Plasma Electromagnetic Diagnostics On Tokamak Systems

(Part I)

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Abstract

In this work, we try to bridge the gaps between electromagnetic theory and the experimental work. In this study, the magnetic field ($\vec{B}$), the plasma current (I), the Ohmic power (P) and finally the plasma electron temperature (T_e) are covered. To simplify the approach, we adhere to plasmas, with circular cross-section. Thus we use the geometry of the torus, where (φ) is in the toroidal direction and (θ) is in the poloidal direction.

INTRODUCTION

Great deal of research has been carried out in plasma physics during the past few decades. Comparable to any other subdiscipline of physics, the field of plasma includes a very substantial body of knowledge covering a wide variety of branches, ranging from the most theoretical to the most practical. In plasma, major quantities confrontation between theory and experiment is possible. This confrontation places strong demands upon theory to do calculations in realistic configuration. But it also requires that the properties of plasmas be measured experimentally as accurately as possible. For this reason much of the effort in experimental plasma physics is devoted to devising,

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developing and proving techniques for diagnosing the properties of plasma-well known as plasma diagnostics [2,6].

The prospect of generating economically significant amounts of power from controlled thermonuclear fusion is the driving major force behind the research on plasma. The overall objective of plasma diagnostic is to deduce information about the state of the plasma from practical observation of physical processes and their effects. The high temperatures sought for fusion frequently eliminate the possibility of internal diagnosing by material probes [5,7].

The aim of this work is to smooth the way for researchers and engineers in this field by casting the theoretical physical laws into a simple quantitive mathematical formulas. To this end the coordinate system is that of toroidal geometry, which is suitable to most promising fusion reactors mainly tokomaks. A simple torus shown in figure (1) depicts the toroidal geometry of a tokomak reactor. Here, the toroidal axis (vertical by convention) is encircled by the magnetic axes. It is a single toroidal field line that generally locates the peak of the plasma current and plasma density profiles. The magnetic axis also identified with the toroidal direction parameter (φ). Similarly closed poloidal curves encircling the magnetic axis, indicate the local poloidal direction (θ). To carry the measurement two coils (loops) are used [12,7].

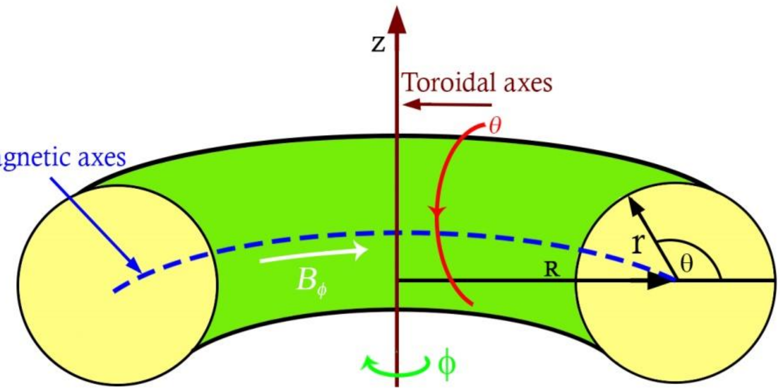

Figure (1): Primitive toroidal coordinates

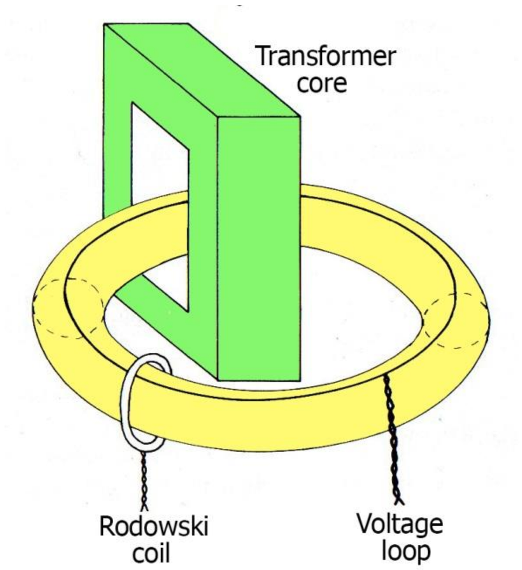

Figure (2): Rogowski coil loop that encircle the magnetic axis. A loop voltage that encircles the toroidal axis as shown in figure (1).

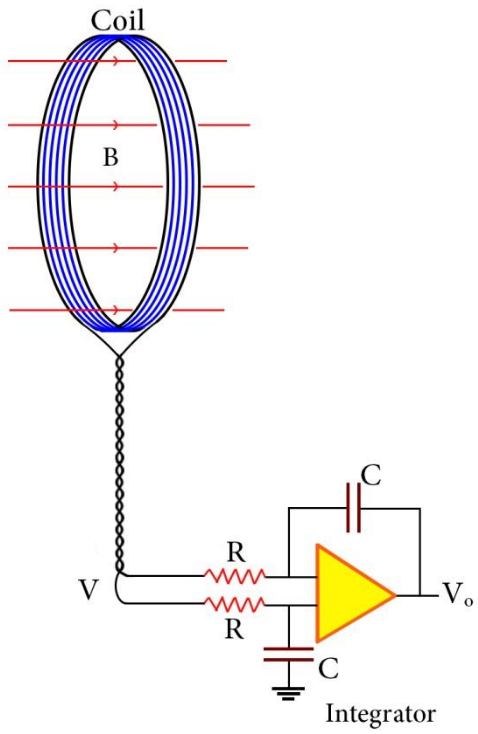

Figure 3: Typical magnetic coil and integrated circuit.

THEORY AND DERIVATIONS

1-Magnetic field Measurement:

We start from the differential form of Maxwell's equation (Faraday's law)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

Integrating both sides of equation (1) over the surface(S) with the differential area vector $d\vec{a} = \hat{n}da$ where $\hat{n}$ is the unit vector normal to the surface (S), we get

$$\int_{s} (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = -\int_{s} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} \tag{2}$$

Using Stokes theorem we change the surface integral on the left side of equation (2) to a line integral to obtain ([1,8]):

$$\int_{line} \vec{E} \cdot d\vec{l} = -\int_{s} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} \tag{3}$$

Here, $d\vec{l}$ is a segment of length vector along the perimeter of the surface area.

The simplest way to measure the magnetic field in the vicinity of a point in space is to use the so called magnetic coil as shown in figure (2). We assume the magnetic field in the pinch as uniform with cylindrical cross-section. A coil with cross-sectional area (A) and (N) number of turns with an integrator and/or oscilloscope (with some non trivial impedance) senses the voltage cross the coil ends. If the coil is good conductor or if the impedance of the electronics is large (∞) the electric field $\left(\vec{E}_{in}\right)$ inside the coil itself is zero and the left side of equation (3) can be written as:

$$\int_{coil} \vec{E}_{in} \cdot d\vec{l} + \int_{end} \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} \tag{4}$$

Since the electric field inside the coil is zero, thus equation (4) reduces to:

$$\int_{end} \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} \tag{5}$$

The magnetic flux (ϕm) is related to the magnetic field ($B\varphi$) in the φ-direction and the electric field ($E\theta$) in the θ – direction is related to the emf ($V\theta$) cross the coil ends respectively [3,10]:

$$\phi_m = \int_s \vec{B}_\varphi da \tag{6}$$

$$V_\theta = \int_{end} \vec{E}_\theta dl \tag{7}$$

If the magnetic field is uniform in space and varying in time, we can relate the emf (Vθ) to the magnetic field flux (ϕm) in the z-direction. Thus:

$$V_\theta = -\frac{d\phi_m}{dt} \tag{8}$$

Using equations (5), (6), (7) and (8) we get:

$$V_\theta = -\frac{d}{dt}\left(\int_s B_\varphi \, da\right) = -\frac{\partial B_\varphi}{\partial t}\int_s da \tag{9}$$

If we have a loop with an area A and N number of turns, equation (9) reduces to:

$$\mathrm{V_0} = |\mathrm{V_\theta}| = \left|-\frac{\mathrm{NAdB_\varphi}}{\mathrm{dt}}\right| \tag{10}$$

Using an analog integrating circuit with resistance(R) and capacitance (C) , we obtain the time constant τ = R C. Therefore, equation (10) becomes:

$$\mathrm{V_0} = \frac{\mathrm{NAB_\varphi}}{\tau} = \frac{\mathrm{NAB_\varphi}}{\mathrm{RC}} \tag{11}$$

Since the quantity (RC/NA) is known and V0 is measured:

$$B_{\varphi} = \left(\frac{RC}{NA}\right) V_0 \qquad (12)$$

The above calculation gives the component of $\vec{B}$ normal to the plane of the coil. If $d\overline{B}/dt$ is non-uniform then it is the mean value over the surface that we measure. The surface integral strictly spans the space between the leads to the coil as well. The leads are usually twisted to make this contribution negligible, [8,4].

The most general case is when we allow a current to flow in the measuring coil. This will change the magnetic field Bφ in its vicinity and thereby changes the measured voltage by induction. The magnetic coil with total resistance Rc is connected to an integrated circuit whose resistance is Re. If the inductance (L) of the coil, the resistance of the measurement electronics Re and the resistance of the coil Rc are included in equation (10) we will have the general form as:

$$\frac{L}{R_e}\frac{dV_{\theta}}{dt} + \left(1 + \frac{R_c}{R_e}\right) V_{\theta} = NA\frac{dB_{\varphi}}{dt} \qquad (13)$$

Here, L is defined as the self- inductance of magnetic coil.

2- Current measurement in the cross-sectional area of the torus:

The so called Rogowski coil is used to measure the total plasma current (I) flowing through the cross-section. This solenoid coil with ends brought around together to form a torus is illustrated in figure (4). If this coil has uniform cross-sectional area (*A*) with constant turns (*n*) per unit length (provided the magnetic field varies little over one turn spacing that is if $\vec{\nabla} B/B << n$) implies that $1/l << n$, where l is the length of the torus. Thus, we can work out the flux linkage per turn as [7,6]:

$$\Phi_i = \int_s B_\varphi \, da \tag{14}$$

where i =1, 2…We can sum this over all turns to get the total flux linkage as

$$\phi = n \int_s da \int_l B_\varphi \, dl \tag{15}$$

The last integral in equation (15) is just Ampere's law

$$\int_l B_\varphi dl = \mu_0 I \tag{16}$$

where I is the total current enclosed by the loop (l) as shown in figure (4). The area of cross-section of the coil is $A = \int_s da$, thus the total flux linkage is

$$\phi = nA\mu I \tag{17}$$

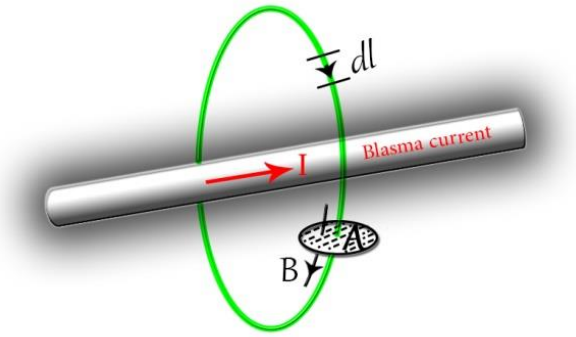

Figure 4: Equivalent geometry of the integral form of flux through a Rodowski coil.

Here n is number of turns of the coil and μ is the magnetic permeability of the medium in the solenoid. Hence the voltage out of the loop may be measured as

$$V = -\frac{d\phi}{dt} = nA\mu\frac{dI}{dt} \tag{18}$$

Which is usually integrated electronically to give a signal proportional to I. This provides a direct measurement of the total current through the centre of the plasma cross-section.

3- Ohmic Power and Conductivity Measurement:

Assuming that the plasma current to be constant during the time of plasma confinement inside the fusion experiment reactor (tokomak), the plasma resistance (R_P) is defined as [12, 8]:

$$R_p = \frac{V_\varphi}{I_\varphi} \tag{19}$$

where V_φ and I_φ are the loop voltage, and the plasma current along the toroidal φ-axis of the reactor torus respectively. The plasma resistance is important because it determines the Ohmic heating input to the plasma and also because it may be used to estimate the electron temperature. However, before moving on to these matters we must consider the more general situation in which the currents are not constant and the inductance makes a significant contribution [9].

Starting from the Poynting vector as applied to a volume bounded by a surface (S) outside the plasma on which the measuring voltage loop lies, see figure (1), we can write:

$$-\int_s \frac{\vec{E}\times\vec{B}}{\mu_0}\cdot\hat{n}da = \int_v (\vec{E}\cdot\vec{J})dv + \frac{1}{2}\int_v \left(\frac{1}{\mu_0}\frac{\partial B^2}{\partial t} + \varepsilon_o\frac{\partial E^2}{\partial t}\right)dv \qquad (20)$$

Here $\vec{J}$ and $\hat{n}da$ are the current density vector and the outward pointing surface element bounding the volume element dv respectively [6].

The first term on the right hand side is the total Ohmic heat dissipated within the volume. The second term is the rate of change of the stored electromagnetic energy within the plasma.

However, the left hand side is known as the Poynting flux or the rate of input of electromagnetic energy from the external circuits. The factor $\frac{1}{2}\varepsilon_0$is proportional to $\frac{1}{2\mu_0}\mu_0\varepsilon_0 = \frac{1}{2\mu_0}\frac{1}{c^2} << 1$. Therefore, according to [12,6] the energy density of the electric field can be dropped from the second term of equation (20). In geometrical description of the torus cross-section, equation (20) can be written as:

$$-\int_{s}\left(\vec{E}\times\frac{\vec{B}}{\mu_0}\right)\cdot d\vec{a}$$

$$=\int_{v}(\vec{E}\cdot\vec{J})dv+\frac{1}{2}\mu_0\int_{v}\left(\frac{\partial B_{\varphi}^2}{\partial t}+\frac{\partial B_{\theta}^2}{\partial t}\right) \qquad (21)$$

The input in the left hand side can be written as:

$$-\int_{s}\left(\vec{E}\times\frac{\vec{B}}{\mu_0}\right)\cdot d\vec{a}=V_{\varphi}I_{\varphi}+V_{\theta}I_{\theta} \qquad (21)$$

where $(V_{\varphi},I_{\varphi})$ are the loop voltage and the current around the torus major axis respectively and (V_{θ},I_{θ}) are the loop voltage and the current around the torus minor axis of the torus respectively. Using equation (21) and equation (22) the Ohmic power (P) can be obtained:

$$P=\int_{v}(\vec{E}\cdot\vec{J})dv=V_{\varphi}I_{\varphi}+V_{\theta}I_{\theta}$$

$$-\frac{1}{2}\mu_0\int_{v}\left(\frac{\partial B_{\varphi}^2}{\partial t}+\frac{\partial B_{\theta}^2}{\partial t}\right)dv \qquad (23)$$

The last two integrals can be put in the following forms:

$$\frac{1}{2\mu_0}\int_{v}\left(\frac{\partial B_{\varphi}^2}{\partial t}\right)dv=\frac{1}{2}\frac{\partial\left(L_{\varphi}I_{\varphi}^2\right)}{\partial t} \qquad (24-a)$$

$$\frac{1}{2\mu_0}\int_v \left(\frac{\partial B_\theta^2}{\partial t}\right) dv = \frac{1}{2}\frac{\partial\left(L_\theta I_\theta^2\right)}{\partial t} \qquad (24-b)$$

These are the energy inductances in the (φ) and (θ) directions of the cross-section of the torus respectively. The inductances can be defined as:

$$L_\varphi = \frac{1}{\mu_0 I_\varphi^2}\int_v B_\varphi^2\, dv \qquad (25-a)$$

$$L_\theta = \frac{1}{\mu_0 I_\theta^2}\int_v B_\theta^2\, dv \qquad (25-b)$$

Substituting the above noted terms in equation (23) we obtain the Ohmic power (P):

$$P = V_\varphi I_\varphi + V_\theta I_\theta - \frac{1}{2}\frac{\partial\left(L_\varphi I_\varphi^2 + L_\theta I_\theta^2\right)}{\partial t} \qquad (23)$$

The first term on the right hand side represents the resistance contribution of the plasma if it is steady. The second term on the right hand side represents the inductance contribution of the plasma if it is time varying.

The plasma current (I_φ) can be found from the magnetic flux (ϕ_m) measurement:

$$V_\varphi = -\frac{d\phi_m}{dt} = NA\mu_0\frac{dI_\varphi}{dt} \qquad (27)$$

The voltage $\left(V_\varphi\right)$ can be measured as:

$$V_\varphi = NA_p\frac{dB_\varphi}{dt} = N\pi a^2\frac{dB_\varphi}{dt} \qquad (28)$$

where A_p is the area of the plasma cross section, which approximately equal πa^2 with (a) defined as the minor radius of the torus cross-section. The voltage V_φ can be measured by the linear loop voltage [3,5]. The current (I_φ) can be written as:

$$I_\varphi = 2\pi r \frac{B_\varphi}{\mu_0} = 2\pi a \frac{B_\varphi}{\mu_0} \tag{29}$$

The plasma radius is approximately equal to ($r \sim a$). Therefore if the plasma is steady the inductance contribution is irrelevant. Thus the Ohmic power can be estimated:

$$P = (V_\theta I_\theta + V_\varphi I_\varphi) + small\ inductive\ term \tag{30}$$

Substituting equations (28) and (29) into equation (20), we obtain

$$P = V_\varphi I_\varphi + \left(N\pi a^2 \frac{dB_\varphi}{dt}\right)\left(2\pi a \frac{B_\varphi}{\mu_0}\right)$$

$$= V_\varphi I_\varphi + \left(N\pi^2 a^3 \frac{1}{\mu_0}\right)\left(\frac{dB_\varphi^2}{dt}\right) \tag{31}$$

All terms on the right hand side of the above equation can be measured experimentally.

Using Ohm's law $\vec{E} = \vec{J}/\sigma$ on the left hand side of the above equation, the plasma conductivity can be estimated if the anisotropy of the conductivity (σ) is ignored, thus:

$$P = \int_v \vec{E} \cdot \vec{J} dv = \frac{1}{\sigma}\int_v \vec{J} \cdot \vec{J} dv = \frac{1}{\langle\sigma\rangle}\left(\frac{I_\varphi}{\pi a^2}\right)^2 (2\pi l)(\pi a^2)$$

$$= \frac{1}{\langle\sigma\rangle}\frac{2l}{a^2} I_\varphi^2 \tag{32}$$

where $\langle\sigma\rangle$ is the average conductivity, l and πa^2 are the length and the area of the torus section respectively. We use equations (31) and (32) to obtain:

$$\langle\sigma\rangle\frac{\pi a^2}{2\pi l}=\frac{I_\varphi^2}{P}=\frac{I_\varphi^2}{V_\varphi I_\varphi+\frac{N\pi^2a^3}{\mu_0}\frac{dB_\varphi^2}{dt}} \tag{33}$$

If B_φ is steady, then we can estimate $\langle\sigma\rangle$ to be:

$$\langle\sigma\rangle\frac{2\pi l}{\pi a^2}=\frac{I_\varphi^2}{V_\varphi I_\varphi}=\frac{2l}{a^2}\frac{I_\varphi}{V_\varphi} \tag{34}$$

4-ElectronTemperature Measurement:

We can make an estimate of the electron temperature if we use the so-called Spitzer conductivity for $\langle\sigma\rangle$ usually used for fully ionized plasmas [1,11] which is given by:

$$\langle\sigma\rangle=1.9\times10^4\frac{T_e^{3/2}}{Z_\sigma\ln\Lambda}\ \Omega^{-1}\mathrm{m}^{-1} \tag{35}$$

Here T_e is the electron temperature in (eV), Z_σ is the resistance anomaly determined by the ion charge, and $\ln\Lambda$ is the Coulomb logarithm. Using equations (34) and (35) we get:

$$T_e^{3/2}=\frac{2l}{a^2}\frac{Z_\sigma\ln\Lambda}{1.9\times10^4}\left(\frac{I_\varphi}{V_\varphi}\right) \tag{36}$$

where $Z_\sigma = Z = 1$ for hydrogen plasmas.

5- References:

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الطرق الكهرومغناطيسية لتشخيص الحالة الرابعة من المادة (البلازما) في منظومة المفاعلات الاندماجية (توكماك)

ملخص

هذه الورقة هي محاولة لتقليص وتقريب المسافة بين ما توصّلت إليه النظريات الكهرومغناطيسية والتجارب المصاحبة لها في مجال الاندماج والمفاعلات الاندماجية. في هذا البحث، المجال المغناطيسي $(\vec{B})$، التيار الكهربي (I) البلازما، طاقة التسخين (T_e) المؤدية للاندماج، وأخيرا درجة حرارة الإلكترونات في هذا الوسط قد تم التعامل معها. ولتبسيط عملية الوصول لهذه النتائج اعتمدنا أن يمون مقطع البلازما دائريا مع الحفاظ على الشكل الهندسي للمفاعل بزواياه المحورية المعروفة (φ, θ).