

ON THE PRECISION OF POPULATION PROJECTIONS WITH REFERENCE TO LIBYAN DATA

By : A. H. Azzam and M. Y. El-Bassiouni *

1. Introduction

A population projection is a prediction of a random vector variable X_t which represents the population by age and sex in year t . (For an illustration of the population projection for Libya, see Venkatacharya (1972), see also Appendix I). The population is assumed to be closed and to grow according to fixed schedules of birth and death probabilities. This can be viewed as a multi-type branching process (Schweder, 1971).

There are three main causes for the deviations in the projection as pointed out by Schweder (1971), “ (a) The projection model is inadequate in that the dynamics of the population development is not correctly described by the given fixed schedules in all years $1, 2, \dots, T$. (b) Even if these schedules are constant throughout the projection period we should expect a deviation simply because we use their *estimates*. (c) Even if the model is adequate and the proper schedules are used we should still expect a non-vanishing deviation because the population development is stochastic and not deterministic in its nature. ”

Sykes (1969) has shown by a specific example that the branching process model does not account for any substantial part of the observed variability in the dynamics of actual populations, i.e., source of

*) Of the Institute of Statistical Studies & Research - Cairo University - Egypt. Currently at the Faculty of Economics and Commerce, Benghazi University Benghazi, Libya.

deviation (c) is not very important. Using Norwegian population data Schweder (1971) has come to the same conclusion as that of Sykes.

These conclusions of Schweder (1971) while true for a country like Norway may not hold entirely for other countries. We wish to examine whether these conclusions are applicable to a developing country like Libya. We also wish to study the impact of changes in the levels of mortality and fertility on the precision of the projections.

2. Methodology

The Model :

The model used in this paper is essentially that of Schweder (1971). The initial female population is represented by the vector e_0 and gives the number of females in various age groups j ($j=0, 1, \dots, m$). (The initial time is $t=0$. Here t is measured in years.) Using the survival and fertility rates P and F , say, we have the following well-known recurrence relations :

$$\begin{aligned} e_{0,t+1} &= \sum_{j=0}^m F_j e_{j,t} \\ e_{j,t+1} &= P_{j-1} e_{j-1,t} \end{aligned} \quad j=1, \dots, m, (1)$$

where F_j are zero for some initial values of j (See Table (1)), and $e_t (= E X_t)$ is the usual projection vector.

In his paper, Schweder (1971) has obtained upper limits of the individual absolute deviations (component deviations) which have a preassigned probability level β , say. These limits are given by the following formula

$$\left| X_{jt} - e_{jt} \right| \leq (SC_{jj}^t)^{\frac{1}{2}} \quad , j=0, 1, \dots, m, (2)$$

where C_{jj}^t are the diagonal elements of the variance-covariance matrix C^t with rank r , say, (the recurrence relations for the matrix C^t are given in appendix I), and S is the β — percentage point in the χ^2 distribution with r degrees of freedom. He has also given the following upper limits for the total absolute deviation :

$$\sum_{j=0}^m |x_{jt} - e_{jt}| \leq V_t \sqrt{S}, \quad \dots \dots \dots (3)$$

where $V_t^2 = \sum_{j=0}^m \sum_{k=0}^m C_{kj}^t$

Schweder has proposed the relative total deviation

$$B_t = V_t \sqrt{S} \sum_{j=0}^m e_{jt} \quad \dots \dots \dots (4)$$

as a measure of the precision of the projection.

Applied to the Norwegian data, the individual upper limits, given by equation (2), constitute higher percentages of the corresponding projected numbers than the relative total deviation. So, we divided the total absolute deviation to various single year age groups according to the proportions held among the individual upper limits, i.e.,

$$D_j = \frac{TD \times CD_j}{\sum_j CD_j} \quad \dots \dots \dots (5)$$

where D_j represents the divided component deviation in age group j , CD_j the component deviation in the same age group, and TD is the upper limit of the total deviation obtained from equation (3).

Further, a cohort analysis is also made to examine how the precision is varying over time. For this we use 5- year age groups.

The Inputs :

Libyan female population alone is considered. The distribution by 5-year age groups is obtained from 1964 Census (Ministry of Economy and Trade, 1964). The number of females at various single year age groups interpolated using Karup-King multipliers (see Venkatacharya 1972) is used as our initial population vector e_0 . Three levels of survival rates are used in the present paper. They are chosen as levels 12, 13, and 17 for females taken from the South Model Life Tables (United Nations, 1966) and will be labelled P_1 , P_2 , and P_3 respectively. These are given for 5-year age groups. Their natural logarithms are interpolated using Karup-King multipliers. The required rates for single year age groups are then obtained by taking their exponential.

Five levels of fertility rates are considered. They will be labelled as F_1 , F_2 , F_3 , F_4 , and F_5 . F_1 , F_2 , F_4 and F_5 are multipliers of F_3 . These are .85, .95, 1.05, and 1.15. The rates at all ages are multiplied by the same number. F_3 is obtained for 5-year age groups from a sample survey in 1969 (Mukherji, S., 1972). The required single year rates are obtained by graphical smoothing which retains the original age pattern of the fertility schedule. These rates are then multiplied by the sex ratio at birth. This sex ratio is taken as 105 for Libya. Table (1) gives the initial population vector e_0 , the survival rates for level 13, and the fertility rates F_3 all for single years of age.

3. Results and Conclusions

Several runs of projected female population for Libya and related errors have been made using the computer program given in appendix II for the following combinations of survival and fertility rates : $P_1 F_1$, $P_1 F_2$, $P_1 F_4$, $P_1 F_5$, $P_2 F_3$, $P_3 F_1$, $P_3 F_2$, $P_3 F_4$, and $P_3 F_5$. Starting with the female age distribution in 1964, the female population is projected for the next 20 years. A sample of the computer output using the inputs of Table (1) are given in appendix III.

Table (2) gives the relative total deviation (B_t), $t=1, 2, \dots, 20$ for the nine combinations of survival and fertility rates mentioned above (Confidence level $\beta = .95$). Examination of Table (2) leads to the following conclusions :

1. The relative total deviation increases by time for all combinations.
2. For the same fertility rates, the relative total deviation decreases when the survival rates increase, that is as mortality declines. This is clear from equation (9) of appendix I.
3. For the same survival rates :
 - (a) In the first 10 years or so, the relative total deviation increases when the fertility rates increase.
 - (b) In the last 10 years or so, the relative total deviation decreases when the fertility rates increase. The magnitude of this change in the second half of the 20 year period is small and this could be due to changing age distribution.
4. Compared with the Norwegian data, the relative total deviation of the projected Libyan female population is relatively high. However, the pure randomness of population dynamics seems to be of minor importance and source of deviation in projection must rest mainly on year-to-year variation in death probabilities and birth distributions, and in the error made when estimating these quantities.

Table (3) presents percentage component deviations for five year age groups. Table (4) presents divided relative deviations by five-year age groups. A study of these tables reveals the following points :

1. The relative deviations and the divided relative deviations increase for a cohort over time.
2. They decrease when the survival rates increase.
3. The relative deviations are not affected by the use of the different

levels of fertility rates. This is due to fact that F_1 , F_2 , F_4 and F_5 are multipliers of F_3 which are cancelled out when calculating the relative deviations (see appendix I). In fact this is a limitation of the individual upper limits proposed by Schweder.

4. The proposed divided relative deviations increase when the fertility rates increase. This is an important merit of using divided relative deviations instead of those suggested by Schweder. However, the divided relative deviations are based on intuitive ground rather than any mathematical proofs.
5. The age pattern of the relative deviations is similar to that of the mortality rates. The same holds true for the divided relative deviations.
6. The component deviations, as obtained by us for Libya as well as those obtained by Schweder, for Norway are very large for individual ages : Some of the upper limits for component deviations are as high as 40% while total deviation is about 3%, and hence the usefulness of such confidence limits are negligible.

For this and other reasons mentioned in the paper, we suggest the use of divided component deviations for the measurement of precision in the population projections.

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References

- 1) Coale, A. J., and Demeny, P. (1966), "Regional Model Life Tables and Stable Populations", Princeton University Press, Princeton, New Jersey.
- 2) Kingdom of Libya, Ministry of Economy and Trade, (1964), "General Population Census 1964", Census and Statistics Department, Tripoli.
- 3) Mukerji, S., (1972), "A Demographic Study in Benghazi", Faculty of Economics and Commerce, University of Libya. (In press).
- 4) Schweder, T. (1971), "The Precision of Population Projections Studied by Multiple Prediction Methods", *Demography*, Vol. (8), pp. 441-450.
- 5) Sykes, Z. M. (1969), "Some Stochastic Versions of the Matrix Model for Population Dynamics", *J. Am. Statist. Ass.*, Vol. (64), pp. 111-130.
- 6) Venkatacharya, K. (1972), "Population Projections of Libya 1964-1984", *Dirassat*, Vols. 8. Faculty of Economics and Commerce, University of Libya, Benghazi, Libya.
- 7) Venkatacharya, K. (1972), "The Study of Future School Enrollments in Libya : 1969-83. (To be published in *DIRASSAT*, Journal of The Faculty of Economics).

TABLE 1

INPUTS

(number of females in hundreds)

J	e_o	P_2	F_3
0	305.	0.951876	0.0
1	289.	0.958171	0.0
2	274.	0.964027	0.0
3	261.	0.969437	0.0
4	248.	0.974392	0.0
5	237.	0.977911	0.0
6	223.	0.983398	0.0
7	212.	0.987441	0.0
8	204.	0.990021	0.0
9	198.	0.991127	0.0
10	192.	0.993042	0.0
11	184.	0.995953	0.0
12	176.	0.997651	0.0
13	170.	0.998132	0.0
14	164.	0.997393	0.0
15	156.	0.996772	0.003800
16	149.	0.996699	0.032700
17	143.	0.996627	0.054600
18	138.	0.996555	0.080000
19	136.	0.996483	0.117100
20	133.	0.996123	0.156100
21	130.	0.995880	0.182000

TABLE 1 (Cont.)

J	e_o	P_2	F_3
22	126.	0.995673	0.191700
23	123.	0.995503	0.188300
24	120.	0.995368	0.183400
25	116.	0.995204	0.177600
26	113.	0.995015	0.171700
27	110.	0.994872	0.165400
28	107.	0.994773	0.159500
29	104.	0.994719	0.152700
30	102.	0.994666	0.146300
31	100.	0.994583	0.139500
32	97.	0.994499	0.134100
33	94.	0.994414	0.127800
34	91.	0.994327	0.121600
35	87.	0.994255	0.114100
36	83.	0.994184	0.106300
37	80.	0.994091	0.097100
38	77.	0.993975	0.086800
39	75.	0.993835	0.073700
40	72.	0.993715	0.062000
41	69.	0.993610	0.052200
42	66.	0.993477	0.043400
43	64.	0.993316	0.034600
44	62.	0.993127	0.027800
45	59.	0.993024	0.021500
46	57.	0.992939	0.016100
47	54.	0.992726	0.010700
48	52.	0.992384	0.006300

TABLE 2

Relative total deviations
(Confidence level $\beta = .95$)

Time	P ₁ F ₁	P ₁ F ₂	P ₁ F ₄	P ₁ F ₅	P ₂ F ₃	P ₃ F ₁	P ₃ F ₂	P ₃ F ₄	P ₃ F ₅
1	0.0229	0.0236	0.0243	0.0248	0.0236	0.0214	0.0221	0.0228	0.0235
2	0.0312	0.0320	0.0328	0.0334	0.0320	0.0291	0.0300	0.0308	0.0316
3	0.0368	0.0377	0.0384	0.0391	0.0376	0.0344	0.0354	0.0362	0.0369
4	0.0411	0.0419	0.0426	0.0432	0.0417	0.0384	0.0394	0.0401	0.0408
5	0.0445	0.0453	0.0459	0.0464	0.0450	0.0416	0.0425	0.0432	0.0437
6	0.0474	0.0481	0.0486	0.0490	0.0477	0.0443	0.0450	0.0458	0.0460
7	0.0498	0.0504	0.0508	0.0510	0.0499	0.0446	0.0471	0.0475	0.0478
8	0.0519	0.0523	0.0526	0.0528	0.0517	0.0483	0.0488	0.0491	0.0493
9	0.0537	0.0540	0.0542	0.0542	0.0533	0.0499	0.0503	0.0505	0.0505
10	0.0553	0.0555	0.0555	0.0554	0.0547	0.0513	0.0515	0.0516	0.0515
11	0.0567	0.0567	0.0567	0.0564	0.0558	0.0525	0.0526	0.0525	0.0523
12	0.0579	0.0579	0.0577	0.0573	0.0568	0.0535	0.0534	0.0532	0.0529
13	0.0591	0.0589	0.0585	0.0581	0.0577	0.0544	0.0542	0.0538	0.0534
14	0.0602	0.0598	0.0593	0.0588	0.0584	0.0552	0.0548	0.0544	0.0538
15	0.0611	0.0606	0.0600	0.0594	0.0592	0.0559	0.0554	0.0548	0.0541
16	0.0620	0.0614	0.0607	0.0599	0.0598	0.0565	0.0558	0.0551	0.0543
17	0.0629	0.0622	0.0613	0.0605	0.0604	0.0570	0.0562	0.0554	0.0545
18	0.0638	0.0629	0.0620	0.0610	0.0610	0.0575	0.0567	0.0556	0.0547
19	0.0647	0.0636	0.0626	0.0616	0.0617	0.0581	0.0571	0.0580	0.0550
20	0.0656	0.0645	0.0634	0.0622	0.0624	0.0587	0.0576	0.0564	0.0553

TABLE 3

Percentage component deviations by five-year age groups
at five year intervals

Year Age Groups		1964	1969	1974	1979	1984
0-4	P ₁ F ₁	0.0				
	P ₁ F ₂	0.0				
	P ₁ F ₄	0.0				
	P ₁ F ₅	0.0				
	P ₂ F ₃	0.0				
	P ₃ F ₁	0.0				
	P ₃ F ₂	0.0				
	P ₃ F ₄	0.0				
	P ₃ F ₅	0.0				
5-9	P ₁ F ₁	0.0	.1983			
	P ₁ F ₂	0.0	»			
	P ₁ F ₄	0.0	»			
	P ₁ F ₅	0.0	»			
	P ₂ F ₃	0.0	.1868			
	P ₃ F ₁	0.0	.1429			
	P ₃ F ₂	0.0	»			
	P ₃ F ₄	0.0	»			
	P ₃ F ₅	0.0	»			
10-14	P ₁ F ₁	0.0	.1283	.2326		
	P ₁ F ₂	0.0	»	»		
	P ₁ F ₄	0.0	»	»		
	P ₁ F ₅	0.0	»	»		
	P ₂ F ₃	0.0	.1187	.2176		
	P ₃ F ₁	0.0	.0808	.1609		
	P ₃ F ₂	0.0	»	»		
	P ₃ F ₄	0.0	»	»		
	P ₃ F ₅	0.0	»	»		

TABLE 3 (Cont.)

Year Age Groups		1964	1969	1974	1979	1984
15-19	P ₁ F ₁	0.0	.0811	0.1493	.2436	
	P ₁ F ₂	0.0	»	»	»	
	P ₁ F ₄	0.0	»	»	»	
	P ₁ F ₅	0.0	»	»	»	
	P ₂ F ₃	0.0	.0753	0.1381	.2275	
	P ₃ F ₁	0.0	.0517	0.0940	.1669	
	P ₃ F ₂	0.0	»	»	»	
	P ₃ F ₄	0.0	»	»	»	
	P ₃ F ₅	0.0	»	»	»	
20-24	P ₁ F ₁	0.0	.0984	0.1209	.1717	.2567
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₃	0.0	.0918	0.1125	.1589	.2394
	P ₃ F ₁	0.0	.0656	0.0788	.1091	.1746
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»
25-29	P ₁ F ₁	0.0	.1197	0.1499	.1583	.1970
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₃	0.0	.1119	0.1400	.1479	.1826
	P ₃ F ₁	0.0	.0810	0.1005	.1047	.1263
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»

TABLE 3 (Cont.)

Year Age Groups		1964	1969	1974	1979	1984
30-34	P ₁ F ₁	0.0	.1366	.1761	.1933	.1938
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₃	0.0	.1281	.1647	.1805	.1807
	P ₃ F ₁	0.0	.0939	.1197	.1302	.1288
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»
35-39	P ₁ F ₁	0.0	.1507	.1981	.2223	.2320
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₃	0.0	.1417	.1858	.2081	.2168
	P ₃ F ₁	0.0	.1054	.1369	.1519	.1571
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»
40-44	P ₁ F ₁	0.0	.1719	.2192	.2495	.2645
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₁	0.0	.1622	.2063	.2342	.2478
	P ₃ F ₁	0.0	.1233	.1547	.1738	.1821
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»

TABLE 3 (Cont.)

Year Age Groups		1964	1969	1974	1979	1984
45-49	P ₁ F ₁	0.0	.1979	.2512	.2773	.2973
	P ₁ F ₂	0.0	»	»	»	»
	P ₁ F ₄	0.0	»	»	»	»
	P ₁ F ₅	0.0	»	»	»	»
	P ₂ F ₃	0.0	.1879	.2377	.2615	.2796
	P ₃ F ₁	0.0	.1478	.1834	.1988	.2100
	P ₃ F ₂	0.0	»	»	»	»
	P ₃ F ₄	0.0	»	»	»	»
	P ₃ F ₅	0.0	»	»	»	»

TABLE 4

Divided relative deviations by five-year age groups at five year intervals

Year Age Groups		1964	1969	1974	1979	1984
0-4	P ₁ F ₁	0.0				
	P ₁ F ₂	0.0				
	P ₁ F ₄	0.0				
	P ₁ F ₅	0.0				
	P ₂ F ₃	0.0				
	P ₃ F ₁	0.0				
	P ₃ F ₂	0.0				
	P ₃ F ₄	0.0				
	P ₃ F ₅	0.0				
5-9	P ₁ F ₁	0.0	0.0441			
	P ₁ F ₂	0.0	0.0449			
	P ₁ F ₄	0.0	0.0457			
	P ₁ F ₅	0.0	0.0463			
	P ₂ F ₃	0.0	0.0436			
	P ₃ F ₁	0.0	0.0362			
	P ₃ F ₂	0.0	0.0368			
	P ₃ F ₄	0.0	0.0374			
	P ₃ F ₅	0.0	0.0379			
10-14	P ₁ F ₁	0.0	0.0285	0.0468		
	P ₁ F ₂	0.0	0.0291	0.0476		
	P ₁ F ₄	0.0	0.0295	0.0482		
	P ₁ F ₅	0.0	0.0300	0.0488		
	P ₂ F ₃	0.0	0.0277	0.0455		
	P ₃ F ₁	0.0	0.0204	0.0354		
	P ₃ F ₂	0.0	0.0208	0.0359		
	P ₃ F ₃	0.0	0.0211	0.0363		
	P ₃ F ₄	0.0	0.0214	0.0367		

TABLE 4 (Cont.)

Year Age Groups		1964	1969	1974	1979	1981
15-19	P ₁ F ₁	0.0	0.0180	0.0301	0.0465	
	P ₁ F ₂	0.0	0.0184	0.0305	0.0471	
	P ₁ F ₄	0.0	0.0187	0.0309	0.0477	
	P ₁ F ₅	0.0	0.0189	0.0313	0.0483	
	P ₂ F ₃	0.0	0.0176	0.0239	0.0447	
	P ₃ F ₁	0.0	0.0131	0.0207	0.0337	
	P ₃ F ₂	0.0	0.0133	0.0210	0.0341	
	P ₃ F ₄	0.0	0.0135	0.0212	0.0344	
	P ₃ F ₅	0.0	0.0137	0.0215	0.0347	
20-24	P ₁ F ₁	0.0	0.0219	0.0243	0.0327	0.0481
	P ₁ F ₂	0.0	0.0223	0.0247	0.0332	0.0488
	P ₁ F ₄	0.0	0.0227	0.0251	0.0336	0.0494
	P ₁ F ₅	0.0	0.0230	0.0254	0.0340	0.0500
	P ₂ F ₃	0.0	0.0214	0.0235	0.0312	0.0460
	P ₃ F ₁	0.0	0.0166	0.0174	0.0220	0.0336
	P ₃ F ₂	0.0	0.0169	0.0176	0.0223	0.0340
	P ₃ F ₄	0.0	0.0172	0.0178	0.0225	0.0343
	P ₃ F ₅	0.0	0.0174	0.0180	0.0227	0.0347
25-29	P ₁ F ₁	0.0	0.0266	0.0302	0.0303	0.0369
	P ₁ F ₂	0.0	0.0271	0.0306	0.0307	0.0374
	P ₁ F ₄	0.0	0.0276	0.0311	0.0311	0.0379
	P ₁ F ₅	0.0	0.0280	0.0315	0.0315	0.0384
	P ₂ F ₃	0.0	0.0261	0.0292	0.0291	0.0350
	P ₃ F ₁	0.0	0.0205	0.0221	0.0211	0.0243
	P ₃ F ₂	0.0	0.0209	0.0224	0.0214	0.0246
	P ₃ F ₄	0.0	0.0212	0.0227	0.0216	0.0249
	P ₃ F ₅	0.0	0.0215	0.0229	0.0218	0.0251

TABLE 4 (Cont.)

Year Age Groups		1964	1969	1974	1979	1984
30-34	P ₁ F ₁	0.0	0.0304	0.0354	0.0369	0.0363
	P ₁ F ₂	0.0	0.0309	0.0360	0.0374	0.0368
	P ₁ F ₄	0.0	0.0315	0.0365	0.0379	0.0373
	P ₁ F ₅	0.0	0.0319	0.0370	0.0383	0.0378
	P ₂ F ₃	0.0	0.0299	0.0344	0.0355	0.0347
	P ₃ F ₁	0.0	0.0238	0.0264	0.0263	0.0248
	P ₃ F ₂	0.0	0.0242	0.0267	0.0266	0.0251
	P ₃ F ₄	0.0	0.0246	0.0270	0.0269	0.0253
	P ₃ F ₅	0.0	0.0249	0.0273	0.0271	0.0256
35-39	P ₁ F ₁	0.0	0.0335	0.0399	0.0424	0.0435
	P ₁ F ₂	0.0	0.0341	0.0405	0.0430	0.0441
	P ₁ F ₄	0.0	0.0347	0.0411	0.0435	0.0447
	P ₁ F ₅	0.0	0.0352	0.0416	0.0440	0.0452
	P ₂ F ₃	0.0	0.0331	0.0388	0.0409	0.0416
	P ₃ F ₁	0.0	0.0267	0.0301	0.0307	0.0303
	P ₃ F ₂	0.0	0.0271	0.0306	0.0310	0.0306
	P ₃ F ₄	0.0	0.0276	0.0309	0.0313	0.0309
	P ₃ F ₅	0.0	0.0279	0.0312	0.0316	0.0312
40-44	P ₁ F ₁	0.0	0.0382	0.0441	0.0476	0.0496
	P ₁ F ₂	0.0	0.0389	0.0448	0.0483	0.0503
	P ₁ F ₄	0.0	0.0396	0.0454	0.0489	0.0509
	P ₁ F ₅	0.0	0.0402	0.0460	0.0494	0.0516
	P ₂ F ₃	0.0	0.0379	0.0431	0.0460	0.0476
	P ₃ F ₁	0.0	0.0312	0.0341	0.0351	0.0351
	P ₃ F ₂	0.0	0.0318	0.0345	0.0355	0.0355
	P ₃ F ₄	0.0	0.0322	0.0349	0.0358	0.0358
	P ₃ F ₅	0.0	0.0327	0.0353	0.0362	0.0362

TABLE 4 (Cont.)

Year Age Groups		1964	1969	1974	1979	1984
45-49	P ₁ F ₁	0.0	0.0440	0.0506	0.0529	0.0557
	P ₁ F ₂	0.0	0.0448	0.0514	0.0536	0.0565
	P ₁ F ₄	0.0	0.0456	0.0521	0.0543	0.0572
	P ₁ F ₅	0.0	0.0462	0.0527	0.0549	0.0580
	P ₂ F ₃	0.0	0.0439	0.0497	0.0513	0.0537
	P ₃ F ₁	0.0	0.0374	0.0404	0.0401	0.0404
	P ₃ F ₂	0.0	0.0381	0.0409	0.0406	0.0409
	P ₃ F ₄	0.0	0.0387	0.0414	0.0410	0.0413
	P ₃ F ₅	0.0	0.0392	0.0419	0.0414	0.0417

APPENDIX I

Equations in the Model

Let the population at time t be characterized by the numbers X_{jt} in each of the $m+1$ disjoint age groups ($j=0, 1, \dots, m$) at time t which are assumed to be random variables. Let e_{jt} be the expectation of X_{jt} ($j=0, 1, \dots, m$), Y_j^i be the number of children that female number i in age group j gives birth to in year t , and X_j^i be 0 if female number i in age group j dies during year t and 1 if she survives. We have

$$P = E(X_j^i),$$

$$F = E(Y_j^i),$$

$$\gamma = \text{Cov}(X_j^i, Y_j^i) = \frac{1}{2} F (1-P), \text{ See Schweder (1971).}$$

$$\tau = \text{Var}(X_j^i) = P_j (1-P_j), \text{ and}$$

$$\sigma = \text{Var}(Y_j^i) = F_j (1-F_j), \text{ since very few women give birth to more than one girl during a year.}$$

The following recurrence relations for the expectation e and the covariance matrix C are well-known :

$$(1) \quad e_{0, t+1} = \sum_{j=0}^m F_j e_{j, t},$$

$$(2) \quad e_{j, t+1} = P_{j-1} e_{j-1, t}, \quad j = 1, \dots, m,$$

$$(3) \quad C_{j+1, j+1}^{t+1} = \tau_j e_{j, t} + P_j^2 C_{j, j}^t, \quad j = 0, \dots, m-1,$$

$$(4) \quad C_{j+1, k+1}^{t+1} = P_k P_j C_{k,j}^t, \quad k \neq j,$$

$$(5) \quad C_{0,0}^{t+1} = \sum_{j=0}^m \sigma_j e_{jt} + \sum_{k=0}^m \sum_{j=0}^m F_k F_j C_{k,j}^t$$

and

$$(6) \quad C_{0,j+1}^{t+1} = f_j e_{j,t} + P_j \sum_{k=0}^m F_k C_{k,j}^t, \quad j=0,1,\dots,m-1,$$

where e_0 is the present population and $c^0 = 0$.

If the matrix C has rank r and S is the β - percentage point in the χ^2 distribution with r degrees of freedom, then we have the following inequalities :

$$(7) \quad |X_{jt} - e_{jt}| \leq \sqrt{SC_{jj}^t}, \quad j=0, \dots, m,$$

and

$$(8) \quad \sum_{j=0}^m |X_{jt} - e_{jt}| \leq V_t \sqrt{S}$$

$$\text{where } V_t^2 = \sum_{j=0}^m \sum_{k=0}^m C_{k,j}^t.$$

Hence, $V_t \sqrt{S}$ can be taken as a measure of the total deviation of the projection at confidence level β . The relative total deviation of the projection may correspondingly be measured by the quantity :

$$(9) \quad B_t = V_t \sqrt{S} / \sum_{j=0}^m e_{j,t},$$

APPENDIX II

A Computer Program for Population Projection

Language : Basic FORTRAN

Structure : The subroutine definition is

SUBROUTINE BPP (M,D,P,F,G,T,S,E,CD,PT,TD,B)

Formal Parameters :

M	integer scalar	input : last age group. output : unchanged.
D	real scalar	input : the β -percentage point in x^2 distribution output : unchanged.
P	real array	input : survival rates. output : unchanged.
F	real array	input : fertility rates. output : unchanged.
G	real array	input : covariances of X_j^i and Y_j^i . output : unchanged.
T	real array	input : variances of X_j^i . output : unchanged.
S	real array	input : variances of Y_j^i . output : unchanged.
E	real array	input : projected population vector at time .. output : projected population vector at time $t+1$.
C	real array	input : variance - covariance matrix at time t . output : variance - covariance matrix at time $t+1$.
CD	real array	output : the vector of component deviations.

- PT real scalar output : projected total population.
TD real scalar output : total deviation.
B real scalar output : relative total deviation.

Restrictions : The variables M,D,P,F,G,T,S,E, and C must not be changed from one call of BPP to the next, since items stored in them are required for the next call. It is the responsibility of the calling program to insure that these arrays remain intact between calls and are of sufficient length. They may, of course, be used for any purpose before the sequence of calls of BPP begins and after the sequence is complete.

THE FORTRAN GENERATOR

SUBROUTINE BPP (M,D,P,F,G,T,S,E,C,CD,PT,TD,B)

DIMENSION P(50), F(50), G(50), T(50), S(50), E(50), C(50,50),
CD(50)

M1 = M + 1

SUM = 0.0

DO 20 K = 1, M

SUM = SUM + S(K) * E(K) + F(K) * F(K) * C(K,K)

K1 = K + 1

DO 20 J = K1, M1

20 SUM = SUM + 2.0 * F(K) * F(J) * C(K,J)

SUM = SUM + S(M1) * E(M1) + F(M1) * F(M1) * C(M1,M1)

DO 50 J = 2, M1

JJ = J - 1

C(J, 1) = G(JJ) * E(JJ)

SUM 1 = 0.0

DO 30 K = 1, JJ

30 SUM 1 = SUM 1 + F(K) * C(K,JJ)

DO 40 K = J, M 1

40 SUM 1 = SUM 1 + F(K) * C(JJ,K)

50 C(J, 1) = C(J, 1) + P(JJ) * SUM 1

DO 60 L = 1, M

L 1 = M 1 - L

L 2 = L 1 + 1

60 C(L2, L2) = T(L1) * E(L1) + P(L1) * P(L1) * C(L1, L1)

MM = M - 1

DO 70 J1 = 1, MM

J = M 1 - J 1

JJ = J - 1

J 2 = J + 1

DO 70 K = J2, M1

KK = K - 1

```
70 C(J, K) = P(JJ)*P(KK)*C (JJ, KK)
   C(1, 1) = SUM
   SUM = F(M 1)*E(M 1)
   DO 80 J = 1, M
   JJ = J + 1
   C(1, JJ) = C(JJ, 1)
   J 1 = M 1 - J
   J 2 = J1+1
   E(J 2) = P(J 1)*E(J 1)

80 SUM = SUM + F(J 1)*E(J 1)
   E(1) = SUM
   PT = 0.0
   V 2 = 0.0
   DO 90 J = 1, M
   X = D*C(J, J)
   CD(J) = SQRT(X)
   PT = PT + E(J)
   V 2 = V 2 + C(J, J)
   JJ = J + 1
   DO 90 K = JJ, M 1

90 V 2 = V 2 + 2.0*C(J, K)
   X = D*C(M 1, M 1)
   CD(M 1) = SQRT(X)
   PT = PT + E(M 1)
   V 2 = V 2 + C(M 1, M 1)
   X = D*V 2
   TD = SQRT(X)
   B = TD/PT
   RETURN
   END
```


APPENDIX III

A sample of print out for the years 1974 and 1984

YEAR 1974

NUMBER OF FEMALES IN HUNDREDS

Age	Projected Vector	Component Deviation	Divided Component Deviation
0	468.	165.	35.
1	433.	159.	33.
2	403.	154.	32.
3	377.	150.	31.
4	356.	146.	30.
5	337.	142.	30.
6	321.	139.	29.
7	308.	136.	28.
8	297.	133.	28.
9	287.	131.	27.
10	236.	60.	13.
11	233.	55.	12.
12	230.	50.	10.
13	227.	45.	9.
14	222.	40.	8.
15	217.	35.	7.
16	208.	31.	6.
17	201.	27.	6.
18	195.	24.	5.
19	190.	22.	5.
20	186.	20.	4.
21	178.	19.	4.
22	171.	19.	4.
23	165.	19.	4.
24	158.	19.	4.
25	150.	19.	4.
26	143.	19.	4.
27	137.		

APPENDIX III (Cont.)

Age	Projected Vector	Component Deviation	Divided Component Deviation
28	132.	19.	4.
29	130.	20.	4.
30	127.	20.	4.
31	124.	20.	4.
32	120.	20.	4.
33	117.	20.	4.
34	114.	20.	4.
35	110.	20.	4.
36	107.	19.	4.
37	104.	19.	4.
38	101.	19.	4.
39	98.	19.	4.
40	96.	19.	4.
41	94.	19.	4.
42	91.	19.	4.
43	88.	19.	4.
44	86.	19.	4.
45	82.	18.	4.
46	78.	18.	4.
47	75.	18.	4.
48	72.	18.	4.
49	70.	18.	4.

Projected Total = 9251.

Total Deviation = 505.

Relative Total Deviation = 0.0546035

YEAR 1984

NUMBER OF FEMALES IN HUNDREDS

Age	Projected Vector	Component Deviation	Divided Component Deviation
0	627.	193.	37.
1	579.	185.	36.
2	538.	179.	34.
3	504.	173.	33.
4	475.	169.	32.
5	450.	164.	32.
6	427.	160.	31.
7	408.	157.	30.
8	391.	154.	29.
9	376.	151.	29.
10	362.	148.	28.
11	349.	145.	28.
12	338.	143.	27.
13	328.	141.	27.
14	318.	139.	27.
15	309.	137.	26.
16	300.	135.	26.
17	292.	133.	25.
18	284.	131.	25.
19	276.	129.	25.
20	228.	62.	12.
21	226.	58.	11.
22	226.	53.	10.
23	219.	49.	9.
24	214.	44.	9.
25	209.	41.	8.
26	200.	37.	7.
27	192.	35.	7.
28	187.	33.	6.
29	182.	32.	6.
30	177.	30.	6.
31	170.	30.	6.

YEAR 1984 (Cont.)

Age	Projected Vector	Component Deviation	Divided Component Deviation
32	162.	29.	6.
33	156.	29.	6.
34	150.	29.	6.
35	142.	29.	6.
36	136.	29.	5.
37	130.	28.	5.
38	125.	28.	5.
39	123.	28.	5.
40	120.	28.	5.
41	117.	28.	5.
42	113.	28.	5.
43	110.	28.	5.
44	107.	28.	5.
45	103.	28.	5.
46	100.	27.	5.
47	98.	27.	5.
48	95.	27.	5.
49	92.	27.	5.

Projected Total = 12539.

Total Deviation = 782.

Relative Total Deviation = 0.0623595