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Structure of the even-even (220-230) Th isotopes within the Framework IBM-2

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ABSTRACT

The specific In this study, some of the nuclear properties of the even-even thorium isotopes in the mass range A=(220–230) were studied within the interacting boson model (IBM-2) framework. The parameters in the Hamiltonian of the IBM-2 model were used to determine which one best fits the experimental spectrum. The NPBOS code was used to diagonalize this Hamiltonian, and the energy level was obtained. Our theoretical calculations and the latest experimental data showed a reasonable degree of agreement. From these calculations, it was possible to determine the dynamic symmetries of these isotopes based on several tests that rely in their calculations on energy levels. Based on the results of the tests used, The isotopes $^{(226-230)}_{90}\mathrm{Th}$ were classified as having rotational symmetry SU(3), $^{224}_{90}\mathrm{Th}$ had X(5) symmetry while the isotope $^{220}_{90}\mathrm{Th}$ had vibrational symmetry U(5), and the isotope $^{222}_{90}\mathrm{Th}$ had unstable gamma symmetry O(6). The phenomenon of backbending of these isotopes was studied, which only appeared in the isotope $^{220}_{90}\mathrm{Th}$.

KEYWORDS: dynamic symmetries IBM-2 model, $^{(226-230)}_{90}\mathrm{Th}$ isotopes.

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1.INTRODUCTION

The study of nuclear collective motion through the spectroscopy of medium-mass and heavy even-even nuclei is one of the most fascinating areas of nuclear physics [1]. The atomic nuclei within the actinide region, particularly thorium isotopes in the mass range A=220–230, provide an excellent testing ground for investigating structural evolution in nuclear architecture. These isotopes reside in a transitional region between well-established nuclear collective spectra.

Many models were developed to study various nuclei because there was no standard theory that could be applied to all of them [2]. One of the main phenomenological approaches used within these models is the IBM-2 model, which effectively describes nuclear collectivity and collective motion in terms of bosons. Arima and Iachello [3] introduced the interacting boson model (IBM-2), one of the algebraic models for studying nuclear structure. Based on group theory, this model characterizes different kinds of nuclear collective states in even-even nuclei.

The bosons, which are pairs of identical valence nucleons with angular momenta of (J=0) for the s state or (J=2) for the d state, are treated as separate constituents, while the nucleus is regarded as an inner core. This model, which is based on algebraic Hamiltonians and is ideal for studying phase transitions, has been widely used to study nuclei from various mass regions. The nuclear shapes in this model that undergo the transition are linked to dynamic symmetries, which allow for the analytic solution of the pertinent observables [4].

The IBM-2 model allows for the replication of three distinct classes of nuclei that, according to standard nomenclature, correspond to axially symmetric deformed rotors, vibrational rotors, and γ-unstable rotor nuclei connected with subgroups in the group reduction process, which begins with the top group U(6) and are designated as SU(3), U(5) and O(6), respectively [5]. Since most collectively structured nuclei have mixed properties rather than just one symmetry, a new class of symmetries that apply to systems localized at critical points was proposed. A recent proposal describes nuclei at the points of phase transitions between various dynamical symmetries using the critical point symmetries E(5) and X(5). The phase transition from U(5) to O(6) is represented by the E(5) critical point symmetry, whereas the transition from U(5) to SU(3) is described by the X(5) critical point symmetry [6].

In the present study, we employed the IBM-2 model to calculate the low-lying energy levels of thorium isotopes in the mass range A = 220–230. The calculated results are compared with available experimental data. Many methods, such as the E-GOS curves, the back-bending, the ratio r(I+2/I) and ratio R(4/2) were used to identify and verify the general characteristics of these isotopes.

The Hamiltonian of the Proton-Neutron Interacting Boson Model (IBM-2)

In the IBM-2 model, the degrees of freedom of protons and neutrons are explicitly considered; therefore, the Hamiltonian can be expressed as [1,3,7]

$$\hat{H} = \varepsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa_{\pi\nu}\hat{Q}_{\pi}^{(2)}.\hat{Q}_{\nu}^{(2)} + \hat{V}_{\pi\pi} + \hat{V}_{\nu\nu} + \hat{M}_{\pi\nu} \quad \textbf{(1)}$$

Where ε is the d-boson energy, for proton $\varepsilon_{d\pi}$ and neutron $\varepsilon_{d\nu}$,are assumed to be equal $(\varepsilon_{\pi} = \varepsilon_{\nu} = \varepsilon)$, $\hat{n}_{d\rho}$ $(\rho = \pi, \nu)$ is number operator of the d-bosons

$\kappa_{\pi\nu}$, is the strength of the quadrupole interaction between neutron and proton bosons, $\hat{Q}_{\rho}^{(2)}$ the quadrupole moment operator is given by:

$$\hat{Q}_{\rho}^{(2)} = (s_{\rho}^{\dagger}\tilde{d}_{\rho} + d_{\rho}^{\dagger}s_{\rho})^{(2)} + \chi_{\rho}(d_{\rho}^{\dagger}\tilde{d}_{\rho})^{(2)} \ldots..(2)$$

χ_{ρ} is the quadrupole deformation parameter for neutrons $(\rho = \nu)$ and protons $(\rho = \pi)$, the term $\hat{V}_{\pi\pi}$ and $\hat{V}_{\nu\nu}$ represent the interaction between like-bosons, and they are written as:

$$\hat{V}_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2}\sqrt{2L+1}\, C_{L}^{\rho} \left[(d_{\rho}^{\dagger}d_{\rho}^{\dagger})^{(L)}(\tilde{d}_{\rho}\tilde{d}_{\rho})^{(L)}\right]^{(0)} + \kappa_{\rho\rho}\hat{Q}_{\rho}^{(2)}.\hat{Q}_{\rho}^{(2)} \ \ldots (3)$$

The last term in equation (3) is quadrupole interaction among similar bosons.

The Majorana operator is contained in the final term of equation (1) and is typically added to eliminate states of mixed proton- neutron symmetry. It is possible to write this term as[8][9]:

$$\widehat{M}_{\pi\nu} = \frac{1}{2}\,\xi_2\left(d_\nu^\dagger s_\pi^\dagger - s_\nu^\dagger d_\pi^\dagger\right)^{(2)}.\left(\tilde{d}_\nu s_\pi - s_\nu \tilde{d}_\pi\right)^{(2)} - \sum_{k=1,3} 2\xi_k\left(d_\nu^\dagger d_\pi^\dagger\right)^{(k)}.\left(\tilde{d}_\nu \tilde{d}_\pi\right)^{(k)} \quad \dots\dots\dots\dots\dots\dots.(4)$$

Energy Ratios and Nuclear Shape Transition

Nuclear shape phases are manifestations of the collective motion modes in nuclei. Consequently, numerous methods have been developed to study and identify these symmetries [6].

Energy Ratio R (4/2)

The energy ratio R (4/2) between the first 2_1^+ and excited states serve as a key indicator of nuclear shape transitions along isotopic chains. It ranges from R (4/2) = 2 for vibrational nuclei near spherical shapes, to R (4/2) = 3.33 for deformed rotor, and R (4/2) = 2.5 for γ-soft nuclei [10].

E-GOS Curves

The relation R=Eγ (I→I−2)/I, known as the E-GOS (Energy Gamma Over Spin) curve, was introduced by Regan et al. [10]. It effectively highlights vibrational and rotational behaviors, as well as transitions between them, without requiring prior structural assumptions.

The information provided by this relation is valuable for understanding structural evolution along the yrast line in even-even nuclei. For the three limiting cases, the E-GOS relations are given by [11]:

$$(\text{Vibrational})\quad R(I) = \frac{\hbar\omega}{I} \xrightarrow{I\to\infty} 0 \quad \dots\dots.(5)$$

$$(\text{Rotational})\ R(I) = \frac{\hbar^2}{2J}\left(4 - \frac{2}{I}\right) \xrightarrow{I\to\infty} 4\left(\frac{\hbar^2}{2J}\right) \dots\dots(6)$$

$$(\gamma - \text{soft})\quad R(I) = \frac{E_{2_1^+}}{4}\left(1 + \frac{2}{I}\right) \xrightarrow{I\to\infty} \frac{E_{2_1^+}}{4} \quad \dots..(7)$$

The Ratio $\mathbf{r}\left(\frac{\mathbf{I+2}}{\mathbf{I}}\right)$.

The symmetry for the excited band of even-even nuclei was defined by constructing the following ratios for a given for each spin.

$$r\left(\frac{I+2}{I}\right) = \left[R\left(\frac{I+2}{I}\right))_{exp} - \frac{I+2}{I}\right] \times \frac{I(I+1)}{I(I+2)} \dots..(8)$$

The properties of each nucleus were ascertained by applying this relationship to a collection of distinct nuclei, where $R(\frac{I+2}{I})_{exp}$ is the experimental value of the ratio in equation (8) [11]. by the limits $0.1 \le r \le 0.35$ for the vibrational nuclei, $0.4 \le r \le 0.6$ for the γ-unstable nuclei, and $0.6 \le r \le 1$ for the rotational nuclei.

The Back-bending Phenomena.

This phenomenon was discovered by Johnson et al. [12]. They observed that a significant increase in the moment of inertia at a specific angular momentum was accompanied by a decrease in the energy of the Gamma-transition connecting states with spin I and I-2 in certain nuclei [13]. This effect causes the value of $\hbar\omega$ to backbend, as quantified in reference [2].

$$\hbar\omega = \frac{\Delta E}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}} \dots\dots.(9)$$

And the immediately of inertia is given by:

$$\frac{2\vartheta}{\hbar^2} = \frac{4I - 2}{\Delta E} \quad \dots\dots.(10)$$

Where $\Delta E = E(I) - E(I - 2)$

2.RESULTS AND DISCUSSION

Energy Levels

The isotopes ${}^{(220-230)}_{90}Th$ have $N_\pi = 4$ and N_ν varies from 2 to 7. Tables 1 and 2 list the Hamiltonian parameters best fit values. Theoretically, one nucleus energy spectrum can be fitted by varying each parameter separately. The values of the parameters were chosen, and one parameter was then allowed to change while the others remained constant until the best fit was achieved. Iteratively, this process was continued until a general fit was obtained. After choosing the proper parameters for each isotope, we applied the Hamiltonian of IBM-2 to our study in order to calculate the theoretical values of energy levels. For numerical calculation, the computer program NPBOS is used to diagonalize the IBM-2 Hamiltonian and generate energy space[14].

Table 1. the best fit values of the Hamiltonian parameters for .

${}^{A}_{Z}X_N$	${}^{220}_{90}Th_{130}$	${}^{222}_{90}Th_{132}$	${}^{224}_{90}Th_{134}$
ε	1.091		
κ	-0.089	0.940	1.096
χ_π	3.460	-0.089	-0.093
χ_ν	2.232	2.924	1.804
$CL_\nu(L=0)$	-0.278	2.350	2.100
$CL_\nu(L=2)$	-0.189	-0.297	-0.353
$CL_\nu(L=4)$	-0.362	-0.091	-0.151
$CL_\pi(L=0)$	1.655	-0.295	-0.267
$CL_\pi(L=2)$	-0.041	0.899	-0.255
$CL_\pi(L=4)$	-0.161	-0.189	-0.173
ξ_1	2.200	0.010	-0.110
ξ_2	-0.089	0.250	0.250
ξ_3	2.209	-0.165	-0.145

Table 2. the best fit values of the Hamiltonian parameters for.

${}^{A}_{Z}X_N$	${}^{226}_{90}Th_{136}$	${}^{228}_{90}Th_{138}$	${}^{230}_{90}Th_{140}$
ε	0.940	0.977	0.977
κ	-0.089	-0.089	-0.089
χ_π	5.741	5.335	5.341
χ_ν	2.459	2.530	2.450
$CL_\pi(L=0)$	0.819	0.940	0.930
$CL_\pi(L=2)$	0.469	0.372	0.311
$CL_\pi(L=4)$	-0.112	-0.152	-0.147
$CL_\nu(L=0)$	0.420	0.690	0.690
$CL_\nu(L=2)$	0.420	0.130	-0.001
$CL_\nu(L=4)$	-0.390	-0.310	-0.295
ξ_1	1.790	0.970	0.950
ξ_2	-0.164	-0.130	-0.131
ξ_3	1.509	1.090	1.090

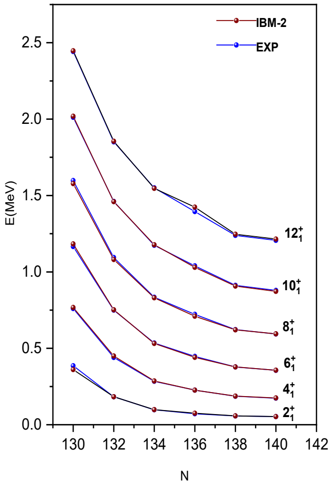

Figure 1: Comparison the experimental and IBM-2 calculated energy levels of ground state band for ${}^{(220-230)}Th$ isotopes [15 − 20] .

The Figure. 1 displays both the experimental energy and the estimated energy levels in the ground states band for ${}^{(220-230)}Th$ isotopes using the parameters in the Table 1As demonstrated, there have a good degree of agreement between experimental and theoretical values of the energy levels.

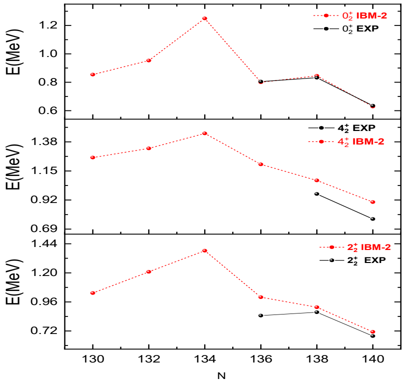

Figure 2: Comparison the experimental and IBM-2 cal-

culated energy levels of $\gamma-$band , and beta band for 2_2^+ , 4_2^+ states and 0_2^+ state respectively. for $^{(220-230)}$Th isotopes [15 − 20] .

The Figure 2: shows the available experimental and calculated energy levels in the beta band for 0_2^+ state and the $\gamma-$band for 2_2^+ , 4_2^+ states for the $^{(220-230)}$Th isotopes , results were obtained that were in reasonable agreement with experimental calculations.

Dynamical Symmetry of Thorium Isotopes $^{(220-230)}_{90}$Th. .

1. Energy Ratios R (4/2).

To characterize the evolution of collectivity in the thorium isotopic chain, we examined the energy ratio R(4/2). The thorium isotopes change from a vibrational structure to a rotational one as the number of neutrons increases, as seen in Figure 3.

The findings showed that isotopes $^{(226-230)}$Th had properties of SU(3) symmetry, whereas ^{222}Th and ^{220}Th follow the O(6) and U(5) limits, respectively. The isotope ^{224}Th lied at the critical point defined by the X(5) symmetry.

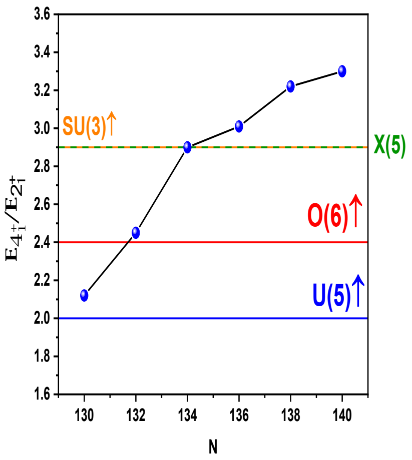

Figure 3: The ratio $\frac{E_{4_1^+}}{E_{2_1^+}}$, $^{(220-230)}$Th of isotopes.

2.E-GOS Curves.

In this study, we plotted $E-GOS$ curves using data for the ground states band of the even-even nuclei extending from $^{220}_{90}$Th to $^{230}_{90}$Th .We compared the $E-GOS$ curves with the ideal limits for a harmonic oscillator, where the first excited state is located at an energy of $E_{2_1^+}$ = 500 Kev , an axisymmetric rotor is located at an energy of $E_{2_1^+}$ = 100 Kev, and the γ-soft at energy $E_{2_1^+}$ =300 Kev .

As shown in Figure 4, the comparison with these ideal symmetry limits reveals a clear pattern. The E-GOS curves for isotopes $^{(224-230)}$Th consistently align with the SU(3) symmetry across all spin (I) values. Furthermore, the curve for ^{222}Th matches the O(6) symmetry, while that of ^{220}Th Th corresponds to the U(5) limit.

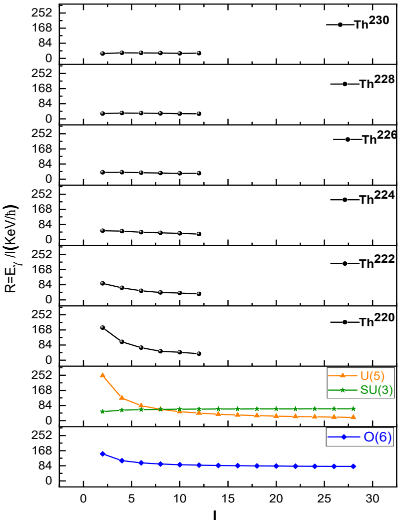

Figure 4: Comparison of the $E-GOS$ curves for the ground states band of $^{(220-230)}_{90}$Th isotopes calculated with the $E-GOS$ curves for the three standard states $U(5), O(6),$ and $SU(3).$

3. The Ratio $r(\frac{I+2}{I})$.

Figure 5 presents the calculated ground-state band for the thorium isotopes $^{(220-230)}$Th . The behavior of the ratio r as a function of spin *I* for each isotope can be observed and interpreted.

For the vibrational-like nuclei ^{220}Th and ^{222}Th, the r ratios begin at a low value and increase with spin *I*. In, ^{220}Th some negative r values are observed. This can be attributed to the isotope's near-spherical magic properties, as its neutron number *N*=130 is close to the magic number *N*=126.

According to our analysis, ^{224}Th is interpreted as a critical-point nucleus. Consequently, its properties are consistent with the X(5) dynamical symmetry, which describes nuclei at the transition between spherical and deformed shapes.

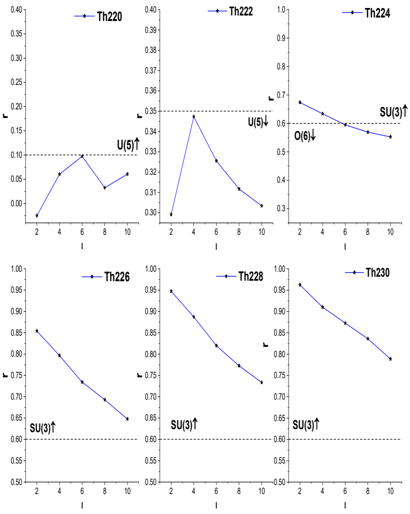

Figure 5: The ratio r((I+2)/I) as a function of I for the even-even isotopes of thorium $^{(220-230)}_{90}\mathrm{Th}$.

For the heavier isotopes $^{(226-230)}\mathrm{Th}$, the r values align with the characteristic pattern for a rotational nucleus. They start with a value close to unity and exhibit a steady decrease with increasing spin *I*.

4. The Back-bending Phenomena

To investigate the phenomenon of back-bending, we plotted the moment of inertia, $\frac{2\vartheta}{\hbar^2}$ as a function of the square of the photon energy $(\hbar\omega)^2$ emitted when the nucleus transitions from state I to state I-2, for the thorium isotopes $^{(220-230)}\mathrm{Th}$. This relationship, shown in Figure. 6, reveals that backbending occurs only in $^{220}\mathrm{Th}$. For this isotope, the backbending phenomenon is observed at spins I = 8 and I = 12.

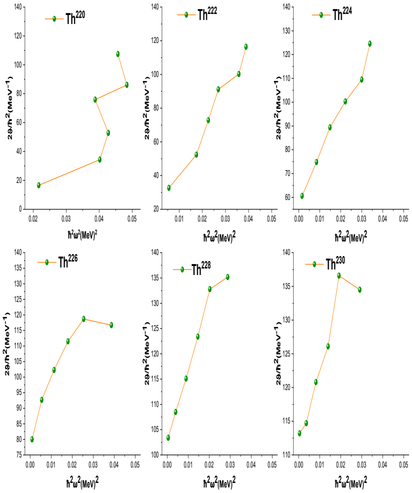

Figure 6: Moment of inertia as a function of the square of the rotational energy for the isotopes $^{(220-230)}_{90}\mathrm{Th}$.

3.CONCLUSION

In this study, we employed the Interacting Boson Model (IBM-2) to systematically investigate the structural properties of thorium isotopes across the mass range (A=220-230). Our analysis revealed that the IBM-2 framework successfully described the nuclear structure evolution throughout this isotopic chain.

The calculations were performed using a neutron-proton boson code (NPBOS). The computed excitation energies showed good agreement with experimental data for members of the ground-state band, the γ-band, and the β-band.

To determine the dynamic symmetries of each isotope, we analyzed the E-GOS curves, the R(4/2) ratio, and the r(I+2/I) ratio. Our analysis indicated that $^{220}_{90}\mathrm{Th}$ exhibits characteristics of the U(5) dynamical symmetry. The $^{222}_{90}Th$ showed O(6) symmetry, $^{224}_{90}Th$ had X(5) symmetry, while the heavier isotopes $^{(226-230)}Th$

rotational symmetry SU(3).

Furthermore, the back-bending phenomenon was observed only in the $^{220}_{90}Th$ isotope, occurring at spin values of I=8 and I=12.

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