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On Convergent Filters in Soft Topological Spaces

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ABSTRACT

This study explains the concept of soft topological spaces in the view soft filters, studying the properties of soft filters in soft topological spaces. The study aims to contribute to the development of soft mathematical concepts and structures, building on the foundations of soft set theory. During the study, we found a relation between the concepts τ-convergence and τ-Hausdorff soft topological spaces.

KEYWORDS: Soft Set, Soft Topological Space, Soft Filter, Soft Ultrafilter, τ-Converges, τ-Hausdorff.

1. INTRODUCTION

Classical tools of Mathematics [19] cannot solve the problems, which are vague rather than precise. To overcome these difficulties Molodtsov [16] initiated the concept of soft set theory which doesn't require the specification of parameters. He applied soft set theory successfully in smoothness of functions, game theory, and operation research and so on. Thereafter so many research works [5] have been done on this concept in different disciplines of Mathematics.

Research on soft sets-based decision-making has received much attention in recent years. Later, in 2003, Maji et al. [14,15] made a theoretical study on soft set theory. They introduced several operations on soft sets and applied soft sets to decision-making problems.

Kharal and Ahmed [8] defined soft mappings. In 2011, Shabir and Naz [17] came up with an idea of soft topological spaces.

Later some researchers [2, 19, 4] and [18] studied on soft topological spaces. The definition of filter and ultrafilter given here are those of Sharma [7].

In our present work, we have introduced the concept of soft topological spaces in the view of soft filters. We also discuss some of the properties of soft filters in soft topological spaces.

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The researcher in this study adopted a theoretical approach, building on the foundations of soft set theory and its applications in topology. The research involves a comprehensive review of existing literature on soft sets, soft topology, and their applications, where he explained the concept of soft filters is defined as a collection of soft sets that satisfy certain properties, including being closed under finite intersections and the superset operation. Soft topology is a extension of classical topology, where soft sets are used to define soft open sets. The study of soft filters and soft topology has many applications, including decision-making and data analysis, the study aims to contribute to the development of soft mathematical concepts and structures, which can have applications in various fields, including fuzzy mathematics, also aims to:

1. Investigate the concept of soft topological spaces in the context of soft filters.
2. Examine the properties of soft filters in soft topological spaces.
3. Study the relationship between soft filters and τ-convergent and τ-Hausdorff soft topological spaces.
4. Contribute to the development of soft mathematical concepts and structures based on soft set theory.

2. PRELIMINARIES

Definition 2.1 [6] A soft set F_A over X is a set defined by the function f_A representing a mapping $f_A: A \longrightarrow P(X)$ such that $f_A = \emptyset$ if $x \notin A$. Here, f_A is called the approximate function of the soft set F_A. A soft set over X can be represented by the set of ordered pairs

$$F_A = \{(x, f_A): x \in A, f_A(x) \in P(X)\}.$$

Definition 2.2 [2] Let F_A and G_A be two soft sets over X. The parallel product of F_A and G_A is defined as $F_A \tilde{\times} G_A = (F \tilde{\times} G)_A$, where $(F \tilde{\times} G)_{(\alpha)} =$

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$F(\alpha) \tilde{\times} G(\alpha), \forall a \in A \subseteq E$. It is clear that $(F \tilde{\times} G)_A$ is a soft set over $X \tilde{\times} X$.

Definition 2.3[17] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if:

1. $\emptyset, X$ are belong to τ.
2. The union of any number of soft sets in τ belong to τ.
3. The intersection of any two soft sets in τ belong to τ.

In this case, the triplet (X, τ, A) is called soft topological space over X, and any member of τ is known as soft open set in X. The complement of a soft open set is called soft closed set over X.

Definition 2.4 [9] A crisp element $x \in X$ is said to be in the soft set F_A over X, denoted by $x \in X$ iff $x \in F(\alpha), \forall \alpha \in A$.

Definition 2.5 [9] A soft set F_A is said to be τ-soft neighborhood of an element $x \in X$ if there exist $G_A \in \tau$ such that $x \in G_A \subseteq F_A$.

Definition 2.6 [10,11] Let (X, τ, A) and (Y, σ, A) be two soft topological spaces. The mapping $f: (X, \tau, A) \longrightarrow (Y, \sigma, A)$ is said to be:

1. Soft continuous if $f^{-1}(F_A) \in \tau, \forall F_A \in \sigma$.
2. Soft home morphism if f is bijective and f, f^{-1} are soft continuous.
3. Soft open if $F_A \in \tau \Longrightarrow f[F_A] \in \sigma$.
4. Soft closed if F_A is soft closed in $(X, \tau, A) \Longrightarrow f[F_A]$ is soft closed in (Y, σ, A).

Definition 2.7 [3] Let(X, τ, A), (Y, σ, A) and (Z, ω, A) be two soft topological spaces. If $f: (X, \tau, A) \longrightarrow (Y, \sigma, A)$ & $g: (Y, \sigma, A) \longrightarrow (Z, \omega, A)$ are soft continuous and $f(X) \subseteq Y$, then the mapping $gf: (X, \tau, A) \longrightarrow (Z, \omega, A)$ is soft.

Definition 2.8 [10] Let τ be a soft topology on X. Then a soft set F_A is said to be τ-soft neighborhood (shortly soft nbh) of the element E_α^X if there exist a soft set $G_A \in \tau$ such that $E_\alpha^X \in G_A \subseteq F_A$. The soft nbh system of a soft element E_α^X in (X, τ, A) is denoted by $N_\tau(E_\alpha^X)$.

Definition 2.9 [9] Let (X, τ, A) be a soft topological space. A subcollection β of τ is said to be an open base of τ if every member of τ can be expressed as the union of some member of β.

Definition 2.10 [10] The soft topology in $X \times Y$ induced by the open base $\beta = \{F_A \times G_A : F_A \in \tau, G_A \in \sigma\}$ is said to be the product soft topology of the soft topologies τ & σ it is denoted by $\tau \times \sigma$. The soft topological space $(X \times Y, \tau \times \sigma, A)$ is said to be the soft topological product of soft topological space (X, τ, A) and (Y, σ, A).

Definition 2.11 [3] A collection $\mathfrak{B}$ of soft neighborhoods of a soft element $E_\alpha^X, \forall \alpha \in A$ is said to be fundamental soft neighborhood system or soft neighborhood base of E_α^X if for any soft nei- ghborhood N_A of $E_\alpha^X, \exists H_A \in B$ such that $H_A \subseteq N_A$.

Definition 2.12 [13] Let (X, τ, A) be a soft topological space. Then $\mathcal{F}$ is called a soft filter on X if $\mathcal{F}$satisfies the following properties:

1. $\emptyset \notin \mathcal{F}$.
2. $\forall F_A, G_B \in \mathcal{F}, F_A \cap G_B \in \mathcal{F}$.
3. $\forall F_A \in \mathcal{F}$ & $F_A \subseteq G_B, G_B \in \mathcal{F}$.

Definition 2.13 [13] A soft filter $\mathcal{F}$ in a toplogical space X is said to converges to a soft point $x_\lambda \in X$ if every soft neighborhood of x_λ belongs to $\mathcal{F}$ for each $\lambda \in A$.

Theorem 2.1 [13] Let (X, τ, A) be a soft filter of a soft topological Hausdorff space X, if $\mathcal{F}$ converges to $x(\lambda) \in X$ also to $y(\lambda) \in X$, for each $\lambda \in A$ then $x = y$.

Theorem 2.2 [13] Let $\mathcal{F}$ be a soft filter of a soft topological space and let $F_A \subseteq X$. Then $x_\lambda \in \overline{F_A}$ iff there exist a soft filter $\mathcal{F}$ of subsets of X such that $F_A \in \mathcal{F}$ and $\mathcal{F}$ converges to $x_\lambda, \forall \lambda \in A$.

Definition 2.14 [12] Let (X, τ, A) be a soft topological space. A filter $\mathcal{F}$ on X is said to be τ-convergent to a point $x \in X$ if every τ-neighborhood N_A of x is a subset of $\mathcal{F}$. We say that x is a τ-limit point of $\mathcal{F}$.

Example 2.1 Let (X, τ, A) be a soft topological space, $X = \{1,2,3\}$ & $A = \{\{1\}, \{1,2\}, \{1,2,3\}\}$, and let $\mathcal{F} = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$ be a filter on X, then the τ-*nbh* are as follows: τ-cl$(\{2\}) = \{1,2\} \in \mathcal{F}, \tau$-cl$(\{1,2\}) = \{1,2\} \in \mathcal{F}$ & τ-cl$(\{2,3\}) = \{1,2,3\} \in \mathcal{F} \Longrightarrow$ since all τ-closure of τ-*nbh* of $x = 2$ belong to $\mathcal{F}$, then every τ-*nbh* N_A of $x = 2$ has a τ-closure (τ-cl(N_A)) belong to $\mathcal{F}, \Longrightarrow x = 2$ is a τ-limit point of the filter $\mathcal{F}$, and $\mathcal{F}$ converges to $x = 2$.

Definition 2.15 [12] Let (X, τ, A) be a soft topological space. A filter base $\mathcal{G}$ on X is said to be τ-converges to a point $x \in X$, if the filter whose base is β^* to a point x, we say that x is a τ-limit point of $\mathcal{G}$.

3. SOFT FILTERS IN SOFT TOPOLOGICAL SPACES

Proposition 3.1 [1] Let (X, τ, A) be a soft topological space and $\mathcal{F}$ be a filter on X then the following statement are equivalent:

1. $\mathcal{F}$ is τ-converges to a point $x \in X$.
2. $\mathcal{F}$ is finer than the collection $\Omega = \{\tau - cl(N_A) : N_A \; is \; \tau - nbh \; of \; x\}$.

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3. For every τ-*nab* N_A of x, there is $F \in \mathcal{F}$ such that $F \subset \tau - cl(N_A)$.

Corollary 3.1 Let (X, τ, A) be a soft topological space. Then the τ-*nbh* filter of a point $x \in X$ is τ-converges to x.

Proof Let N_A be a τ-*nbh* of x. By definition of a τ-*nbh*, there exists a set V in A such that $x \in V$ & $V \subseteq N_A$. Now, consider the τ-*nbh* filter of x, denoted by $F_\tau(x)$. Since $V \in A$ & $x \in V$, we have $V \in F_\tau(x)$. Since $V \subseteq N_A$, we conclude that $F_\tau(x)$ is τ-converges to x.

Proposition 3.2 Let (X, τ, A) be a soft topological space, let τ be an indiscrete topology on X. Then every filter on X is τ-converges to a point $x \in X$.

Proof Suppose that $\mathcal{F}$ is an arbitrary filter on X. Since the only open sets in τ are X and the empty set. So, we have two cases:

1. If $\mathcal{F}$ contains the empty set, then the filter $\mathcal{F}$ is trivial and we can say that it τ-converges to every point in X.
2. If $\mathcal{F}$ does not contains the empty set, then $\mathcal{F}$ contains X. Since X is the only non-empty open set in τ, every subset of X is a τ-*nbh* of every point in X. Therefore, $\mathcal{F}$ is τ-converges to every point x in X.

Proposition 3.3 Let (X, τ, A) be a soft topological space. If a filter $\mathcal{F}$ on X is τ-converges to a point x in X, then every filter $\mathcal{G}$ is finer than $\mathcal{F}$ also τ-converges to a point x.

Proof To prove this, assume that $\mathcal{F}$ is τ-converges to x in X, this means that for every τ-*nbh* N_A of x, there exists a set A in $\mathcal{F}$ such that $A \subseteq N_A$. Now, consider a filter $\mathcal{G}$ that is finer than $\mathcal{F}$, this means that every set in $\mathcal{G}$ is also a set in $\mathcal{F}$. Let take a τ-*nbh* N_A of x. Since $\mathcal{F}$ is τ-converges to x, there exist a set A in $\mathcal{F}$ such that $A \subseteq N_A$. Since $\mathcal{G}$ is finer than $\mathcal{F}$, A is also in $\mathcal{G}$. Therefore, we have $A \subseteq N_A$ for every τ-*nbh* N_A of x, and every set A in $\mathcal{G}$. This implies that $\mathcal{G}$ is τ-converges to x in X.

Proposition 3.4 Let Ω be the collection of all filters on a soft topological space (X, τ, A) which is τ-converges to the same point x in X. Then the intersection of all filters in Ω also τ-converges to a point x.

Proof Assume that Ω is the collection of all filters on a soft topological space (X, τ, A) which τ-converges to the same point x in X. Now, let $\mathcal{F}$ be the intersection of all filters in Ω. By the definition of τ-converges, a filter $\mathcal{F}$ is converges to x if for every τ-open set U containing x, there exists a set A in $\mathcal{F}$ such that for every element y in A, (x, y) is in $\tau(U)$. Let U be a τ-open set containing x, and since $\mathcal{F}$ is the intersection of all filters in Ω, it means that for every filter $\mathcal{G}$ in Ω, U is in $\mathcal{G}$. Therefore, U is also in $\mathcal{F}$. Now y any element in $\mathcal{F}$, and since $\mathcal{F}$ is a filter, it means that it contains the intersection of any two sets in $\mathcal{F}$. Thus, if A is any set in $\mathcal{F}$, then y is also in A. This implies that (x, y) is in $\tau(U)$, as U is a τ-open set containing x. Therefore, for any τ-open set U containing x, there exists a set A in $\mathcal{F}$ such that for every element y in A, (x, y) is in $\tau(U)$. This fulfills the definition of τ-converges, Then the intersection of all filters in Ω also τ-converges to a point x.

Proposition 3.5 Let (X, τ, A) be a soft topological space. A filter $\mathcal{F}$ on X is τ-converges to a point x in X if and only if every ultrafilter containing $\mathcal{F}$ is τ-converges to a point x.

Proof Suppose that $\mathcal{F}$ does not τ-converges to x. This means that there exists a τ-open set U containing x such that for every set A in $\mathcal{F}$, there exists an element y in A such that (x, y) is not in $\tau(U)$. Now, consider the collection Ω of all ultrafilters containing $\mathcal{F}$, and since every ultrafilter containing $\mathcal{F}$ is τ-converges to x, it follows that for every τ-open set U containing x, there exist a set A in every ultrafilter $\mathcal{H}$ in Ω such that for every element y in A, (x, y) is in $\tau(U)$. However, we have found a τ-open set U containing x such that for every set A in $\mathcal{F}$, there exists an element y in A such that(x, y) is not in $\tau(U)$. This implies that there exists at least one ultrafilter in Ω (specifically, the one containing the sets that do not satisfy the τ-convergence condition) that does not τ-converge to x. This contradicts our assumption that every ultrafilter containing $\mathcal{F}$ is τ-converges to x. Therefore, if every ultrafilter containing $\mathcal{F}$ is τ-converges to x, then $\mathcal{F}$ must indeed τ-converge to x.

Example 3.1 Let (X, τ, A) be a soft topological space where $X = \{1,2,3\}$ & $A = \{\{1\}, \{1,2\}, \{1,2,3\}\}$, and let $\mathcal{F} = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$ be a filter on X, if $x = 2 \implies$ the τ-*nbh* are as follows: for $\{2\} \implies \{2\} \in \mathcal{F}$, for $\{1,2\} \implies \{1,2\} \in \mathcal{F}$, for $\{2,3\} \implies \{1,2,3\} \in \mathcal{F}$. Since all τ-*nbh* of $x = 2$ belong to $\mathcal{F} \implies \mathcal{F}$ is τ-converges to $x = 2$. Suppose that $\mathcal{H}$is an ultrafilter containing $\mathcal{F} \implies \mathcal{H} = \{\emptyset, \{1,2,3\}\}$. For the ultrafilter $\mathcal{H}$ is τ-converges to a point x, for the same point $x = 2 \implies$ for $\{2\} \implies \{2\} \in \mathcal{H}$, for $\{1,2\} \implies \{1,2\} \notin \mathcal{H}$, for $\{2,3\} \implies \{1,2,3\} \in \mathcal{H}$. Since there exist a τ-*nbh* $\{1,2\} \notin \mathcal{H} \implies \mathcal{H}$ does not τ-converge to $x = 2$.

Definition 3.1 A soft topological space (X, τ, A) is said to be τ-Hausdorff if for every two distinct points x & y in X, there exist two τ-open sets U & V such that τ-$cl(U) \cap \tau$-$cl(V) = \emptyset$.

Proposition 3.6 If a soft topological space (X, τ, A) is τ-Hausdorff, then every τ-converges filter on X has a unique τ-limit point.

Proof Suppose that (X, τ, A) is a τ-Hausdorff soft topological space, and let $\mathcal{F}$ be a τ-convergent filter on X. Assume, for contradiction, that $\mathcal{F}$ has more than one τ-limit point and let x, y be two distinct τ-limit point in $\mathcal{F}$, then by definition of a τ-Hausdorff space, there exist two

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τ-open sets U & V such that $x \in U, y \in V$ & $\tau\text{-}cl(U) \cap \tau\text{-}cl(V) = \emptyset$. Since $\mathcal{F}$ is a τ-convergent filter, for any τ-open set containing x,(in this case U), there exists an element z in $\mathcal{F}$ such that $z \in U$. Similarly, there exists an element s in $\mathcal{F}$ such that $s \in V$. Now, consider the set $W = U \cap V$. Since $\tau\text{-}cl(U) \cap \tau\text{-}cl(V) = \emptyset$, it follows that the $\tau\text{-}cl(W) \cap \mathcal{F} = \emptyset$. However, this contradicts the assumption that x & y are τ-limit points in $\mathcal{F}$, as W is a τ-open set containing both x & y, and there is no element in $\mathcal{F}$ contained in W. Therefor, our assumption that $\mathcal{F}$ has more than one τ-limit point is false. This implies that $\mathcal{F}$ has a unique τ-limit point.

Definition 3.2 Let (X, τ, A) a soft topological space, A filter (filter base) $\mathcal{G}$ on a set X is said to be τ-accumulates at a point $x \in X$ if every τ- *nbh* N_A of x, and every $F \in \mathcal{F}$, the intersection $\tau\text{-}cl(N_A) \cap F \neq \emptyset$. We say that x is a τ-cluster point of $\mathcal{F}$.

Proposition 3.7 Let (X, τ, A) be a soft topological space and let $\mathcal{F}$ be a filter on X. If a point $x \in X$ is τ-limit point of $\mathcal{F}$, then it is τ-cluster point of $\mathcal{F}$.

Proof Suppose that a point x is a τ-limit point of $\mathcal{F}$, i.e., for every τ-*nbh* N_A of $x, N_A \cap \mathcal{F} \neq \emptyset$. Let W be any τ-*nbh* of x, then by definition of τ-cluster point, for every τ-*nbh* N_A of x,

$N_A \cap \mathcal{F} \neq \emptyset$. Since τ- *nbh* N_A of x, and since x is a τ-limit point of $\mathcal{F}$. Then $N_A \cap \mathcal{F} \neq \emptyset$. Now, consider the intersection $(N_A \cap W) \cap \mathcal{F} \neq \emptyset$. Since $N_A \cap \mathcal{F} \neq \emptyset, N_A \cap W \neq \emptyset$, we have $(N_A \cap W) \cap \mathcal{F} \neq \emptyset$. Thus, X is a τ-cluster point of $\mathcal{F}$.

Proposition 3.8 Let (X, τ, A) be a soft topological space. If a filter base $\mathcal{G}$ on X is τ-converges to $x \in X$, then it is τ-accumulates at $x \in X$ and in a τ-Hausdorff space, at no point other that x.

Proof Assume a filter $\mathcal{G}$ on X τ-converges to x. This means that every τ-*nbh* N_A of x, $N_A \in \mathcal{G}$. Since $\mathcal{G}$ is τ-converges to x, we have $N_A \in \mathcal{G}$ for every τ-*nbh* N_A of x. Therefore, $N_A \cap \mathcal{G} \neq \emptyset$, as N_A it self is in $\mathcal{G}$. Now, let y be a point in X other than x.We need to show that $\mathcal{G}$ does not τ-accumulate at y. Since X is a τ-Hausdorff space, then there exists disjoint τ-*nbh* N_A& M_A of x&y, respectively. Since $\mathcal{G}$ τ-converges to x, we know that $N_A \in \mathcal{G}$. However, since N_A & M_A are disjoint, $N_A \cap M_A = \emptyset$, which implies that $N_A \cap (M_A \cap \mathcal{G}) = \emptyset$. Therefore, $\mathcal{G}$ dose not τ-accumulate at y in a τ-Hausdorff space.

Proposition 3.9 Let (X, τ, A) be a soft topological space and let $\mathcal{F}$ is subordinate to $\mathcal{G}$, if $\mathcal{G}$ is τ-converges to $x \in X$, then $\mathcal{F}$ is τ-converges to x.

Proof By definition of τ-convergence, we know that for any *nbh* N_A of x in A, $\mathcal{G}$ eventually enters N_A with respect to τ. That is, there exist some index n such that for all $k \geq n, \mathcal{G}(k) \in N_A$. Now, since $\mathcal{F}$ is subordinate to $\mathcal{G}$, it follows that for every all $k \geq n, \mathcal{F}(k) \subseteq \mathcal{G}(k)$. Therefore, $\mathcal{F}(k)$ is also in N_A for all $k \geq n$. Thus, we have shown that for every *nbh* N_A of $x, \mathcal{F}$ eventually enters $N_A$with respect to τ. Therefore, $\mathcal{F}$ is τ-converges to x in the soft topological space (X, τ, A).

Proposition 3.10 Let (X, τ, A) be a soft topological space and let $\mathcal{F}$ is subordinate to $\mathcal{G}$, if $\mathcal{F}$ is τ-accumulates at $x \in X$, then $\mathcal{G}$ is τ-accumulates at x.

Proof By definition of τ-accumulation, we know that for any *nbh* N_A of x in A, $\mathcal{G}$ intersects N_A with respect to τ at infinitely many indices. That is, there exist infinitely many indices k such that $\mathcal{G}(k) \in N_A$. Now, since $\mathcal{F}$ is subordinate to $\mathcal{G}$, it follows that for every $k, \mathcal{F}(k) \subseteq \mathcal{G}(k) \in N_A$ for those same indices. Thus, we have show that for every *nbh* N_A of $x, \mathcal{F}$ intersects N_A with respect to τ at infinitely many indices. Therefore, $\mathcal{F}$ is τ-accumulates at x in the soft topological space (X, τ, A).

Definition 3.3 Let $\mathcal{F}$ & $\mathcal{G}$ be two soft filters in soft topological space (X, τ, A), we say that $\mathcal{F}$ is finer than $\mathcal{G}$ or $\mathcal{G}$ is coarser than $\mathcal{F}$ if $\mathcal{G} \subset \mathcal{F}$. If $\mathcal{F} \neq \mathcal{G}$, then we say that $\mathcal{F}$ is strictly finer than $\mathcal{G}$ or $\mathcal{G}$ is strictly coarser than $\mathcal{F}$.

Proposition 3.11 Let $(\mathcal{F}_i)_{i \in I}$ be any non-empty family of soft filters on X. Then $\mathcal{F} = \bigcap_{i \in I} \mathcal{F}_i$ is a soft filter on X.

Proof Let $(\mathcal{F}_i)_{i \in I}$ be any non-empty collection of soft filters on X. Let $\mathcal{F}$ be the intersection of all elements in $\mathcal{F}_i$ i.e., $\mathcal{F} = \bigcap\{A: A \in \mathcal{F}_i\} = \bigcap_{i \in I} \mathcal{F}_i$. Since $\mathcal{F}_i$ is non-empty, $\mathcal{F}$ is guaranteed to exist. To do this, we need to verify the following properties:
1- $\mathcal{F}$ is non-empty: since $\mathcal{F}_i$ is non-empty, each element in $\mathcal{F}_i$ is non-empty. So, the intersection $\mathcal{F}$ will also be non-empty.
2- $\mathcal{F}$ is upward closed: let $A \in \mathcal{F}$ & $A \subseteq B$, since each element in $\mathcal{F}_i$ is upward closed we have $B \in A$ for all $A \in \mathcal{F}_i$. So, $B \in \mathcal{F}$.
3- $\mathcal{F}$ is closed under finite intersection: Let $A, B \in \mathcal{F}$. Then $A, B \in \mathcal{F}_i$. Since each element in $\mathcal{F}_i$ is closed under finite intersection, we have $A \cap B \in \mathcal{F}_i$. Therefore, $A \cap B \in \mathcal{F}$. 4- As $\mathcal{F}$ satisfies all the properties of soft filter, $\mathcal{F}$ is indeed a soft filter on X.

Remark 3.1 The soft filter induced by the single set X, is the smallest element of the order set of all soft filters on X.

Theorem 3.1 Let A be a set in X. Then there exists a soft filter on X containing A if for any given finite subset $\{S_1, S_2, \dots, S_n\}$ of A, the intersection $\bigcap_{i \in I} S_i \neq \emptyset$. In fact $\mathcal{F}_A$ is the coarsest soft filter containing A.

Proof Suppose that there exists a soft filter $\mathcal{F}_A$ on X containing A. Let B be the set of all finite intersections of members of A. Then by conditions of soft filter we have

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$B \subseteq \mathcal{F}_A$ & $\mathcal{F}_A \neq \emptyset$. Suppose that $\mathcal{F}_A = \{A \in \mathcal{F}^X : A \text{ contains a member of } B\}$, where B is the family of finite intersections of A. Then $\mathcal{F}(A)$ satisfies the conditions, $\Longrightarrow \mathcal{F}(A)$ is generated by A.

Corollary 3.2 Let $\mathcal{F}$ be a soft filter in a set X, and $A \subseteq X$. Then, there is a soft filter $\acute{\mathcal{F}}$ which is finer than $\mathcal{F}$, and such that $A \in \acute{\mathcal{F}}$ iff $A \cup U = \emptyset$ for each $U \in \mathcal{F}$.

Proof $\Longrightarrow$ Assume that $A \subseteq \acute{\mathcal{F}}$, where $\acute{\mathcal{F}}$ is a soft filter finer than $\mathcal{F}$. i.e. that $\acute{\mathcal{F}}$ satisfies the three conditions for being a soft filter, we can consider an arbitrary U in $\mathcal{F}$. Since $\acute{\mathcal{F}}$ is finer than $\mathcal{F}$, we know that U is a subset of $\acute{\mathcal{F}}$. Therefore, the intersection of A & U is non-empty, as A is a subset of $\acute{\mathcal{F}}$. $\Longleftarrow$ Conversely, $A \cap U \neq \emptyset$ is non-empty for every U in $\mathcal{F}$. We want to show that there exists a soft filter $\acute{\mathcal{F}}$ that finer than $\mathcal{F}$, and such that A is a subset of $\acute{\mathcal{F}}$. To construct such an $\acute{\mathcal{F}}$, we define the family of subsets $\mathcal{R}$ of X as follows:

$\mathcal{R} = \{V \subseteq X, V \cap A \neq \emptyset \text{ for every } U \text{ in } \mathcal{F}\}$ we will show that $\mathcal{R}$ is a soft filter finer than $\mathcal{F}$, and A is a subset of $\mathcal{R}$.

$*$ $\mathcal{R} \neq \emptyset$, since $A \cap U \neq \emptyset, \forall\, U \in \mathcal{F}$, we have that X is in $\mathcal{R}$.

$*$ $\mathcal{R}$ closed, let $V \subseteq X$, such that V in $\mathcal{R}$, and let W be a subset of X such that V is a subset of W. We want to show that W is in $\mathcal{R}$. For every U in $\mathcal{F}$ $A \cap U \neq \emptyset$. Since $V \subseteq W, A \cap V \subseteq A \cap W$. Thus, $A \cap W \neq \emptyset, \forall\, U \in \mathcal{F} \xrightarrow{yields} W \in \mathcal{R}$.

$*$ Let V_1 & $V_2 \subseteq X \ni V_1$ & $V_2 \in \mathcal{R}$. We want to show that $V_1 \cap V_2 \neq \emptyset$. Since V_1 & $V_2 \in \mathcal{R} \xrightarrow{yields} A \cap V_1 \neq \emptyset$ & $A \cap V_2 \neq \emptyset$. Therefore, the $A \cap (V_1 \cap V_2) \neq \emptyset, \forall\, U$ in $\mathcal{F}$, which means that $V_1 \cap V_2 \in \mathcal{R}$

Corollary 3.3 a set F of a soft filter on a non-empty set X, has a least upper bound in the set of all soft filters on X, if for all finite sequence $(\mathcal{F}_i)_{i\in I}, 0 \leq i \leq n$ of elements of F and all $A_i \in \mathcal{F}_i, 1 \leq i \leq n, \bigcap_{i=1} A_i \neq \emptyset$.
Note that the above corollary is not true in general case.

Example 3.2 Let $X \neq \emptyset$ & $\mathcal{F}$ be a soft filter such that $\mathcal{F} = \{A, B\}$, where A & $B \subseteq X$. Suppose that $A \cap B \neq \emptyset$. In this case, we can see that there is no soft filter $\acute{\mathcal{F}}$in $\mathcal{F}$ that is a subset of every upper bound of $\mathcal{Q}$. Then any upper bound of $\mathcal{Q}$ moust be contain both A & B but there is no soft filter contain A & B.

Theorem 3.2 Let β be a set of X. Then the set of X containing an element of β is a soft filter on X if β possesses the following conditions:
1- β_1: The intersection of two members of β contain a member of β. 2- β_2: $\beta \neq \emptyset, \emptyset \notin \beta$.

Proof Let X, be a non-empty and $\mathcal{G} \subseteq X$. We need to show that $\mathcal{G}$ satisfies the properties of a soft filter:

1. $\mathcal{G}$ is non-empty: since β is non-empty, there exists at least one subset A in β. Since A is a non-empty subset of X, it must contain at least one element. Therefore, $\mathcal{G}$ is non-empty.
2. Any subset of X that contains an element of $\mathcal{G}$ is also an element of $\mathcal{G}$: let B be a subset of X that contains an element of $\mathcal{G}$. This means that there exists an element G in β such that G is a subset of B. Since G is a member of β, it satisfies the conditions of β. Therefore, B also contains at least one element of β, making it a member of $\mathcal{G}$.
3. The intersection of any two members of $\mathcal{G}$ is also a member of $\mathcal{G}$: let D & E be two members of $\mathcal{G}$. This means that there exist elements N & M in β such that $N\mathcal{F}D$ & $M \subseteq E$. Since β satisfies the condition that the intersection of any two members contains a member of β, the intersection of N & M is a non-empty subset of X. Let us call this intersection H. Since H is a non-empty subset of X, & H is contained in both D & E, then H is an element of $\mathcal{G}$. Therefore, $\mathcal{G}$ satisfies all the properties of a soft filter on X.

Definition 3.4 Let A & B be two sets on X satisfying conditions β_1 & β_2. We call them base of soft filters they generate. We consider two bases equivalent, if they generate the same soft filter.

Remark 3.2 Let A be a subset of soft filter $\mathcal{F}$. Then the set β of finite intersections of members of A is a base of soft filter $\mathcal{F}$.

Proposition 3.12 A subset β of a soft filter $\mathcal{F}$on X is a base of $\mathcal{F}$ if every member of $\mathcal{F}$ contains a member of β.

Proof Let $\mathcal{F}$ be a soft filter on a set X, and let β be a subset of $\mathcal{F}$. To prove this, we need to show two conditions:

1. Every member of β is also a member of β: since β is a subset of $\mathcal{F}$, every member of β is also a member of $\mathcal{F}$. This ensures that the members of β are contained in $\mathcal{F}$.
2. For every element A in $\mathcal{F}$, there exists an element B in β such that B is contained in A: given any element A in $\mathcal{F}$. We know that A is a member of $\mathcal{F}$ and thus, by the given condition, A contains a member of β. Let denote this member as B. Since B is a member of β and β is a subset of $\mathcal{F}$, B is also a member of $\mathcal{F}$. Moreover, since A contains B, it follows that B is contained in A. Therefore, for every element A in $\mathcal{F}$, there exists an element B in β such that B is contained in A. By satisfying both conditions, we can conclude that β is indeed a base of the soft filter $\mathcal{F}$.

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Proposition 3.14 On a set X, a soft filter $\acute{\mathcal{F}}$ with base $\acute{\beta}$ is a finer than a soft filter with base β if every member of β contains a member of $\acute{\beta}$.

Proof Let us consider an element x in $\acute{\mathcal{F}}$. By definition of a soft filter with base $\acute{\beta}$, there exists a member y in β such that y is a subset of x. Now, since every member of β contains a member of $\acute{\beta}$, there exists a member z in $\acute{\beta}$ such that z is a subset of y. Since y is a subset of x and z is a subset of y, it follows that z is a subset of x. Hence, every element in $\acute{\mathcal{F}}$ is also in $\mathcal{F}$, which implies that $\acute{\mathcal{F}}$ with base $\acute{\beta}$ is finer than $\mathcal{F}$ with base β.

Proposition 3.15 Two soft filter bases β and $\acute{\beta}$ on a set X are equivalent if every member of β contains a member of $\acute{\beta}$ and every member of $\acute{\beta}$ contains a member of β.

Proof Let us consider an element x in the soft filter $\acute{\mathcal{F}}$ with base $\acute{\beta}$. By definition of a soft filter with base $\acute{\beta}$, there exists a member y in β such that y is a subset of x. Since every member of β contains a member of with base $\acute{\beta}$, there exists a member z in β such that z is a subset of y. Now, let us consider an element $\acute{x}$ in the soft filter $\mathcal{F}$ with base β. By definition of a soft filter with base β, there exists a member $\acute{y}$ in β such that $\acute{y}$ is a subset of x. Since every member of β contains a member of $\acute{\beta}$, there exists a member $\acute{z}$ in $\acute{\beta}$ such that $\acute{z}$ is a subset of y. From the above, we can conclude that for every element in $\acute{\mathcal{F}}$ there exists an element in $\mathcal{F}$ that is a subset of it, and for every element in $\mathcal{F}$, there exists an element in $\acute{\mathcal{F}}$ that is a subset of it. Therefore, the soft filters with bases β and $\acute{\beta}$ are equivalent.

4. SOFT ULTRAFILTER

Definition 4.1 A soft ultrafilter on a set X, is a soft filter $\mathcal{F}$such that for any soft filter $\acute{\mathcal{F}}$ on X, if $\acute{\mathcal{F}}$ is strictly finer than $\mathcal{F}$, then $\acute{\mathcal{F}} = \mathcal{F}$.

Definition 4.2 A soft ultrafilter on a set X, is a soft filter $\mathcal{F}$such that there is no soft filter on X which is strictly finer than $\mathcal{F}$.

Theorem 4.1 Let $\mathcal{F}$ be any soft ultrafilter on a set X, then there exists a soft ultrafilter other than $\mathcal{F}$.

Proof Assume that $\mathcal{F}$is a soft ultrafilter on X. Let $\acute{\mathcal{F}}$ be the all subsets A of X such that A^C is in $\mathcal{F}$, then there exists a non-empty set G, which is a complement of the empty set, i.e., $X \in \acute{\mathcal{F}}$. Now, we will show that $\acute{\mathcal{F}}$ satisfies the three conditions required for a soft filter to an ultrafilter: finiteness, upward closure and downward closure.

Condition 1: Since, $\mathcal{F}$ is a soft ultrafilter, $\mathcal{F}$ does not contain the empty set, and it is closed under finite intersections and supersets. Let us assume that there exists a finite collection Ω of subsets of X such that each element in Ω or their complements is not in $\mathcal{F}$. We can consider the intersection of all sets in Ω, denoted as $A = \bigcap(\Omega)$. Since Ω is a finite collection, A is a finite intersection of subsets of X. If A is in $\mathcal{F}$, then A^C is also in $\mathcal{F}$ (as $\mathcal{F}$ is an ultrafilter). Since each set in Ω or their complements is not in $\mathcal{F}$, A orA^C is not in $\mathcal{F}$, leading to a contradiction. Therefore, the assumption that such a finite collection Ω exists is false, and $\acute{\mathcal{F}}$ satisfies the finiteness condition.

Condition 2: Let A be a subset of X such that A is in $\acute{\mathcal{F}}$ and B is a subset of X such that B is a superset of A. Since A is in $\acute{\mathcal{F}}$, A^C is in $\mathcal{F}$ (by definition of $\acute{\mathcal{F}}$). Since B is a superset of A, B^C is a subset of A^C, by the upward closure property of $\mathcal{F}$ (as it is an ultrafilter), if A^C is in $\mathcal{F}$, then B^C is also in $\mathcal{F}$. Therefore, B is in $\acute{\mathcal{F}}$, satisfying the upward closure condition.

Condition 3: Let A be a subset of X such that A is in $\acute{\mathcal{F}}$ and B is a subset of X such that B is a subset of A. Since A is in $\acute{\mathcal{F}}$, A^C is in $\mathcal{F}$ (by definition of $\acute{\mathcal{F}}$). By the downward closure property of $\mathcal{F}$ (asit is ultrafilter) if A^C is in $\mathcal{F}$, then B^C is also in $\mathcal{F}$. Therefore, B is in $\acute{\mathcal{F}}$, satisfying the downward closure condition. From conditions 1, 2&3, we conclude that $\acute{\mathcal{F}}$ is a soft filter on X.

Now, let us verify that $\acute{\mathcal{F}}$ is an ultrafilter. $\acute{\mathcal{F}}$ does not contain the empty set because X is not in $\mathcal{F}$, $\acute{\mathcal{F}}$ is proper because it contains all complements of sets in $\mathcal{F}$. Snice $\mathcal{F}$ satisfies the definition of a soft ultrafilter, it is an ultrafilter. As we have constructed a soft ultrafilter $\acute{\mathcal{F}}$ other than $\mathcal{F}$.

Example 4.1 Let $X = \{1,2,3,4\}$ and the soft filters $\mathcal{F} = \{\emptyset, X, \{1,2\}, \{1,3\}, \{2,4\}, \{3,4\}\}$, and $\acute{\mathcal{F}} = \{\emptyset, X, \{1,2\}, \{3,4\}\}$, we note that every element of $\acute{\mathcal{F}}$ is also an element of $\mathcal{F}$, but $\acute{\mathcal{F}}$ strictly contains a subset of $\mathcal{F}$. Therefore, $\acute{\mathcal{F}}$ is a soft filter that is finer than $\mathcal{F}$.

Proposition 4.1 Let $\mathcal{F}$ be a soft ultrafilter on a set X. If A & B are two soft subsets, such $A \cup B \in \mathcal{F}$, then $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

Proof Suppose $\mathcal{F}$ is a soft ultrafilter on X, and let A & B be two soft subsets of X such $A \cup B \in \mathcal{F}$, we will show that either $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

Case 1: $A \cup B \in \mathcal{F}$ & neither A nor $B \in \mathcal{F}$ leads to a contradiction because $\mathcal{F}$ is a soft ultrafilter, and by definition it is closed under finite intersections. Let us consider that the intersection of the complements of A & B denoted as A^C & B^C, respectively. Since neither A nor $B \in \mathcal{F}$, it follows that A^C & B^C are not in $\mathcal{F}$. According to the definition of a soft ultrafilter, if $\mathcal{F}$ is closed under finite intersections, that the intersection of A^C & B^C denoted as $A^C \cap B^C = (A \cup B)^C$, must also be

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in $\mathcal{F}$. Since the empty set does not belong to $\mathcal{F}$ (as $\mathcal{F}$ is proper), this leads to a contradiction. Therefore, our assumption that neither $A \in \mathcal{F}$ nor $B \in \mathcal{F}$ must be incorrect.

Case 2: Either $A \in \mathcal{F}$ nor $B \in \mathcal{F}$. Assuming $A \cup B \in \mathcal{F}$ and either $A \in \mathcal{F}$ nor $B \in \mathcal{F}$ holds true based on the logical negation of case 1. In other words, if neither $A \in \mathcal{F}$ nor $B \in \mathcal{F}$, so $A^C \cap B^C$ would belong to $\mathcal{F}$, leading to a contradiction. Hence, $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

Corollary 4.1 Let $\mathcal{F}$ be a soft ultrafilter on a set X, and let $(F_i)_{i\in I}$ be a finite sequence of soft sets in X, if $\bigcup_{i\in I} F_i \in \mathcal{F}$, then at least one of the $F_i \in \mathcal{F}$.

Proof Suppose that $\mathcal{F}$ is a soft ultrafilter on X, let F_i be a finite sequence of soft sets in X. Suppose that $\bigcup_{i\in I} F_i \in \mathcal{F}$. Suppose that none of the sets in F_i belong to $\mathcal{F}$. Since none of the sets in $F_i \in \mathcal{F}$, their complements $(F_i)^C$ must be belong to $\mathcal{F}$. By the definition of soft ultrafilter, if $\mathcal{F}$ is closed under finite intersections, then the intersection of the complements of the sets in F_i, denoted by $\bigcap(F_i)^C$ must also belong to $\mathcal{F}$. However, $\bigcap(F_i)^C = (\bigcup F_i)^C$ which is the complement of the union of all the sets in F_i. Since we assumed that $\bigcup F_i \in \mathcal{F}$, it follows that $(\bigcup_{i\in I} F_i)^C$ does not belong to $\mathcal{F}$. This contradicts the fact that $\bigcap(F_i)^C$ must also belong to $\mathcal{F}$. Therefore, our assumption that none of the sets in F_i belong to $\mathcal{F}$ must be incorrect, Hence at least one of the sets in F_i belong to $\mathcal{F}$, as stated in the theorem.

Definition 4.3 Let A be a soft set in a set X. If U is any soft set in X, then the soft set $A \cap U$ is called trace of U on A, and it is denoted by U_A. For all soft sets U & V in X, we have $(U \cap V) = U_A \cap V_A$.

Definition 4.4 Let A be a soft set in a set X. Then the set Λ_A of traces $A \in \mathcal{F}^X$ of member of Λ is called the trace of Λ on A.

Definition 4.5 Let A be a soft set in a set X, and let Λ be any soft set in X. The set of traces denoted by Λ_A, consists of all members of Λ for which the intersection with A is non-empty. In other words, Λ_A is the set of all elements in Λ that have a non-empty intersection with A.

Definition 4.6 Let $\mathcal{F}$ be a soft filter on a set X, and $A \in \mathcal{F}^X$. If $\mathcal{F}_A$ is trace of $\mathcal{F}$ on A, then $\mathcal{F}_A$ is said to be induced by $\mathcal{F}$ & A.
Note that: the trace of a soft filter $\mathcal{F}$ on A is the set of all elements $x \in X$ such that every soft set $U \in \mathcal{F}$ contains x whenever it contains A. Mathematically, it can be written as $\mathcal{F}_A = \left\{x \in X: \forall\, U \in \mathcal{F}, A \subseteq U \xrightarrow{yields} x \in U\right\}$.

Proposition 4.2 Let $\mathcal{F}$ be a soft filter on a set X inducing a soft filter $\mathcal{G}_A$ on $A \in \mathcal{F}^X$, then the trace β_A on A of a base β of $\mathcal{F}$ is a base of $\mathcal{G}_A$.

Example 4.2 Let $X = \{1,2,3\}, \mathcal{F} = \{\emptyset, X, \{1,2\}, \{1,3\}, \{2,3\}\}$, & $A = \{1,2\}$, then the soft filter induced on A by $\mathcal{F}$ is $\mathcal{G}_A = \{\emptyset, X, \{1,2\}\}$. Now take a base for $\mathcal{F}$ as $\beta = \{\{1,2\}, \{2,3\}\}$ and the trace of β is $\beta_A = \{\{1,2\} \cap A, \{2,3\} \cap A\} = \{\{1,2\}, \{2\}\}$. If we examine $\mathcal{G}_A$ the soft filter induced on A,the trace $\beta_A = \{\{1,2\}\}$ is not a base for $\mathcal{G}_A$ under A since it does not satisfy the properties of being non-empty sets, and closed under finite intersection.

5. CONCLUSION

The study highlights the significance of soft topological spaces in dealing with uncertainty and their potential applications in various fields. The results of this research demonstrate the importance of soft filters in soft topological spaces and their research demonstrates the relationship with τ-convergent and τ-Hausdorff soft topological spaces. The findings of this study open up avenues for further research on soft topological spaces and their connections with other topological spaces. The most important expected outcomes of this study are the following:

1. A deeper understanding of soft topological spaces and their properties.
2. Insights into the relationship between soft filters and τ-convergent and τ-Hausdorff soft topological spaces.
3. Contributions to the development of soft mathematical concepts and structures.
4. Identification of potential applications of soft topological spaces in various fields.

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