Classification Statistics Z* and Z

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1. INTRODUCTION

Suppose we have an observation x from one of two p-variate populations $N(\mu_1, \Sigma_{11})$ and $N(\mu_2, \Sigma_{11})$ where the parameters μ_1, μ_2 and Σ_{11} are unspecified but Σ_{11} is positive definite. Given independent random samples from these populations, the problem of classification of x into its relevant population can be tackled by using the discriminant function Z proposed by Kudo (5) and John (3, 4) and studied by Memon (6), Memon and Okamoto (8). Sometimes there occur situations in texanomical problems when in addition to the knowledge of discriminators, information is available on a covariate y whose mean is known to be the same in both multivariate populations π_1 and π_2, that is, $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$ has population π_i: $N\left[\begin{pmatrix} \mu_i \\ \nu \end{pmatrix}, \Sigma\right]$, $i = 1, 2$, where $(x_i', y_i') = (x_{i1}, \ldots, x_{ip}, y_{i\,p+1}, \ldots, y_{i\,p+q})$ and the covariance matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

is positive definite. Let

$$\begin{pmatrix} x_{i1} \\ y_{i1} \end{pmatrix}, \ldots, \begin{pmatrix} x_{iN_i} \\ y_{iN_i} \end{pmatrix} \quad i = 1, 2$$

be independent random samples from π_i. Although the covariate has no discriminating power by itself, Memon (6, 9) still proposes, like as in Cochran and Bliss (1), to utilize the additional information y in replacing x by $\overset{*}{x} = x - \hat{\beta}y$ in the statistic Z where $\hat{\beta}$ is a sample estimate of regression matrix β of x on y. According to this the modified criterion is

$$\overset{*}{Z} = \frac{N_1}{N_1 + 1}(\overset{*}{x} - \overset{*}{\bar{x}}_1)' \underline{S}^{-1} (\overset{*}{x} - \overset{*}{\bar{x}}_1) - \frac{N_2}{N_2 + 1}(\overset{*}{x} - \overset{*}{\bar{x}}_2)' \underline{S}^{-1} (\overset{*}{x} - \overset{*}{\bar{x}}_2), \qquad (1.1)$$

and the procedure of classification proposed by him is to assign $\binom{x}{y}$ to π_1 if $\overset{*}{Z} \geqq 0$ and to π_2 if $\overset{*}{Z} > 0$, where $\overset{*}{\bar{x}}_i = \bar{x}_i - \hat{\beta}\bar{y}_i$, $\hat{\beta} = S_{12} S_{22}^{-1}$, $\bar{x}_i$ and $\bar{y}_i$ denote the sample means, and $\underline{S} = S_{11} - S_{12}S_{22}^{-1} S_{21}$ with

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

as the best unbiased estimator of Σ. Memon and Okamoto (7) study properties of the $\overset{*}{W}$ statistic and give asymptotic expansions of its distribution function and probabilities of misclassification that arise in using it. This paper follows the same approach in studying the $\overset{*}{Z}$ statistic when the sample from each population has the same size N, and thus extends the case $N_1 = N_2 = N$ of the paper by including the information on covariate in the discriminant function.

2. THE MAIN RESULT

We can easily see that as $N_1, N_2 \to \infty$, the limiting distribution of $\overset{*}{Z}$ is $N(-D^2, 4D^2)$ or $N(D^2, 4D^2)$ according as $\binom{x}{y} \in \pi_1$ or π_2, D^2

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cerned, all the responding Libyan accounting firms reported that they were able to hire only 75.5 percent of their stated demand in 1965. Only 68.4 percent of the indicated employment needs in 1970 were met. In addition the responding firms as a whole were able to hire 62.0 percent of their expressed demand for accountants in 1975. In summary, one can conclude that there has been a continuing shortage of accountants in Libya.

4. CONCLUSION AND RECOMMENDATIONS

As a result of the rapid changes in the Libyan environment, many professional accounting services are expected to be provided by the public accounting firms in Libya. The finding of this research indicate that the need for professional accounting services in Libya exceeds their availability to the business community. These services include bookkeeping, auditing, liquidations, tax services, management advisory services and systems design and installation.

However, on the basis of the tested operational hypotheses, the author accepts the major hypothesis of this study. Many professional accounting services are not widely provided by the public accounting profession in Libya because of the characteristics of the Libyan accounting firms, the size of their clients and the shortage of qualified accountants. Although the majority of the responding firms expressed their desire to provide a variety of accounting services, elimination of certain deterrents must be undertaken in order to accomplish this objective. On the basis of these findings, the following recommendations can be offered:

1. Although an Organization of Libyan Accountants and Auditors was established, most of its work was of an administrative nature. At this time, the organization should engage in a continuing effort of professional developments. A code of professional ethics should be enacted. Committees for the development and codification of accounting and auditing standards should be formed. In addition seminars, conferences, research projects and continuing education courses in related to accounting subjects should be carried out in order to improve the profession's body of knowledge.

2. In order to overcome the shortage of qualified accountants facilities of the Libyan accounting educational programs at the university and pre-university levels should be expanded. More students should be accepted to these programs each year.

3. Training of individuals, particularly those with some business education, may be considered among the means of solving the employment problem of the Libyan accounting firms. The Libyan accounting firms should assume the responsibility of training non-accountants whenever possible. Also, the Libyan Organization of Accountants and Auditors and the Department of Accountancy at the Faculty of Economics and Commerce should initiate training and educational programs in this respect.

4. This study demonstrated that the size of an accounting firm is one of the most important factor which tends to influence its ability to provide a variety of professional accounting services for the community. Therefore, small Libyan public accounting firms should consider carefully the advantages and potential of associating or merging with other firms. Optimum advantages would occur in situations where talents and strengths of one firm complement the talents and strengths of the other. Such a merger would increase specialization among members of new firm and would reduce the duplication of expertise to be maintained by each member before the merger.

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sional accounting services by the Libyan accounting firms and the size of their clients in terms of total employment.

The second chi-square test was used to evaluate the relationship between the size of the clients served by the responding Libyan public accounting firms in terms of total assets and the extent of rendering professional accounting services in Libya. Table 7 and its chi-square calculations provide the necessary information for this kind of statistical analysis. In this table, the responding public accounting firms were classified into two groups. The first group includes accounting firms with a majority of clients having LD. 99,999 of total assets or lower. The second group consists of accounting firms with a majority of clients having LD. 100,000 of total assets or higher. The professional accounting services rendered were classified into three classes as mentioned earlier in this study.

Examination of the data presented in Table 7 reveals that the null hypothesis (H_0) of independence is rejected. The derived chi-square value of 6.96 is greater than the table chi-square value of 4.605 with two degrees of freedom and at the .10 level of significance. Also, the null hypothesis can be rejected even at the .05 level of significance. Therefore, there is a statistically significant association between the size of clients served by the Libyan public accounting firm and the extent of professional accounting services provided by these firms.

As a result, the findings of this research support the existence of a connection between the extent of providing professional accounting services in Libya and the size of clients receiving these services. The size of clients may be considered as one of the most important determinants in this respect. Libyan accounting firms which are serving large-sized clients should plan to expand their services in order to satisfy the increasing demands of their clients.

TABLE 7. *Size of Clients Served by the Responding Libyan Public Accounting Firms in Terms of Total Assets and the Extent of Rendering Professional Accounting Services.*

Number of Accounting Services Rendered	Size of Clients: Majority of Clients Had LD.0–99,999	Size of Clients: Majority of Clients Had LD.100,000 or More	Total
1–2	12	5	17
3–4	7	6	13
5 or more	4	12	16
Totals	23	23	46

The derived chi-square value	= 6.96
The critical chi-square values with two degrees of freedom at:	
The .10 level of significance	= 4.605
The .05 level of significance	= 5.991

5. *The Adequacy of Qualified Accountants*

The lack of qualified accountants needed by the Libyan public accounting firms is one of the most serious problems facing the Libyan accounting profession today. The shortage of accountants deterred many Libyan public accounting firms from rendering a variety of professional accounting services.

In order to determine the availability of needed accountants, recipients of the questionnaire were asked to indicate the number of accountants demanded and hired by their firms in 1965, 1970 and 1975. The responding firms desired to hire 53 accountants in 1965, 158 in 1970 and 455 in 1975. The demand for accountants increased by 198.1 percent between 1965 and 1970 and by 758.5 percent from 1965 to 1975. One can conclude that much of this increased demand came from the expanding number of public accounting firms. In addition, the rapid growth of the Libyan economy created a great deal of pressure upon old accounting firms to hire additional accountants. The attraction of new accountants would help these firms to satisfy the expanding needs for more professional accounting services.

As far as the actual employment is con-

pendent of the responding accountants' age.

H_1: There is a relationship between the extent of rendering professional accounting services in Libya and the age of the responding accountants.

Examination of Table 5 and its supplementary chi-square calculations reveal that the null hypothesis cannot be rejected. The calculated chi-square value of 0.46 is extremely small to reject H_0 at the .10 level of significance with two degrees of freedom. In fact, even at the .25 level of significance, the chi-square value should reach as high as 2.773 before the null hypothesis can be rejected. Therefore, there is no significant relationship between the extent to which professional accounting services have been provided in Libya and the age of the responding accountants.

4. *Size of Clients Served by the Libyan Public Accounting Firms and the Extent of Rendering Professional Accounting Services*

Clients served by the Libyan public accounting profession is one of the most important variables, and cannot be ignored by this research. Therefore, an attempt was made to examine the possibility of a relationship between the size of clients served by the Libyan public accounting firms and the extent of rendering professional accounting services for these clients. For the purpose of this study, the size of clients was determined in terms of their total employment and their total assets. The chi-square test was used to examine the following hypotheses:

H_0: The extent of rendering professional accounting services by the responding Libyan accounting firms is independent of the size of their clients.

H_1: There is an association between the extent of rendering professional accounting services in Libya and the size of clients served by the responding Libyan accounting firms.

The first chi-square test was applied to test the relationship between the size of clients in terms of their total employment and the extent of rendering professional accounting services. In order to make the data more appropriate for this test, the responding accounting firms were classified into one group with a majority of clients who had 99 or fewer employees and another group with a majority of clients who had a total employment of 100 or more employees. The number of accounting services were categorized into three groups as indicated in the preceeding sections of this study.

The required information for applying the chi-square test is provided by Table 6 and its chi-square values. Examination of this table reveals that the null hypothesis (H_0) of independence can be rejected since the calculated chi-square value of 6.96 is greater than the critical chi-square value of 4.605 with two degrees of freedom at the .10 level of significance. In fact, the null hypothesis can be rejected at the .05 level of significance. Therefore, one can conclude that there is a statistically significant relationship between the extent of providing profes-

TABLE 6. *Size of Clients Served by the Responding Libyan Public Accounting Firms in Terms of Total Employment and the Extent of Rendering Professional Accounting Services.*

Number of Accounting Services Rendered	Size of Clients: Majority of Clients Had 0–99 Employees	Size of Clients: Majority of Clients Had 100 or More	Total
1–2	14	3	17
3–4	8	5	13
5 or more	6	10	16
Totals	28	18	46

The calculated chi-square value	= 6.96
The critical chi-square value with two degrees of freedom at:	
The .10 level of significance	= 4.605
The .05 level of significance	= 5.991

accounting profession has any relationship to the extent of providing accounting services, Table 4 was prepared. For purposes of chi-square analysis, the responding firms were classified into one group with five years or less in practice and another group with more than five years in practice. The number of accounting services rendered was categorized into three groups: the first group included accounting firms rendering one or two services; the second group included firms providing three or four services; and the third group included firms providing five or more professional accounting services. Then, the chi-square test was applied to test the following hypotheses:

H_0: There is no relationship between the extent of providing professional accounting services and the age of the responding Libyan public accounting firms.

H_1: There is a relationship between the extent of providing professional accounting services and the age of the responding Libyan public accounting firms.

As a result of examining the data presented in Table 4, an individual can conclude that there is a statistically significant association between the extent of rendering professional accounting services and the length of time the responding public accounting firms have been practicing the public accounting profession in Libya. The null hypothesis (H_0) of independence is rejected since the calculated chi-square value of 5.47 is greater than the critical chi-square value of 4.605 with two degrees of freedom at the .10 level of significance. Therefore, the age of the Libyan public accounting firms has influenced the extent of rendering professional accounting services in Libya. One can expect to find that old Libyan public accounting firms provide more services than newly established ones. However, this relationship is not as statistically significant as the established relationship in the preceeding section. In fact, the null hypothesis cannot be rejected at the .05 percent level of significance.

3. *Age of the Responding Accountants and the Extent of Rendering Professional Accounting Services.*

Table 5 and its chi-square values present information concerning the relationship between age of the responding accountants and the extent to which professional accounting services have been rendered by the Libyan accounting firms. In order to make this information more appropriate for the chi-square statistical test, the responding accountants were classified according to those who were between 29 and 39 years old, and those who were 40 years of age or older. Again, the number of services are categorized into three groups: the first group includes firms which were providing one or two services; the second group consists of firms which were rendering three or four services; and the third group is composed of firms which were providing five or more accounting services. The chi-square test was applied to this data in order to test the following hypotheses:

H_0: The extent of rendering professional accounting services in Libya is inde-

TABLE 5. *Age of the Responding Accountants and the Extent of Rendering Professional Accounting Services.*

Number of Services Rendered	Age of Responding Accountants		
	29–39	40 or More	Total
1–2	12	5	17
3–4	9	4	13
5 or more	9	6	15
Totals	30	15	45*

The calculated chi-square value	= 0.46
The critical chi-square values with two degrees of freedom at:	
The .10 level of significance	= 4.605
The .05 level of significance	= 5.991

*One accountant did not indicate his age.

TABLE 2. *The Size of Responding Libyan Public Accounting Firms in Terms of Full-time Employment and the Extent of Rendering Professional Accounting Services.*

Number of Services Rendered	Size Classification		Total
	1–7	More than 7	
1–2	13	4	17
3–4	6	7	13
5 or More	5	11	16
Totals	24	22	46

The computed chi-square value = 7.02
The critical chi-square values with two degrees of freedom at:
The .10 level of significance = 4.605
The .05 level of significance = 5.991

hypothesis (H_0) of no relationship is rejected since the derived chi-square value of 7.02 is greater than the critical chi-square value of 4.605 for two degrees of freedom at the .10 level of significance. In fact, the null hypothesis might be rejected at .05 level of significance since the critical chi-square value at this level of significance is only 5.991.

The Contingency Table 3 and its chi-square values include information concerning size of the Libyan public accounting firms in terms of annual gross fees and the extent of rendering accounting services by these firms. Analysis of this information reveals that there is statistically significant dependency between size of the Libyan accounting firms and the number of accounting services provided for their clients. The calculated chi-square value of 7.28 is high enough to reject the null hypothesis both at the .10 and the .05 level of significance.

In general, one can conclude from the above analysis that there is a statistically significant association between size of the public accounting firms in Libya and the extent to which professional accounting services have been provided by these firms. The larger the size of an accounting firm, the greater the number of professional accounting services which can be rendered. Therefore, this research demonstrated that the size of an accounting firm is one of the most important factors in evaluating its ability to provide a variety of professional accounting services for the community.

TABLE 3. *The Size of the Responding Libyan Public Accounting Firms in Terms of Annual Gross Fees and the Extent of Rendering Professional Accounting Services.*

Number of Services Rendered	Size Classification		Total
	LD.0–24,999	LD.25,000 and More	
1–2	12	5	17
3–4	5	8	13
5 or More	4	12	16
Totals	21	25	46

The derived chi-square value = 7.28
The critical chi-square values with two degrees of freedom at:
The .10 level of significance = 4.605
The .05 level of significance = 5.991

2. *Age of the Accounting Firms and the Extent of Rendering Accounting Services*

Age of the responding Libyan public accounting firms is one of the considerable variables in this study. In order to determine whether the length of time the responding firms have been practicing the public

TABLE 4. *Age of the Responding Libyan Public Accounting Firms and the Extent of Rendering Professional Accounting Services.*

Number of Services Rendered	Age of Firms		Total
	1–5 Years	More than 5 Years	
1–2	11	6	17
3–4	5	8	13
5 or More	4	12	16
Totals	20	26	46

The calculated chi-square value = 5.47
The critical chi-square values with two degrees of freedom at:
The .10 level of significance = 4.605
The .05 level of significance = 5.991

riers. Although a number of respondents reported that their firms did not render some of the accounting services for various reasons, the majority felt that the shortage of qualified accountants who are equipped with adequate knowledge is the major constraint on providing such services. However, it seems that many of the Libyan accounting firms are aware of the need for professional accounting services and are prepared to render the needed services currently or in the near future. These respondents reported that elimination of the Libyan environmental barriers must be undertaken in order to accomplish this desire.

In order to gather additional information needed for this study, the recipients of the questionnaire were asked to indicate the extent to which professional accounting services have been provided by their firms. Answers of respondents on this subject are summarized in Table 1.

Three, or 6.5 percent, of the responding Libyan public accounting firms were rendering only one service. Fourteen accounting firms, or 30.4 percent, were providing two services; six firms, or 13.1 percent, engaged in providing three services; seven firms, or 15.2 percent, rendered four services; and ten firms, or 21.7 percent, were involved in rendering five services. Five, or 10.9 percent, among the responding Libyan accounting firms were rendering six accounting services. Only one firm reported that it was providing seven accounting services for its clients.

In addition, the date presented in Table 1 shows that the majority of responding accounting firms (65.2 percent) were providing four services or less. On the other hand, only sixteen Libyan public accounting firms were engaged in rendering five or more accounting services for their clients.

TABLE 1. *The Extent of Providing Professional Accounting Services by the Responding Libyan Accounting Firms.*

Number of Services	Number of Firms	Percent of Total
Rendering one service	3	6.5
Rendering two services	14	30.4
Rendering three services	6	13.1
Rendering four services	7	15.2
Rendering five services	10	21.7
Rendering six services	5	10.9
Rendering seven services	1	2.2
Totals	46	100.00

3. HYPOTHESES TESTED

In order to draw a conclusion concerning the study's major hypotheses, the proposed five operational hypotheses will be investigated statistically as follows:

1. *The Relationship between Size of the Accounting Firms and the Extent of Providing Accounting Services*

The size classification of responding public accounting firms is determined in term of full-time employment and annual gross fees. The chi-square nonparametric statistical method of analysis was used to test if there is any significant relationship between size of the accounting firms in terms of full-time people employed, and the extent of providing accounting services. Specifically, the null hypothesis (H_0) in this situation is that there is no relationship between the size of accounting firms and the number of professional accounting services provided. The corresponding research hypothesis (H_1) states that a relationship does exist between the size of accounting firms and the extensiveness of rendering accounting services. Rejection of the null hypothesis would indicate that a relationship exists.

The Contingency Table 2 and its chi-square values provide the needed information for the proposed statistical test. Evaluation of this data indicates that there is a statistically significant relationship between the size of Libyan public accounting firms in terms of full-time employment and the extent to which accounting services have been provided by these firms. The null

The chi-square nonparametric statistical method of analysis will be used whenever possible to test the research operational hypotheses and to make some inferences from the study. The chi-square test was judged to be more appropriate than other statistical methods in this situation since the sample of N observations is a random sample and the data can be categorized into a contingency table.

The general formula to be used in computing the chi-square test statistic is as follows[1]:

$$T = \sum_{i=1}^{r} \sum_{j=1}^{C} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} = observed number of cases categorized in i^{th} row of j^{th} column,

E_{ij} = number of cases expected to be categorized in i^{th} row of j^{th} column,

$\sum_{i=1}^{r} \sum_{j=1}^{C}$ means that all (r) rows and all (c) columns should be summed in order to sum all cells.

In order to find the number of expected cases for each cell (E_{ij}), one must multiply the two marginal totals related to a particular cell in the contingency table, and then divide this result by the total number of cases in the study's sample (n). Therefore, $E_{ij} = R_i C_j / N$. The value of the test statistic obtained by the general chi-square formula has always $(r-1)(c-1)$ degrees of freedom.[2]

For purposes of this study, the .10 percent level of significance is used. However, greater or lesser degrees of significance are also considered.

[1] W. J. Conover, *Practical Nonparametric Statistics* (New York: John Wiley & Sons, Inc., 1971), p. 155.
[2] *Ibid.*, pp. 155–156. Also see Sidney Siegel, *Nonparametric Statistics for the Behavioral Sciences* (New York: McGraw-Hill Book Company, 1956), pp. 104–110.

2. SERVICES RENDERED BY LIBYAN ACCOUNTING FIRMS

Proprietors, Managing Partners or Accountants in charge of each firm in the sample were asked to answer the questionnaire. Information such as types of services rendered, length of time for each service, need for and availability of services, Constraints on rendering accounting services and awareness of the need for these services were provided by respondents.

Although a trend to shift toward other services was noticeable, auditing was the major source of revenue for all the responding Libyan public accounting firms. In relation, other services had a secondary importance as a source of revenue. Respondents reported that all their firms were rendering auditing services. Thirty-four firms, or 73.9 percent of the responding firms, performed bookkeeping. Thirty firms, or 65.2 percent of the respondents, were engaged in tax practices. Less than half of the responding Libyan accounting firms provided liquidation services. Less than 25 percent of the responding firms rendered systems design and installations, and management advisory services. Other services, such as court and bank consultations, were provided by ten of the responding Libyan public accounting firms.

Auditing and bookkeeping services have been widely provided by most of the responding Libyan accounting firms since their origin. Tax services and liquidations have been introduced only during the last ten years. On the other hand, management advisory services and systems work have had a short history in Libya. Most of the responding public accounting firms which were engaged in rendering these services started doing so during the last five years.

The findings of this study indicate that the need for professional accounting services in Libya exceeds their availability to the business community. Some of the needed accounting services could not be provided because of the Libyan environmental bar-