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Meta-Model Based Scaling Laws of a Two-Winding Transformer

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ABSTRACT

Recently, there has been growing interest in system-based global optimization techniques for the design of small systems such as micro-grids. To reduce the search space of the global optimization technique, a meta-model based scaling law was introduced. In this paper, a scaling technique was derived in which a transformer mass and power loss were expressed in terms of rated power, specified current density, and frequency. Curve-fitting techniques were used to derive a meta-model for the scaled mass and power loss. To achieve more generality, the meta-model was also defined as a function of frequency.

Keywords: Scaling laws, Meta-model, integrated energy resources, micro grids.

1. Introduction

Meta-model based algorithms are used to capture the performance of a component in a multiple component system such as a transformer in a micro-grid system [1]-[3]. To achieve that, the concept of scaling laws is introduced. By deriving the meta-model for each component, system-based optimization can be conducted to obtain efficient and compact systems with the possibility of varying frequency over ratings.

Global optimization techniques such as genetic algorithms may be utilized to derive the meta-model on the basis of the trade-off between competing performance equations [4]-[8]. This trade-off may be between mass and power loss or between volume and cost. To set the stage, a scaled analytical model of the two winding transformer is first derived in terms of general quantities such as current density, frequency, and rated power. Such quantities do not depend on the transformer size or ratings and thus the meta-model will be general to a wide range of transformer ratings.

To achieve an optimum design of a multi-component system, the coupling between the components of the system should be taken into account. Therefore, it is not accurate to optimize each component separately since the performance of one component affects the performance of another component in the system [1]. For instance, the temperature of a transformer has an affect on the performance of a cascaded converter. In other words, an optimum design of an individual device may not be necessarily the optimum design if the device is a component of a multi-component system.

In another perspective, when all components are considered as a single optimization problem this will lead to a large number of parameters and thus large number of degrees of freedoms. This may preclude the optimization based design of the system to converge to the desired solution. By introducing component's meta-model based scaling laws this issue may be resolved.

The objective of the work presented in this paper is to explore the possibility of developing meta-model based scaling laws for a power transformer. Such scaling laws enable one to approximate key performance metrics, i.e. loss and mass, based upon device power ratings without requiring one to perform a component optimization. Often, large degree of freedom component-level optimization cannot be performed when system-level optimization is considered.

This paper is organized as follow. In Section 2, a model is derived for the two-winding transformer. The scaling laws are then defined in Section 3. The scaled design process is considered in Section 4. In Section 5, the meta-model which represent the transformer optimum designs as function of rated power, frequency, and current density is obtained. Finally, Section 6, concludes the work of this paper.

2. Two-Winding Transformer Model

To explore scaling laws, a simplified two winding, core type transformer shown in Fig. 1. The α-winding (lighter orange) is wound on the left leg and the β-winding (darker orange) is wound on the right leg. For simplicity, the two windings are assumed to have the same dimensions and the clearances between the windings and the core are neglected; therefore,

$$w_\alpha = w_\beta = \frac{w_{ci}}{2} \tag{1}$$

and

$$h_\alpha = h_\beta = \frac{h_{ci}}{2} \tag{2}$$

where w_α, w_β, and w_{ci} are the widths of α-winding, β-winding, and core interior window respectively and h_α, h_β, and h_{ci} are the heights of α-winding, β-winding, and core interior window respectively.

Prior to considering scaling, it is useful to define and describe several key parameters of the transformer. The rms current density for winding j is expressed as

$$J_j = \frac{N_j I_j}{A_j k_{pf}} \tag{3}$$

where N_j and I_j are winding x number of turns and rms current respectively, k_{pf} is the winding packing factor, and A_j is the area of winding j. The winding area is represented by

$$A_j = w_j h_j \tag{4}$$

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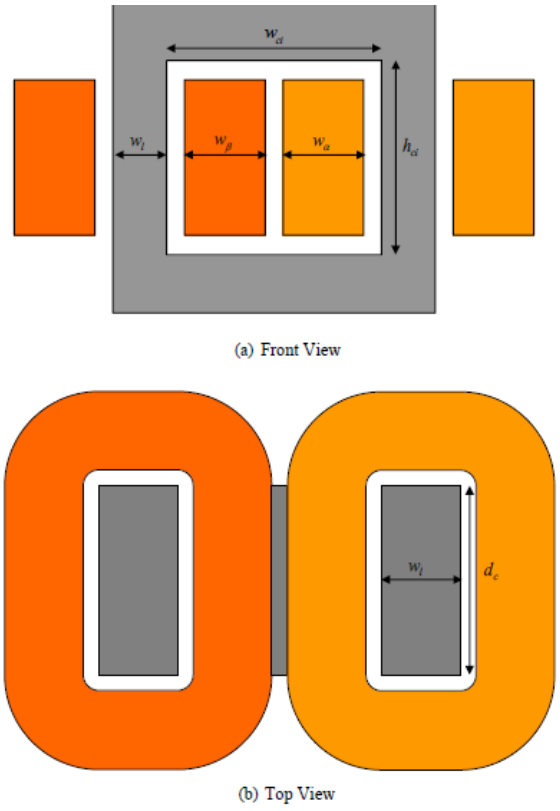


Fig. 1: Two Winding Core Type Transformer Cross-Section

In the work herein, the α-winding and β-winding rms current densities are assumed to be equal

$$J_\alpha = J_\beta = J \tag{5}$$

The mass is another quantity of interest and it is given by

$$M = 2d_c w_l \left(w_{ci} + h_{ci} + 2w_l\right)\rho_c + \sum_{j=\alpha,\beta} k_{pf} U_j \rho_{jc} \tag{6}$$

where ρ_c and ρ_{jc} are the mass density of core material and j-winding conductor respectively and U_j is the volume of winding x which is calculated by

$$U_j = h_j w_j \left(2\left(d_c + w_l\right) + \pi w_j\right) \tag{7}$$

Typically, it is convenient to utilize a T-equivalent circuit when analyzing transformers. The T-equivalent circuit shown in Fig. 2 is considered herein. Within the circuit, the referred (primed) α-winding rms voltage, rms current and resistance are expressed as

$$V_\alpha^{'} = \frac{N_\beta}{N_\alpha} V_\alpha \tag{8}$$

$$I_\alpha^{'} = \frac{N_\alpha}{N_\beta} I_\alpha \tag{9}$$

$$r_\alpha^{'} = \left(\frac{N_\beta}{N_\alpha}\right)^2 r_\alpha \tag{10}$$

As shown, the leakage flux is neglected in the model considered for scaling. The flux path inside the core is assumed to be the average path. The peak flux density is expressed

$$B_{pk} = \frac{\sqrt{2} N_\beta I_m P}{A_c} \tag{11}$$

where I_m is the rms magnetizing current, A_c is the core cross-sectional area, and P is the core permeance which is calculated using the relationship

$$P = \frac{\mu A_c}{l_p} \tag{12}$$

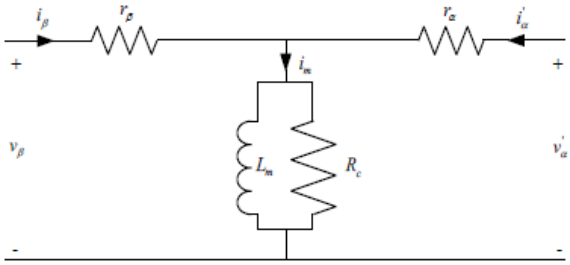


Fig. 2: Transformer T-equivalent Circuit

In (12), μ is the core material permeability and l_p is the flux path average length inside the core which is expressed as

$$l_p = 2\left(w_{ci} + h_{ci} + 2w_l\right) \tag{13}$$

Neglecting the core loss resistance and the voltage drop on the resistance of the β-winding, r_β; the rms magnetizing current can be approximated

$$I_m = \frac{V_\beta}{\omega_e L_m} \tag{14}$$

where V_β is the rms value of the β-winding terminal voltage, ω_e is the angular frequency of the sinusoidal primary voltage, and L_m is the magnetizing inductance which is defined as

$$L_m = N_\beta^2 P \tag{15}$$

Using (3), (12)-(15) and (11) and simplifying one can approximate the peak flux density using:

$$B_{pk} = \frac{\sqrt{2} P_r}{J_\beta A_c A_\beta \omega_e k_{pf}} \tag{16}$$

where P_r is the transformer rated power.

Typically, the magnetizing current is required to be much less than the rated current. This can be achieved by enforcing this constraint

$$I_m < \frac{P_r}{V_\beta} k_m \tag{17}$$

where k_m is a constant which is much less than 1. Substituting equations (3), (14), and (15) into (17) and simplifying yields

$$J_\beta^2 > \frac{P_r}{\omega_e k_m A_\beta^2 k_{pf}^2 P} \tag{18}$$

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It is very interesting to consider (17) and (18). Although the magnetizing current is equal to the sum of the α-winding and the β-winding currents as in Fig. 2, its upper limit can be enforced by setting a lower limit on the β-winding current density.

3. Scaling Laws

The objective of this section is to set the stage for the normalization process by defining the normalization base. The goal is to scale all quantities tied to ratings (i.e. dimensions) and not those that are rating independent (i.e. flux density and field intensity).

One can note from the previous section that many of the key constraints can be expressed in terms of current density. This makes the current density a good candidate to be a parameter in the scaling laws (in addition to power and frequency). Another advantage of selecting the current density as a parameter is that it is a general quantity. In other words, a particular value of the current density may correspond to a wide range of transformer sizes, power ratings, and voltage levels.

3.1. Geometrical Quantities

To establish the meta-model, the linear dimensions are scaled as [1]

$$\hat{x} = x / D \tag{19}$$

In (19), the notation '^' denotes the scaled quantity and D is the normalization base. The area and volume are scaled accordingly using [1]

$$\hat{a} = a / D^2 \tag{20}$$

$$\hat{U} = U / D^3 \tag{21}$$

Substituting (19) and (21) into (6), normalized mass is expressed as

$$\hat{M} = 2\hat{l}_c \hat{w}_l \left(\hat{w}_{ci} + \hat{h}_{ci} + 2\hat{w}_l \right) \rho_c + \sum_{j=\alpha,\beta} k_{pf} \hat{U}_j \rho_{jc} \tag{22}$$

where

$$\hat{M} = M / D^3 \tag{23}$$

3.2. Electrical Quantities

It is desired not to scale the flux density when deriving the meta-model. Considering (11), (12), (19), and (20), to keep B_m unscaled the current must be scaled as [1]

$$\hat{i} = i / D \tag{24}$$

From (3), (20), and (24), the current density is expressed

$$\hat{J} = JD \tag{25}$$

The flux linkage associated with winding j is expressed as

$$\lambda_j = N_j \int_{S_j} B \cdot ds \tag{26}$$

where B is the flux density and S_j is the surface.

Since the flux density is not scaled [1], then from (20), the scaled flux linkage can be expressed as

$$\hat{\lambda} = \lambda / D^2 \tag{27}$$

The j- winding instantaneous voltage is calculated using

$$v_j = \frac{l_j N_j}{\sigma a_j} i_j + \frac{d\lambda_j}{dt} \tag{28}$$

where l_j and a_j are the winding j wire length and area respectively, i_j is winding j instantaneous current and σ is the winding conductor material conductivity.

If time is scaled as [1]

$$\hat{t} = t / D^2 \tag{29}$$

then from (1), (2), (5), (10), (11) and (12), the voltage can be expressed in terms of scaled quantities as [1]

$$v_j = \frac{\hat{l}_j N_j}{\sigma \hat{a}_j} \hat{i}_j + \frac{d\hat{\lambda}_j}{\hat{dt}} \tag{30}$$

From which one can observe that voltage is not scaled.

The frequency is the reciprocal of time and therefore, from (29) the frequency is scaled as

$$\hat{f} = fD^2 \tag{31}$$

Since the relationship between the angular frequency and the frequency is

$$\omega = 2\pi f \tag{32}$$

then

$$\hat{\omega} = \omega D^2 \tag{33}$$

From (16), (19), (25), and (33), the flux density is expressed in terms of the scaled quantities as

$$B_{pk} = \frac{\sqrt{2}\hat{P}_r}{\hat{J}_\beta \hat{A}_c \hat{A}_\beta \hat{\omega}_e k_{pf}} \tag{34}$$

where the scaled rated power is defined as [1]

$$\hat{S}_r = S_r / D \tag{35}$$

From (12), (19), and (20) the scaled permeance is

$$\hat{P} = \frac{\mu \hat{A}_c}{\hat{l}_p} \tag{36}$$

where

$$\hat{P} = P / D \tag{37}$$

The constraint on current density (18) can be expressed in terms of scaled quantities

$$\hat{J}_\beta^2 > \frac{\hat{P}_r}{\hat{\omega}_e k_m \hat{A}_\beta^2 k_{pf}^2 \hat{P}} \tag{38}$$

3.3. Voltage Regulation

Due to the winding resistances and leakage inductances, the secondary voltage of a transformer varies with load condition. It is desired in practice to keep this variation within a specified margin which depends on the type of the load and its sensitivity to voltage variations. During normal operation of a transformer, the largest variation in the secondary voltage occurs when the load condition changes from no-load to full-load. Thus, the voltage regulation is defined as the absolute difference between the secondary voltage at full-load and the one at no-load relative to the voltage at no-load:

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$$\chi = \left| \frac{V'_{\alpha,fl} - V'_{\alpha,nl}}{V'_{\alpha,nl}} \right| \quad (39)$$

To simplify analysis, the leakage inductances are neglected in the initial scaling derivations as shown by the transformer electric equivalent circuit in Fig. 2. The leakage inductances will be accounted for in the future research. In addition, the voltage drop on the primary resistance is neglected at no-load since the magnetizing impedance is relatively large compared to the primary resistance. The magnetizing current is neglected at full-load since it is much smaller than the rated load current as enforced by (17). Therefore, the transformer voltage regulation can be approximated as

$$\chi = \left(r_\beta + r'_\alpha \right) \frac{I'_\alpha}{V_\beta} \quad (40)$$

Using (3), (9), (10), and (40) the voltage regulation can be expressed as

$$\chi = \frac{J_\alpha J_\beta k_{pf}}{\sigma P_r} \left(\frac{A_\alpha}{A_\beta} U_\beta + \frac{A_\beta}{A_\alpha} U_\beta \right) \quad (41)$$

Although voltage is not scaled, the voltage regulation can be expressed in terms of scaled quantifies

$$\chi = \frac{\hat{J}_\alpha \hat{J}_\beta k_{pf}}{\sigma \hat{P}_r} \left(\frac{\hat{A}_\alpha}{\hat{A}_\beta} U_\beta + \frac{\hat{A}_\beta}{\hat{A}_\alpha} \hat{U}_\beta \right) \quad (42)$$

3.4. Loss

Transformer power loss is comprised of transformer winding electrical resistance loss and core loss. The resistive power lost in winding j is calculated using

$$P_{loss_j} = I_j^2 N_j^2 \frac{U_j}{k_{pf} A_j^2 \sigma} \quad (43)$$

From (3) and (43), the resistive power lost due to winding*j* may be formulated in terms of the rms current density as

$$P_{loss_j} = \frac{U_j k_{pf} J_j^2}{\sigma} \quad (44)$$

It is noted that the resistive power loss in both windings are equal since the current density and the winding dimensions are assumed to be the same for both windings. Thus, the total resistive loss is twice that in (44). Expressed in terms of scaled quantities using, (21), (25), and (35) to (44) yields

$$\hat{P}_{loss_j} = \frac{\hat{U}_j k_{pf} \hat{J}_j^2}{\sigma} \quad (45)$$

Core loss includes hysteresis loss and eddy current loss. To demonstrate the hysteresis loss, Fig. 3 is first considered. At each cycle, the flux density follows the lower path when it is increasing and it follows the upper path when it is decreasing. Therefore, the trajectory of the flux density forms a loop and the area of this loop represents energy lost in the core in form of heat. This lost energy is referred to as hysteresis loss. Typically, the flux density waveform is not a pure sinusoidal function due to the effect of saturation. In the analysis herein, the flux density waveform is assumed to be sinusoidal by neglecting the saturation effect. Thus the hysteresis loss is estimated using

$$P_h = k_h B_{\max}^{\beta_h} f^{\alpha_h} U_c \quad (46)$$

where k_h, α_h, and β_h are the hysteresis loss constants.

The eddy current loss is also approximated using MSE [8]

$$P_e = k_e B_{\max}^2 f^2 U_c \quad (47)$$

where k_e is the eddy current loss constant.

The total core loss is the sum of the hysteresis and eddy current loss; thus,

$$P_{cl} = P_h + P_e \quad (48)$$

To enable scaling of the hysteresis loss in (46), the constant α_h must be an integer. Typically α_h is very close to 1 and thus it is herein approximated to be 1. The hysteresis loss is thus modeled

$$P_h = k_h B_{\max}^{\beta_h} f U_c \quad (49)$$

Applying (21), (31), and (35) to (49) yields a scaled loss

$$\hat{P}_h = k_h B_{\max}^{\beta_h} \hat{f} \hat{U}_c \quad (50)$$

To obtain the scaled eddy current loss, (21), (31), and (35) are substituted into (47) which yields

$$\hat{P}_e = \left(k_e B_{\max}^2 \hat{f} \hat{U}_c \right) f \quad (51)$$

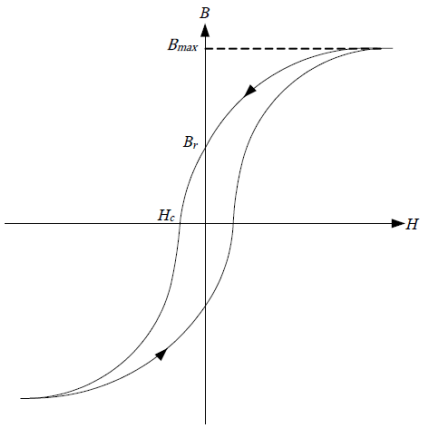


Fig. 3: Magnetizing Curve for a Soft Magnetic Material.

The scaled eddy current loss is a function of the nominal frequency which is undesired in the scaling process. This problem can be addressed by not scaling the frequency or by considering only the hysteresis loss to represent the total core loss. The latest resolution is acceptable in the low frequency range where the hysteresis loss is the dominant core loss. Therefore, if only low frequencies are of interest then only the hysteresis loss is used to represent the total core loss.

3.5. Nominal Design Performance

Before starting the scaled design process, it is useful to explain how one can apply the equations derived thus far to a specific design. If the voltage of winding *j* and transformer rated power are defined, then the winding *j* rated current is calculated using

$$I_j = \frac{P_r}{V_j} \quad (52)$$

If the winding *j* current density is defined and the winding dimensions are known, then the number of turns for the

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corresponding winding is calculated using (3). After calculating the current density, the transformer performance equations can be evaluated.

3.6. Normalization Base Selection

The selection of the normalization base is a very crucial step. Since transformers are typically defined in terms of the rated power, the base of normalization is selected to be the rated power; thus,

$$D = P_r \tag{53}$$

4. Scaled Design Process

Using the scaled model defined by equations (19)-(51), transformer design is considered to establish Pareto-optimal fronts from which a meta-model can be proposed.

The first step in the design process is to define the design vector as

$$\hat{\boldsymbol{\theta}} = \left[\hat{J}\ \hat{h}_{ci}\ r_{ci}\ r_l\ r_c\right]^T \tag{54}$$

where the ratios r_{ci}, r_l, and r_c are defined as

$$r_{ci} = \frac{\hat{w}_{ci}}{\hat{h}_{ci}} \tag{55}$$

$$r_l = \frac{\hat{w}_l}{\hat{h}_{ci}} \tag{56}$$

$$r_c = \frac{\hat{l}_{ci}}{\hat{h}_{ci}} \tag{57}$$

The second step is to implement the design constraints. The less-than and greater-than functions are used to represent the scaled design constraints. [7]

The first constraint is the constraint on the current density

$$c_1 = \mathrm{gte}\left(\hat{J}, \hat{J}_{mnr}\right) \tag{58}$$

where the minimum required current density $\hat{J}_{mnr}$ is

$$\hat{J}_{mnr} > \sqrt{\frac{\hat{P}_r}{\hat{\omega}_e k_m \hat{A}_\beta^2 k_{pf}^2 \hat{P}}} \tag{59}$$

The second constraint is imposed on the voltage regulation as

$$c_2 = \mathrm{lte}\left(\chi, \chi_{mxa}\right) \tag{60}$$

In the analysis used to develop the scaled model, the magnetic material is assumed to be linear. Therefore, a constraint is imposed on the flux density as

$$c_3 = \mathrm{lte}\left(B_{pk}, B_{mxa}\right) \tag{61}$$

A final constraint is imposed on the total power loss $\hat{P}_l$ as follows

$$c_4 = \mathrm{lte}\left(\hat{P}_l, \hat{P}_{l,mxa}\right) \tag{62}$$

The fitness function used for the performance evaluations is defined as:

$$f(\theta) = \begin{cases} \left[\dfrac{1}{\hat{M}}\ \dfrac{1}{\hat{P}_l}\right]^T & c = 1 \\ (c-1)\left[1\ 1\right]^T & c < 1 \end{cases} \tag{63}$$

where c is defined as

$$c = \frac{1}{n_c}\sum_{i=1}^{n_c} c_i \tag{64}$$

and n_c is the number of constraints.

The fitness function is calculated using the Pseudo-code as illustrated in Fig. 4.

Step 1- define $\hat{\omega}$, k_{pf}, the material parameters, χ_{mxa}, and B_{mxa}
Step 2-determine the parameters of the design vector in (54)
evaluate transformer dimensions
evaluate χ using (42) and B_{pk} using (34)
calculate $\hat{J}_{mnr}$ using the RHS of (38).
3-evaluate c_1, c_2, c_3, and c_4 using (58) and (60)-(62).
evaluate c using (64)
4-calculate $\hat{M}$ using (22)
calculate $\hat{P}_{loss_f}$ using (45)
calculate $\hat{P}_h$ using (50)
evaluate the total loss $\hat{P}_l = 2\hat{P}_{loss_f} + \hat{P}_h$
use (63) to evaluate the fitness function
return to step 2
end

Fig. 4: Multi-Objective Optimization Pseudo-Code.

To define the search space of the multi-objective optimization process, the range of the scaled parameters is defined as follows: $10^5 \le \hat{J} \le 10^{20}$ AW/m^2, $10^{-12} \le \hat{h}_{ci} \le 0.1$ m/W, $0.1 \le r_{ci} \le 10$, $10^{-2} \le r_l \le 10$, and $0.1 \le r_c \le 10$, where r_{ci}, r_l, r_c, and are unit-less. The packing factor k_{pf} is selected to be 0.6, the maximum allowed ratio between the magnetizing and the rated current k_m is chosen to be 0.05, the maximum voltage regulation χ_{mxa} is set to 0.05, the upper limit on the flux density B_{mxa} is 1.4 T, the winding conductor is selected to be copper which has a conductivity σ of 5.959*10^7 S/m and a mass density of 8890 Kg/m^3, and the steel material is chosen to be linear with relative permeability μ_r that is equal to 5000, mass density of 7402 Kg/m^3, and the hysteresis loss constants are chosen to be 64.064 J/m^3 for k_h and 1.7991 for β_h. After defining the design parameters, specifications, and constraints, a multi-objective optimization is conducted with a population size of 2000 and for 2000 generations.

5. Multi-Objective Optimization Results

The normalized loss versus normalized mass when the normalized frequency is 3.75*10^{10} HzW^2 is shown in Fig. 5. This value corresponds to a nominal frequency of 60 Hz at rated power of 25 kW. As shown by Fig. 5, the relationship between normalized loss and normalized mass is composed of two linear regions in the log-log scale. Typically, transformers tend to operate around the knee of the magnetization curve. Since the steel material in the work herein is assumed to be linear, the operating point of the transformer will tend to be against the upper flux density limit. Therefore, the region where the designs are against the upper flux density limit (plotted in red) is selected to obtain the meta-model based scaling law.

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In order to construct meta-model based scaling laws that relate normalized mass and normalized loss to normalized frequency and normalized current density, the multi-objective optimization is conducted at several values of the normalized frequency. Then the values of $\hat{J}$ at each frequency is evaluated and used to obtain plots of the normalized mass versus normalized current density and normalized loss versus normalized current density at each normalized frequencies. These are depicted in Fig. 6 and Fig. 7 respectively.

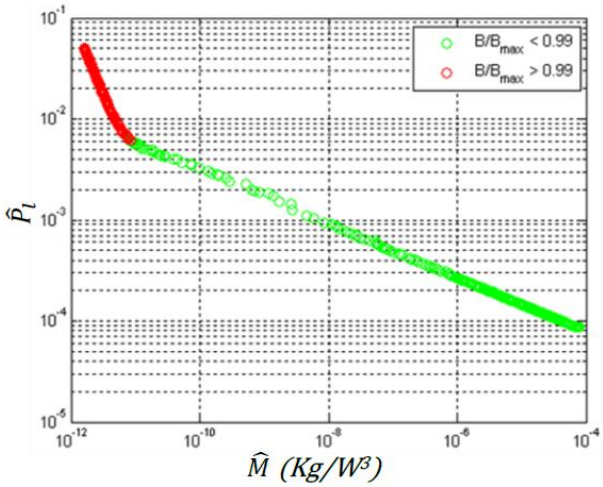


Fig. 5: Normalized Pareto-Optimal Front

By using curve-fitting techniques, a meta-model based scaling law can be constructed from the results shown in Fig. 6 and Fig. 7. The goal is to express the normalized mass and loss as functions of normalized frequency and current density. Relationships of the form

$$\hat{M} = C_M \hat{f}^{nfM} \hat{J}^{nJM} \tag{65}$$

$$\hat{P}_l = C_l \left(\hat{J}\hat{f}^{nfl1} + b_{Jl} \right)^{nJl} \hat{f}^{nfl2} \tag{66}$$

are considered herein.

The parameters of the meta-model expressed by (65) and (66) are calculated using curve fitting techniques and listed in Table. 1 The resulting curves are plotted with the original data in Fig. 6 and Fig. 7. Comparing values, one can see that the meta-model obtained by the curve fitting techniques represents the normalized mass and loss for different values of normalized frequency and current density very well.

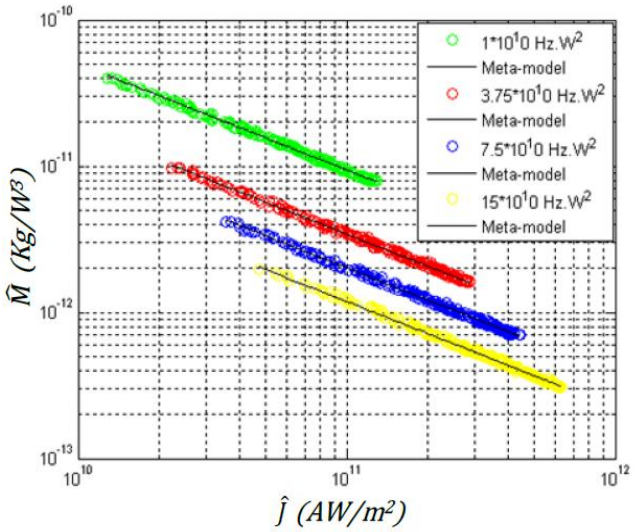


Fig. 6: Normalized Mass versus Normalized Current Density.

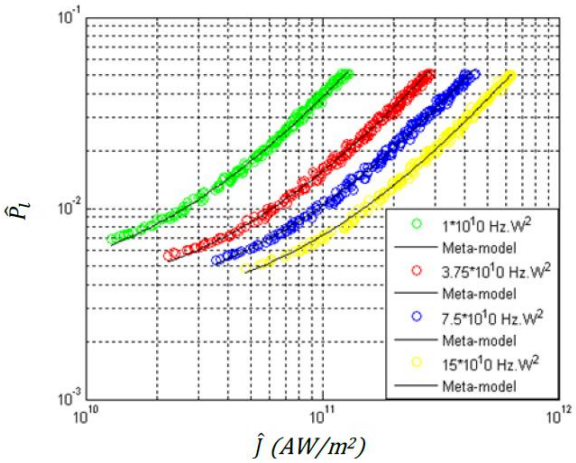


Fig. 7: Normalized Loss versus Normalized Current Density

Table 1. Meta-Model Parameters.

Parameter	Value	Parameter	Value
C_M	4.0298*10^{4}	b_{Jl}	1.9054*10^{5}
n_{fM}	-0.7656	n_{Jl}	1.5276
n_{JM}	-0.7251	n_{fl1}	-0.5142
C_l	3.5328*10^{-10}	n_{fl2}	-0.1069

In practice, it is most useful to express the meta-model in terms of the physical quantities. This can be achieved by applying (25), (33), (35), and (52) to (65) and (66) which yields

$$M = C_M P_r^3 \left(f P_r^2 \right)^{nfM} \left(J P_r \right)^{nJM} \tag{67}$$

$$P_l = C_l P_r \left(J P_r \left(f P_r^2 \right)^{nfl1} + b_{Jl} \right)^{nJl} \left(f P_r^2 \right)^{nfl2} \tag{68}$$

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Equations (67) and (68) can be used to generate the pareto-optimal front for transformers whith specified power rating, (low) operating frequency, and current density. Thus, for any transformer with a defined operating voltage, rated power, and frequency, pareto-optimal front for that transformer can be obtained by sweeping the range of the current density values.

6. Conclusion

In the work herein, meta-model that capture the optimum designs of wide range of two-winding transformer ratings and frequencies is derived using scaling laws. First, the transformer performance equations are derived in terms of general quantities such as rated power, frequency, and current density. Then scaling laws are applied to the transformer model. Using genetic algorithms, a pareto-optimal front which represent the trade-off between scaled mass and loss is obtained. Using curve fitting techniques, a meta-model which relate the transformer mass and loss to its general parameters is derived.

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