = p_3 / (1 - p_3)$, we find that

$$\ddot{Z}_{a_1} < \ddot{Z}_{a_2} < \ddot{Z}_{a_0} < 0.$$

That is, $\hat{Z}_{a_0}$ yields minimum absolute bias in the family $\{\hat{Z}_a : 0 < a < a_0\}$.

Also, $\ddot{Z}_{a_0} \rightarrow$ as $p_3 \rightarrow 1/2$. At $p_3 = 1/2$ the procedures $\hat{Z}_{a_0}$ and $\hat{Z}_1$ are identical. When $p_3 < 1/2$, the procedure $\hat{Z}_{a_0}$ underestimates y. To determine the procedure $\hat{Z}_{a_0}$ for estimating y, we consider Eq. (4.2). Rearranging terms and using $p_1 + p_2 + p_3 = 1$, we can express (4.2) as

$$\begin{aligned}&[wY_2(1 - a_0) + wY_3(1 + a_0) + (1 - 2w)\,Y_4]\,p_1\\ &+[wY_1(1 - a_0) + wY_3(1 + a_0) + (1 - 2w)\,Y_4]\,p_2\\ &+[wY_1(1 - a_0) + wY_2(1 + a_0) + (1 - 2w)\,Y_4]\,p_3\\ &-y.\end{aligned}$$

So,

$$\hat{Z}_{a_0} = w\,(1 - a_0)\,(\text{minimum of available scores}) + w\,(1 + a_0)\,(\text{maximum of available scores}) + (1 - 2w)\,Y_4. \tag{4.4}$$

THE PROCEDURE $\hat{Z}_{1R}$

For estimation of the final score y of a student we may make use of

$$\hat{Z}_{1R} = \hat{Z}_1 + w\,\hat{R}\,(1 - 2p_3), \tag{5.1}$$

where $\hat{R}$ denotes the mean difference of the two best scores Y_2 and Y_3 for the group of his classmates attending all the tests held during the course. Here, the estimate of y is based on scores of other stu-