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Egwaider

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The Egwaider Type-I Distribution

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الملخص

في هذه الورقة تمّ التعريف بتوزيع قويّدر من النوع الأوّل، الذي هو عبارة عن توزيع احتمالي مُنفصل تمّ توليده كحالة خاصّة من توزيع قويّدر المحدود (المُنتهي)، وتمّت مُناقشة خواصّ هذا التوزيع واستعراض بعض الحالات الخاصّة منه، كما تمّ توليد بيانات من هذا التوزيع عند قيم مُختلفة لمعالمه واستخدمت هذه البيانات لتقدير معلمة الشكل للتوزيع باستخدام طريقتين للتقدير هما طريقة دالّة الإمكان الأعظم وطريقة العزوم، وأخيرًا تمّت مُناقشة النتائج وإعطاء خُلاصة العمل.

وكان الهدف من عرض هذا التوزيع هو التعريف بخصائصه كتوزيع جديد، على أمل تطبيقه مُستقبلًا لتمثيل بعض الظواهر الطبيعيّة ودوّال زمن الحياة لبعض العناصر.

الكلمات المفتاحية: توزيع قويّدر، التقدير باستخدام طريقة دالّة الإمكان الأعظم، التقدير باستخدام طريقة العزوم.

Abstract

In this paper the Egwaider Type-I distribution is introduced. It is a discrete distribution generated as a special case of the finite Egwaider distribution. The properties of the resulted discrete distribution are discussed. Some special cases from the resulted discrete distribution are also introduced and some remarks about it are given. Data from this discrete distribution are simulated and used to estimate its shape parameter. Finally, the conclusion and discussion are given.

Keywords: Egwaider distribution, Maximum likelihood estimation, Moment estimation.

1. INTRODUCTION

The Egwaider (EGW) distribution is a finite three parameters, univariate, unimodal discrete probability distribution obtained by **Muiftah (2018)** as an analogue of the power function distribution using a well-known method of discretization and is given by the pmf:

$$P(Y=y)=\begin{cases}\left(\dfrac{y-a}{s}\right)^{\beta}-\left(\dfrac{y-a+1}{s}\right)^{\beta}, & \dfrac{s}{\beta}<0\\[2ex] \left(\dfrac{y-a+1}{s}\right)^{\beta}-\left(\dfrac{y-a}{s}\right)^{\beta}, & \dfrac{s}{\beta}>0\end{cases};$$

with a finite or semi-infinite support defined according to the signs of both β and s as follows:

$$\begin{aligned} & y\in[a,a+s-1], && s>0,\beta>0;\\ & y\in[a+s,\ a-1], && s<0,\ \beta>0;\\ & y\in[a+s,\infty), && s>0,\beta<0\\ & y\in(-\infty,a+s], && s<0,\beta<0 \end{aligned}$$

where, a and a+s are the end points of the support of the distribution, and β is the shape parameter.

Thus, the EGW distribution can be re-written as:

$$P(Y=y)=\begin{cases}\dfrac{(y-a)^{\beta}-(y-a+1)^{\beta}}{s^{\beta}}, & \dfrac{s}{\beta}<0\\[2ex] \dfrac{(y-a+1)^{\beta}-(y-a)^{\beta}}{s^{\beta}}, & \dfrac{s}{\beta}>0\end{cases};$$

$$\begin{aligned} & y=a,a+1,\cdots,a+s-1; && s>0,\beta>0\\ & y=a+s,a+s+1,\cdots,a-1; && s<0,\beta>0\\ & y=a+s,a+s+1,\cdots; && s>0,\beta<0\\ & y=\cdots,a+s-1,a+s; && s<0,\beta<0 \end{aligned}$$

(1)

2. DERIVATION OF THE DISTRIBUTION

The Egwaider Type-I (EGW-I) distribution arises when, in the pmf (1), $\beta>0$ and $s>0$, and is given by the following pmf:

$$P(Y=y)=\frac{(y-a+1)^{\beta}-(y-a)^{\beta}}{s^{\beta}}$$

$$,y=a,a+1,\cdots,a+s-1;s>0;\quad \beta>0$$

(2)

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where, both a and s are integers.

It may be proved that the function (2) is a pmf as follows:

$$\sum_{y=a}^{a+s-1}\frac{(y-a+1)^{\beta}-(y-a)^{\beta}}{s^{\beta}}$$
$$=\frac{1}{s^{\beta}}\{[(1)^{\beta}-(0)^{\beta}]+[(2)^{\beta}-(1)^{\beta}]+[(3)^{\beta}-(2)^{\beta}]$$
$$+\cdots+[(s-1)^{\beta}-(s-2)^{\beta}]+[(s)^{\beta}-(s-1)^{\beta}]\}$$
$$=\frac{1}{s^{\beta}}[s^{\beta}-(0)^{\beta}]=\frac{s^{\beta}}{s^{\beta}}=1 \quad \#$$

2.1. Cumulative Distribution Function:

The cumulative distribution function (cdf) of the EGW-I distribution is given by:

$$F(y)=\frac{(y-a+1)^{\beta}}{s^{\beta}}, \quad y=a,\cdots,a+s-1; \quad s>0, \tag{3}$$

Proof:

$$F(y)=\sum_{u=a}^{y}\frac{(u-a+1)^{\beta}-(u-a)^{\beta}}{s^{\beta}}$$
$$=\frac{\{[(1)^{\beta}-(0)^{\beta}]+[(2)^{\beta}-(1)^{\beta}] +\cdots+[(y-a+1)^{\beta}-(y-a)^{\beta}]\}}{s^{\beta}}$$
$$=\frac{(y-a+1)^{\beta}-(0)^{\beta}}{s^{\beta}}=\frac{(y-a+1)^{\beta}}{s^{\beta}}, \quad y=a,\cdots,a+s-1, \quad \#$$

2.2. Survival Function:

The survival function (sf) of the EGW-I distribution is hence given by:

$$S(y)=1-F(y)=1-\frac{(y-a+1)^{\beta}}{s^{\beta}}, y=a,\cdots,a+s-1; \quad s>0, \tag{4}$$

It may be observed that the EGW-I distribution reserve the same (cdf / sf) of the continuous power function distribution when $\beta > 0$ and $s > 0$.

2.3. Failure Rate Function:

The failure rate function (frf) of the EGW-I distribution is given by:

$$h(y)=\frac{(y-a)^{\beta}-(y-a+1)^{\beta}}{(y-a)^{\beta}-s^{\beta}}, y=a,\cdots,a+s-1; \quad s>0, \tag{5}$$

Proof:

$$h(y)=\frac{S_1(y-1)-S_1(y)}{S_1(y-1)}$$
$$=\frac{\left[1-\frac{(y-a)^{\beta}}{s^{\beta}}\right]-\left[1-\frac{(y-a+1)^{\beta}}{s^{\beta}}\right]}{\left[1-\frac{(y-a)^{\beta}}{s^{\beta}}\right]}$$

$$=\frac{\left[\frac{(y-a+1)^{\beta}-(y-a)^{\beta}}{s^{\beta}}\right]}{\left[\frac{s^{\beta}-(y-a)^{\beta}}{s^{\beta}}\right]}=\frac{(y-a+1)^{\beta}-(y-a)^{\beta}}{s^{\beta}-(y-a)^{\beta}}$$

$$=\frac{(y-a)^{\beta}-(y-a+1)^{\beta}}{(y-a)^{\beta}-s^{\beta}}, \quad y=a,\cdots,a+s-1, \quad \#$$

2.4. Moments:

The rth moment of the EGW-I distribution is given by:

$$\mu_r'=E(Y^r)=\frac{1}{s^{\beta}}\sum_{i=0}^{s-1}(a+i)^r[(i+1)^{\beta}-(i)^{\beta}]; \quad s>0, \tag{6}$$

Proof:

$$E(Y^r)=\frac{1}{s^{\beta}}\sum_{y=a}^{a+s-1}y^r[(y-a+1)^{\beta}-(y-a)^{\beta}]$$

$$=\frac{1}{s^{\beta}}\{a^r[(1)^{\beta}-(0)^{\beta}]+(a+1)^r[(2)^{\beta}-(1)^{\beta}]$$
$$+(a+2)^r[(3)^{\beta}-(2)^{\beta}]+(a+3)^r[(3)^{\beta}-(2)^{\beta}]$$
$$+\cdots+(a+s-1)^r[s^{\beta}-(s-1)^{\beta}]\}$$

$$=\frac{1}{s^{\beta}}\sum_{i=0}^{s-1}(a+i)^r[(i+1)^{\beta}-(i)^{\beta}] \quad \#$$

It may clearly observed that $\mu_0' = E(Y^0) = 1$, *as:* $(a+i)^0 = 1 \; \forall i$ and $\sum_{i=0}^{s-1}[(i+1)^\beta - (i)^\beta] = s^\beta$,

Proof:

$$\sum_{i=0}^{s-1}[(i+1)^\beta - (i)^\beta] = [(1)^\beta - (0)^\beta] + [(2)^\beta - (1)^\beta] + [(3)^\beta - (2)^\beta] + \cdots + [(s)^\beta - (s-1)^\beta] = s^\beta - (0)^\beta = s^\beta \quad \#$$

2.4.1. Mean of the distribution:

The mean of the EGW-I distribution is given by:

$$\mu_1' = \mu_y = E(Y) = a + \frac{\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta}; \quad s > 0 \tag{7}$$

Proof:

$$\mu_1' = E(Y) = \frac{1}{s^\beta}\sum_{i=0}^{s-1}(a+i)[(i+1)^\beta - (i)^\beta]$$

$$= \frac{1}{s^\beta}\sum_{i=0}^{s-1}[a(i+1)^\beta - a(i)^\beta + i(i+1)^\beta - (i)^{\beta+1}]$$

$$= \frac{1}{s^\beta}\{[a(1)^\beta - a(0)^\beta + 0(1)^\beta - (0)^{\beta+1}] + [a(2)^\beta - a(1)^\beta + 1(2)^\beta - (1)^{\beta+1}] + [a(3)^\beta - a(2)^\beta + 2(3)^\beta - (2)^{\beta+1}] + \cdots + [a(s)^\beta - a(s-1)^\beta + (s-1)(s)^\beta - (s-1)^{\beta+1}]\}$$

$$= \frac{1}{s^\beta}[1(2)^\beta - (1)^{\beta+1} + 2(3)^\beta - (2)^{\beta+1} + 3(4)^\beta - (3)^{\beta+1} + \cdots + a(s)^\beta + (s-1)(s)^\beta - (s-1)^{\beta+1}]$$

$$= \frac{a(s)^\beta + \sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta}$$

$$= a + \frac{\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta} \quad \#$$

2.4.2. Second Moment:

The second moment of the EGW-I distribution is given by:

$$\mu_2' = E(Y^2) = a^2 + \frac{\sum_{i=1}^{s-1}\{(2a+i)[i(i+1)^\beta - (i)^{\beta+1}]\}}{s^\beta}; \quad s > 0 \tag{8}$$

Proof:

$$\mu_2' = E(Y^2) = \frac{1}{s^\beta}\sum_{i=0}^{s-1}(a+i)^2[(i+1)^\beta - (i)^\beta]$$

$$= \frac{1}{s^\beta}\sum_{i=0}^{s-1}(a^2 + 2ai + i^2)[(i+1)^\beta - (i)^\beta]$$

$$= \frac{1}{s^\beta}\sum_{i=0}^{s-1}[a^2(i+1)^\beta - a^2(i)^\beta + 2ai(i+1)^\beta - 2a(i)^{\beta+1} + i^2(i+1)^\beta - (i)^{\beta+2}]$$

$$= \frac{1}{s^\beta}[a^2 + a^2(2)^\beta - a^2 + 2a(2)^\beta - 2a + (2)^\beta - 1 + a^2(3)^\beta - a^2(2)^\beta + 4a(3)^\beta - 2a(2)^{\beta+1} + 4(3)^\beta - (2)^{\beta+2} + \cdots + a^2(s)^\beta - a^2(s-1)^\beta + 2a(s-1)(s)^\beta - 2a(s-1)^{\beta+1} + (s-1)^2(s)^\beta - (s-1)^{\beta+2}]$$

$$= \frac{\{a^2(s)^\beta + [2a(2)^\beta - 2a + (2)^\beta - 1 + 4a(3)^\beta - 2a(2)^{\beta+1} + 4(3)^\beta - (2)^{\beta+2} + \cdots + 2a(s-1)(s)^\beta - 2a(s-1)^{\beta+1} + (s-1)^2(s)^\beta - (s-1)^{\beta+2}]\}}{s^\beta}$$

$$= a^2 + \frac{\sum_{i=1}^{s-1}\{2ai(i+1)^\beta - 2a(i)^{\beta+1} + i^2(i+1)^\beta - (i)^{\beta+2}\}}{s^\beta}$$

$$= a^2 + \frac{\sum_{i=1}^{s-1}\{(2a+i)[i(i+1)^\beta - (i)^{\beta+1}]\}}{s^\beta} \quad \#$$

2.5. Variance:

Using the first and second moments given by (7) and (8) respectively, the variance of the EGW-I distribution will be:

$$Var(Y) = \frac{\sum_{i=1}^{s-1}[i^2(i+1)^\beta - (i)^{\beta+2}]}{s^\beta} - \frac{\left[\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]\right]^2}{s^{2\beta}}; \quad s>0$$

(9)

Proof:

$$Var(Y) = E(Y^2) - [E(Y)]^2$$

$$Var(Y) = a^2 + \frac{\sum_{i=1}^{s-1}\{(2a+i)[i(i+1)^\beta - (i)^{\beta+1}]\}}{s^\beta} - \left(a + \frac{\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta}\right)^2$$

$$= a^2 + \frac{\sum_{i=1}^{s-1}\{(2a+i)[i(i+1)^\beta - (i)^{\beta+1}]\}}{s^\beta} - a^2 - \frac{2\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta} - \left(\frac{\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]}{s^\beta}\right)^2$$

Let $$k_i = i(i+1)^\beta - (i)^{\beta+1},$$

(10)

then,

$$Var(Y) = a^2 + \frac{\sum_{i=1}^{s-1}(2a+i)k_i}{s^\beta} - a^2 - \frac{2a\sum_{i=1}^{s-1}k_i}{s^\beta} - \left(\frac{\sum_{i=1}^{s-1}k_i}{s^\beta}\right)^2$$

$$= \frac{2a\sum_{i=1}^{s-1}k_i}{s^\beta} + \frac{\sum_{i=1}^{s-1}ik_i}{s^\beta} - \frac{2a\sum_{i=1}^{s-1}k_i}{s^\beta} - \frac{\left(\sum_{i=1}^{s-1}k_i\right)^2}{s^{2\beta}}$$

$$\therefore \; Var(Y) = \frac{\sum_{i=1}^{s-1}ik_i}{s^\beta} - \frac{\left(\sum_{i=1}^{s-1}k_i\right)^2}{s^{2\beta}}$$

(11)

replacing k_i by its value given by (10) in (11),

$$Var(Y) = \frac{\sum_{i=1}^{s-1}[i^2(i+1)^\beta - (i)^{\beta+2}]}{s^\beta} - \frac{\left[\sum_{i=1}^{s-1}[i(i+1)^\beta - (i)^{\beta+1}]\right]^2}{s^{2\beta}}$$

$$= \frac{\sum_{i=1}^{s-1}[(i)^{\beta+2} - i^2(i+1)^\beta]}{s^\beta} - \frac{\left[\sum_{i=1}^{s-1}[(i)^{\beta+1} - i(i+1)^\beta]\right]^2}{s^{2\beta}} \quad \#$$

2.6. Mode of the distribution:

*The mode of the EGW-I distribution is analogues to that of the continuous power function distribution given by **Crooks (2017)** when s > 0 and is given as:*

$$Mode(Y) = \begin{cases} a, & \beta < 1 \\ a+s-1, & \beta > 1 \end{cases};$$

(12)

Proof:

As s>0, the pmf (6) attains its maximum value if the nominator $[(y-a+1)^\beta - (y-a)^\beta]$ is maximum for $a \le y \le a+s-1$.

For $\beta < 1:$ $[(y-a+1)^{\beta} - (y-a)^{\beta}]$ is maximum if Y takes its minimum value (y = a),

$$\Rightarrow \underset{\beta<1}{Max}\, P(Y=y) = P(Y=a) = \frac{(a-a+1)^{\beta} - (a-a)^{\beta}}{s^{\beta}}$$

$$= \frac{(1)^{\beta} - (0)^{\beta}}{s^{\beta}} = \frac{1}{s^{\beta}}, \quad s>0 \quad \#$$

For $\beta > 1:$ $[(y-a+1)^{\beta} - (y-a)^{\beta}]$ is maximum if Y takes its maximum value (y = a+s-1),

$$\Rightarrow \underset{\beta>1}{Max}\, P(Y=y) = P(Y=a+s-1)$$

$$= \frac{(a+s-1-a+1)^{\beta} - (a+s-1-a)^{\beta}}{s^{\beta}}$$

$$= \frac{(s)^{\beta} - (s-1)^{\beta}}{s^{\beta}}, \quad s>0 \quad \#$$

2.7. Practical Applications:

2.7.1. Table (1) represents the pmf [P(y)], the cdf [F(y)], the sf [S(y)], and the frf [h(y)], of the EGW-I (0, 10, 0.2) distribution. Whereas, figures (1) and (2) represent the distribution of the pmf [P(y)] and the frf [h(y)] of the EGW-I (0, 10, 0.2) respectively.

The p.m.f. of the EGW-I (0, 10, 0.2) is given by:

$$P(Y=y) = \frac{(y+1)^{0.2} - y^{0.2}}{10^{0.2}} \qquad ,y = 0,1,\cdots,9$$

Table (1): The EGW-I (0, 10, 0.2) distribution

y	P(y)	F(y)	S(y)	h(y)
0	0.630957	0.630957	0.369043	0.630957344
1	0.093822	0.72478	0.27522	0.254231639
2	0.061223	0.786003	0.213997	0.222452391
3	0.04655	0.832553	0.167447	0.21752707
4	0.037997	0.870551	0.129449	0.226921969
5	0.03233	0.90288	0.09712	0.249749161
6	0.028269	0.93115	0.06885	0.291079027
7	0.025203	0.979148	0.020852	0.522271893
8	0.022796	0.979148	0.020852	0.522271893
9	0.020852	1	0	1
Σ	**1.000000**			
E(Y) = 1.3856, Var(Y) = 5.6545				

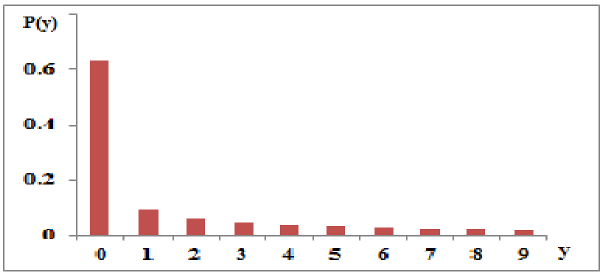

Fig (1): The pmf of the EGW-I (0, 10. 0.2) distribution

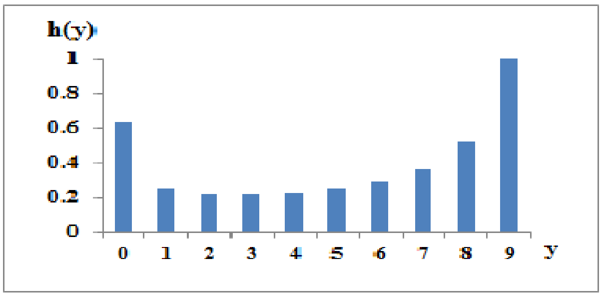

Fig (2): The frf of the EGW-I (0, 10. 0.2) distribution

2.7.2. Table (2) represents the pmf, the cdf, the sf, and e frf, of the EGW-I (5, 8, 0.5) distribution. Whereas, figures (3) and (4) represent the distribution of the pmf and the frf of the EGW-I (5, 8, 0.5) respectively.

The p.m.f. of the EGW-I (5, 8, 0.5) is given by:

$$P(Y=y) = \frac{(y-4)^{0.5} - (y-5)^{0.5}}{8^{0.5}} \qquad ,y = 5,6,\cdots,12$$

Table (2): The EGW-I (5, 8, 0.5) distribution

y	P(y)	F(y)	S(y)	h(y)
5	0.353553391	0.353553	0.646447	0.353553391
6	0.146446609	0.5	0.5	0.22654092
7	0.112372436	0.612372	0.387628	0.224744871
8	0.094734345	0.707107	0.292893	0.244395276
9	0.083462634	0.790569	0.209431	0.284959256
10	0.075455989	0.866025	0.133975	0.360291162
11	0.069388943	0.935414	0.064586	0.517926121
12	0.064585653	1	0	1
Σ	**1.000000**			
E(Y) = 7.2350, Var(Y) = 5.3380				

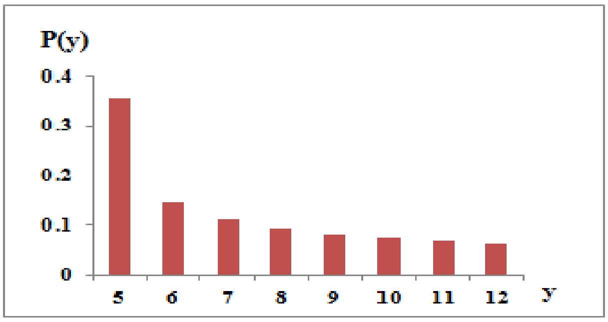

Fig (3): The pmf of the EGW-I (5, 8. 0.5) distribution

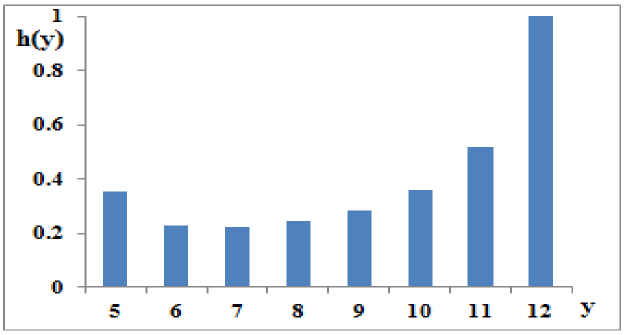

Fig (4): The frf of the EGW-I (5, 8. 0.5) distribution

2.7.3. Table (3) represents the pmf, the cdf, the sf, and the frf, of the EGW-I (0, 10, 1.3) distribution. Whereas, figures (5) and (6) represent respectively the pmf and the frf of the EGW-I (0, 10, 1.3) distribution.

The p.m.f. of the EGW-I (0, 10, 1.3) is given by:

$$P(Y=y)=\frac{(y+1)^{1.3}-y^{1.3}}{10^{1.3}} \qquad ,y=0,1,\cdots,9$$

Table (3): The EGW-I (0, 10, 1.3) distribution

y	P(y)	F(y)	S(y)	h(y)
0	0.05011872	0.050119	0.949881	0.0501187
1	0.07328805	0.123407	0.876593	0.0771550
2	0.08564682	0.209054	0.790946	0.0977042
3	0.09480953	0.303863	0.696137	0.1198685
4	0.10226308	0.406126	0.593874	0.1469008
5	0.10862412	0.514750	0.485250	0.1829078
6	0.11421609	0.628966	0.371034	0.23537598
7	0.11923235	0.748199	0.251801	0.32135189
8	0.12379878	0.871998	0.128002	0.49165288
9	0.12800246	1	0	1
Σ	**1.000000**			
E(Y) = 5.1435, Var(Y) = 7.4107				

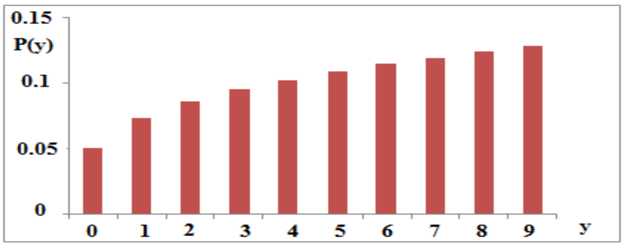

Fig (5): The pmf of the EGW-I (0, 10. 1.3) distribution

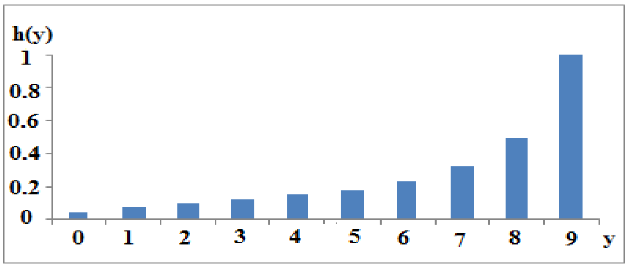

Fig (6):The frf of the EGW-I (0, 10. 1.3) distribution

2.7.4. Table (4) represents the pmf, the cdf, the sf, and the frf, of the EGW-I (3, 10, 3.5) distribution. Whereas, figures (7) and (8) represent the distribution of the pmf and the frf of the EGW-I (3, 10, 3.5) respectively.

The p.m.f. of the EGW-I (0, 10, 1.3) is given by:

$$P(Y=y)=\frac{(y-3+1)^{3.5}-(y-3)^{3.5}}{10^{3.5}} \qquad ,y=0,1,\cdots,9$$

Table (4): The EGW-I (3, 10, 3.5) distribution

y	P(y)	F(y)	S(y)	h(y)
3	0.0003162	0.000316	0.999684	0.000316
4	0.0032615	0.003578	0.996422	0.003263
5	0.0112108	0.014789	0.985211	0.011251
6	0.0256886	0.040477	0.959523	0.026074
7	0.0479112	0.088388	0.911612	0.049932
8	0.0789245	0.167313	0.832687	0.086577
9	0.1196615	0.286974	0.713026	0.143705
10	0.1709723	0.457947	0.542053	0.23978
11	0.2336434	0.69159	0.30841	0.431034
12	0.3085000	1	0	1
Σ	**1.000000**			
E(Y) = 10.2486, Var(Y) = 3.0962				

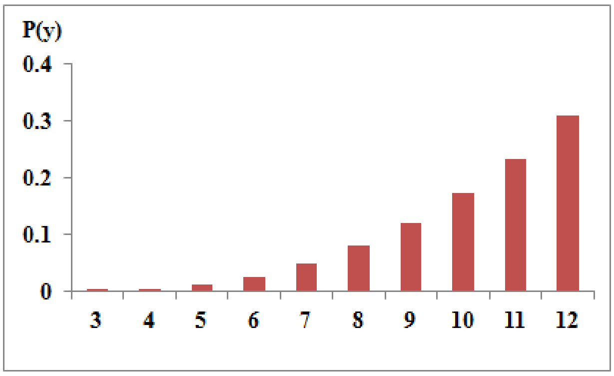

Fig (7): The pmf of the EGW-I (3, 10. 3.5) distribution

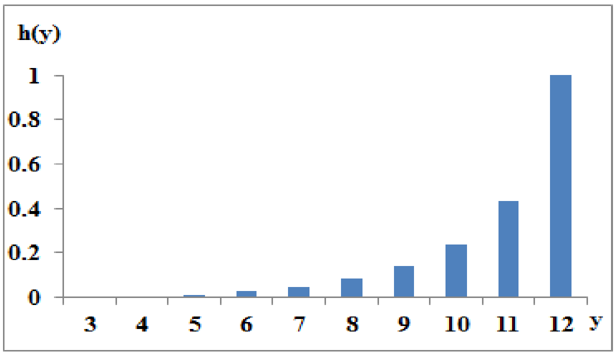

Fig (8): The frf of the EGW-I (3, 10. 3.5) distribution

2.7.5. Table (5) represents the pmf, the cdf, the sf, and the frf, of the EGW-I (-3, 10, 1.5) distribution. Whereas, figures (9) and (10) represent the distribution of the pmf and the frf of the EGW-I (-3, 10, 1.5) respectively.

The p.m.f. of the EGW-I (-3, 10, 1.5) is given by:

$$P(Y=y)=\frac{(y+4)^{1.5}-(y+3)^{1.5}}{10^{3.5}} \quad ,y=-3,-2,\cdots,6$$

Table (5): The EGW-I (-3, 10, 1.5) distribution

y	P(y)	F(y)	S(y)	h(y)
-3	0.031623	0.031623	0.968377	0.0316228
-2	0.057820	0.089443	0.910557	0.0597081
-1	0.074874	0.164317	0.835683	0.0822288
0	0.088665	0.252982	0.747018	0.1060993
1	0.100571	0.353553	0.646447	0.1346302
2	0.111205	0.464758	0.535242	0.1720244
3	0.120904	0.585662	0.414338	0.2258866
4	0.129880	0.715542	0.284458	0.313463
5	0.138273	0.853815	0.146185	0.4860932
6	0.146185	1	0	1
Σ	**1.000000**			
E(Y) = 2.4883, Var(Y) = 6.8302				

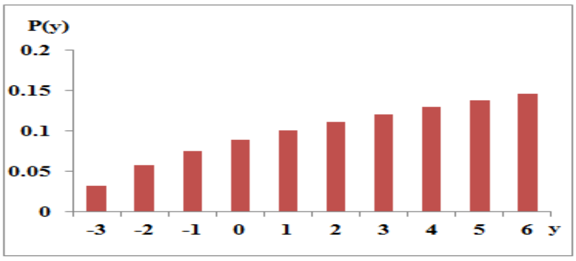

Fig (9): The pmf of the EGW-I (-3, 10. 1.5) distribution

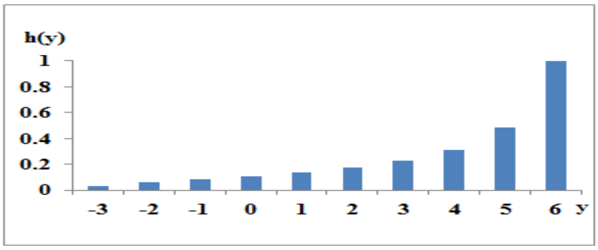

Fig (10): The frf of the EGW-I (-3, 10. 1.5) distribution

2.7.6. Table (6) represents the pmf, the cdf, the sf, and the frf, of the EGW-I (-8, 5, 0.2) distribution. Whereas, figures (11) and (12) represent the distribution of the pmf and the frf of the EGW-I (-8, 5, 0.2) respectively.

The p.m.f. of the EGW-I (-8, 5, 0.2) is given by:

$$P(Y=y)=\frac{(y+9)^{0.2}-(y+8)^{0.2}}{5^{0.2}} \qquad ,y=-8,-7,\cdots,-4$$

Table (6): The EGW-I (-8, 5, 0.2) distribution

y	P(y)	F(y)	S(y)	h(y)
-8	0.724780	0.7247800	0.2752200	0.7247800
-7	0.107774	0.8325532	0.1674468	0.3915900
-6	0.070327	0.9028805	0.0971195	0.4199976
-5	0.053472	0.9563525	0.0436475	0.5505797
-4	0.043647	1	0	1
Σ	**1.000000**			
E(Y) = -7.4166, Var(Y) = 1.2283				

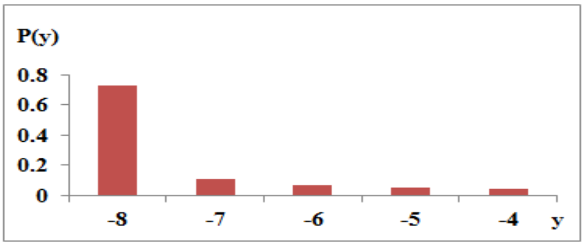

Fig (11):The pmf of the EGW-I (-8, 5. 0.2) distribution

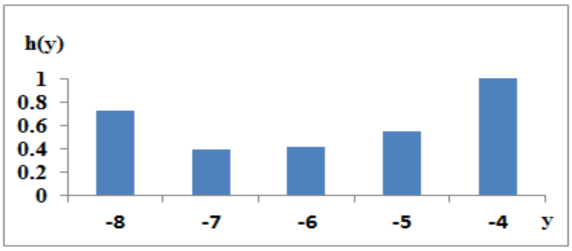

Fig (12): The frf of the EGW-I (-8, 5. 0.2) distribution

2.7.7. Table (7) represents the pmf, the cdf, the sf, and the frf, of the EGW-I (8, 2, 0.4) distribution, whereas, figures (13) and (14) represent respectively the distribution of the pmf and the frf of the EGW-I (8, 2, 0.4).

The p.m.f. of the EGW-I (-8, 5, 0.2) is given by:

$$P(Y=y)=\frac{(y-7)^{0.4}-(y-8)^{0.4}}{2^{0.4}} \qquad ,y=8,9.$$

Table (7): The EGW-I (8, 2, 0.4) distribution

y	P(y)	F(y)	S(y)	h(y)
8	0.75785828	0.75785828	0.24214172	0.7578583
9	0.24214172	1	0	1
Σ	1.000000			
E(Y) = 8.24214172, Var(Y) = 0.18350911				

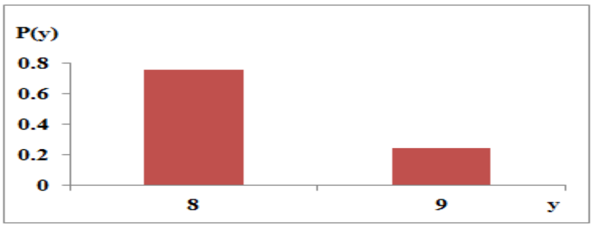

Fig (13): The pmf of the EGW-I (8, 2. 0.4) distribution

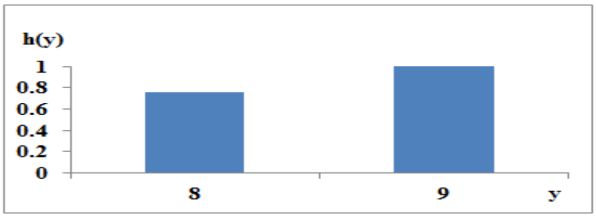

Fig (14): The frf of the EGW-I (8, 2. 0.4) distribution

The plots of the pmf 's given by figures (1), (3), (5), (7), (9), (11) and (13) show either increasing or decreasing shapes according to the values of a, s, and β. It shows that the mode of the EGW-I distribution, may be given as in equation (15).

The plots given by figures (2), (4), (6), (8), (10), (12), and (14), show the different shapes of the frf of the EGW-I distribution according to the values of a, s, and β. Some shapes of the frf as illustrated include bathtub and j shape frf.

3. SOME SPECIAL CASES FROM EGW-I DISTRIBUTION

Analogues to the special cases from the continuous power function distribution Crooks (2017), some discrete distributions may be considered as special cases from the EGW-I distribution as follows:

3.1. For β = 1, the EGW-I distribution given by the pmf (2) is reduced to the discrete uniform distribution in the interval [a, a+s-1], with the pmf ':

$$P(Y=y)=\frac{(y-a+1)-(y-a)}{s}=\frac{1}{s}, \quad y=a,\cdots,a+s-1; \quad s>0,$$

3.2. For β = 2, the EGW-I distribution given by the pmf (2) is reduced to the discrete ascending wedge (right rectangular) distribution in the interval [a, a+s-1], with a pmf:

$$P(Y=y)=\frac{(y-a+1)^2-(y-a)^2}{s^2}$$
$$=\frac{2y-2a+1}{s^2},\qquad y=a,\cdots,a+s-1;\quad s>0,$$

3.3. For a = 0, 0 < β < 1, the EGW-I distribution given by the pmf (2) is reduced to the discrete analogue of the first type of the Pearson type VIII distribution with a pmf:

$$P(Y=y)=\frac{(y-0+1)^{\beta}-(y-0)^{\beta}}{s^{\beta}}$$
$$=\frac{(y+1)^{\beta}-y^{\beta}}{s^{\beta}},y=0,\cdots,s-1;\quad s>1;\quad 0<\beta<1,$$

3.4. For a = 0, β > 1, the EGW-I distribution given by the pmf (2) is reduced to the discrete analogue of the first type of the Pearson type IX distribution with a pmf:

$$P(Y=y)=\frac{(y+1)^{\beta}-y^{\beta}}{s^{\beta}},y=0,\cdots,s-1;\quad s>1;\quad \beta>1,$$

4. ESTIMATION OF THE PARAMETERS

The values of a and s are usually pre-assigned (i.e., can be obtained from the data). Therefore, β (the shape parameter) is the only parameter that is estimated.

4.1. Maximum likelihood estimation:

Given a random sample Y1, Y2, ..., Yn from the EGW-I distribution (2), then the likelihood function of this sample is:

$$L(a,s,\beta)=\prod_{j=1}^{n}\left[\frac{(y_j-a+1)^{\beta}-(y_j-a)^{\beta}}{s^{\beta}}\right],\quad s>0,$$

The log - likelihood function is then given by:

$$ln\,L(a,s,\beta)=\sum_{j=1}^{n}\{ln\,[(y_j-a+1)^{\beta}-(y_j-a)^{\beta})]\}-n\beta\,ln(s),\quad s>0,$$

The first partial derivative of $ln\,\mathrm{L}(a,s,\beta)$ w.r.t β is:

$$\frac{\partial ln L}{\partial\beta}=\sum_{j=1}^{n}\frac{[(y_j-a)^{\beta}ln(y_j-a)-(y_j-a+1)^{\beta}ln(y_j-a+1)]}{(y_j-a)^{\beta}-(y_j-a+1)^{\beta}}-n\,ln(s)$$

Equating this partial derivative with zero will yield the normal equation $\frac{\partial\, ln\,L}{\partial\beta}=0$ that need numerical technique to be solved in order to get the MLE $\hat{\beta}$ of the parameter β.

4.2. Moment estimation:

Given a random sample $Y_1, Y_2, ..., Y_n$ from the EGW-I distribution (2), then, the moment estimate $\tilde{\beta}$ of the parameter β may be estimated using the method of moments by solving the equation $\mu_1'=m_1$, where, $\mu_1'=E(Y)$, and m_1 is the first moment of the sample (sample mean) given by: $m_1=\frac{1}{n}\sum_{i=1}^{n}y_i$, *then, for the EGW-I distribution:*

$$a+\frac{1}{s^{\tilde{\beta}}}\sum_{i=1}^{s-1}[\,i(i+1)^{\tilde{\beta}}-(i)^{\tilde{\beta}+1}]=\frac{1}{n}\sum_{i=1}^{n}y_i$$

This equation also need a numerical technique to be solved to get the moment estimate $\tilde{\beta}$ of the parameter β.

4.3. Simulation study:

A Mathcad program is used to simulate data from the EGW-I distribution for some values of a, s, and β, the ML and moment estimates of β ($\hat{\beta}$ and $\tilde{\beta}$). Their corresponding variances and MSEs are obtained for sample sizes (n = 25, 50, and 100) using 1000 replications. The obtained results are given in table (8).

5. DISCUSSION AND CONCLUSION

The aim of this paper was to illustrate the EGW-I distribution as a new discrete probability distribution that may have some practical applications in reality. The properties of the distribution were discussed. Some practical applications (examples) consider different values of the distribution parameters that were given. The ML and moment estimates of the shape parameter β and their corresponding variances and MSEs are obtained for different sample sizes using simulated data from EGW-I distribution. A consideration on table (8) may show that the variances and the MSEs of the ML estimates are smaller in most cases than those of the moment estimates. This indicates that the ML estimates seems to be better to use than the moment estimates.

6. FUTURE WORK

6.1. The second type of the EGW distribution, namely EGW-II distribution is under study and to be obtained as a discrete analogue of the continuous finite power function distribution when s < 0.

6.2. Searching some natural phenomena and lifetimes that may be expressed using the EGW-I distribution.

7. REFERENCES

1. Crooks, G. E. (2017). Field guide to continuous probability distributions. http:// threeplusone.com/ field guide. v 0.11. Accessed on 4/7/2017.
2. Muiftah, M. S. A. (2018). On Expressing Continuous Distributions with Discrete Distributions. A PhD Thesis, FSSR, Cairo University, Egypt.

Table(8): ML and moment estimation of the parameter β of the EGW-I (a, s, β) distribution

a = 0 , s = 9							
β	n	$\hat{\beta}$	Var ($\hat{\beta}$)	MSE ($\hat{\beta}$)	$\tilde{\beta}$	Var ($\tilde{\beta}$)	MSE ($\tilde{\beta}$)
0.2	25	**0.218**	0.024	0.025	**0.286**	0.005	0.020
	50	**0.206**	0.020	0.020	**0.288**	0.002	0.018
	100	**0.179**	0.013	0.014	**0.286**	0.001	0.016
0.5	25	**0.887**	0.005	0.306	**0.511**	0.015	0.015
	50	**0.894**	0.003	0.314	**0.516**	0.007	0.008
	100	**0.893**	0.001	0.311	**0.512**	0.004	0.004
0.8	25	**0.979**	0.014	0.079	**0.754**	0.027	0.031
	50	**0.975**	0.008	0.069	**0.741**	0.013	0.021
	100	**0.979**	0.004	0.064	**0.738**	0.006	0.014
1.0	25	**1.107**	0.023	0.046	**0.902**	0.038	0.057
	50	**1.124**	0.013	0.044	**0.895**	0.019	0.041
	100	**1.139**	0.006	0.045	**0.839**	0.009	0.032
1.2	25	**1.406**	0.009	0.094	**1.059**	0.050	0.090
	50	**1.420**	0.002	0.099	**1.055**	0.024	0.066
	100	**1.424**	0.0005	0.101	**1.044**	0.011	0.060
1.5	25	**1.713**	0.003	0.093	**1.291**	0.072	0.159
	50	**1.712**	0.001	0.091	**1.275**	0.035	0.136
	100	**1.712**	0.001	0.091	**1.268**	0.018	0.126
2.0	25	**1.861**	0.041	0.080	**1.662**	0.117	0.346
	50	**1.856**	0.026	0.067	**1.648**	0.054	0.302
	100	**1.853**	0.017	0.061	**1.641**	0.028	0.286
2.5	25	**2.321**	0.061	0.125	**2.029**	0.162	0.605
	50	**2.331**	0.032	0.090	**2.026**	0.073	0.522
	100	**2.317**	0.022	0.088	**2.003**	0.036	0.530
3.0	25	**2.779**	0.076	0.174	**2.370**	0.221	1.015
	50	**2.802**	0.042	0.121	**2.377**	0.109	0.884
	100	**2.807**	0.021	0.096	**2.355**	0.048	0.881

Table(8) continued: ML and moment estimation of the parameter β of the EGW-I (a, s, β) distribution

a = 5 , s = 10							
β	n	$\hat{\beta}$	Var ($\hat{\beta}$)	MSE ($\hat{\beta}$)	$\tilde{\beta}$	Var ($\tilde{\beta}$)	MSE ($\tilde{\beta}$)
0.2	25	**0.388**	0.000123	0.070	No solution found		
	50	**0.387**	0.000014	0.070	**0.135**	0.003	0.011
	100	**0.386**	0.000006	0.070	**0.135**	0.001	0.010
0.5	25	**1.008**	0.000001	0.516	**0.435**	0.015	0.026
	50	**1.008**	0.000001	0.516	**0.427**	0.008	0.019
	100	**1.008**	0.000001	0.517	**0.424**	0.004	0.015
0.8	25	**1.603**	0.000024	1.291	**0.674**	0.029	0.061
	50	**1.604**	0.000006	1.293	**0.677**	0.015	0.045
	100	**1.604**	0.000003	1.294	**0.673**	0.007	0.039
1.0	25	**1.001**	0.000173	0.00017	No solution found		
	50	**1.000**	0.000000	0.00000	**0.325**	0.015	0.927
	100	**1.000**	0.000000	0.00000	**0.325**	0.007	0.918
1.2	25	**1.489**	0.000083	0.167	**0.998**	0.052	0.134
	50	**1.489**	0.000009	0.167	**1.005**	0.026	0.102
	100	**1.489**	0.000004	0.167	**0.993**	0.012	0.098
1.5	25	**1.806**	0.000495	0.188	**1.261**	0.074	0.188
	50	**1.804**	0.000222	0.185	**1.245**	0.034	0.164
	100	**1.801**	0.000095	0.181	**1.234**	0.017	0.159
2.0	25	**2.295**	0.000215	0.176	**1.651**	0.114	0.357
	50	**2.295**	0.001559	0.175	**1.624**	0.054	0.337
	100	**2.296**	0.000712	0.176	**1.624**	0.026	0.309
2.5	25	**2.635**	0.027000	0.064	**2.040**	0.155	0.579
	50	**2.659**	0.011000	0.062	**2.006**	0.079	0.567
	100	**2.676**	0.001651	0.064	**1.997**	0.039	0.544
3.0	25	**3.002**	0.011000	0.01100	**2.072**	0.302	2.026
	50	**2.998**	0.003365	0.00337	**2.045**	0.147	1.969
	100	**3.000**	0.000293	0.00029	**2.020**	0.072	1.993