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Effect of Change In Infection Rate on SIRS Model of Infection Disease using MATLAB Tools

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Abstract

In this paper, we study SIRS model of infectious rate, where we use the Euler's method to unravel the differential equations, the model contains 3 ordinary differential equations, we'll conduct numerical simulation by assuming values for the parameters, then change the infection rate β and study the effect of this on the behavior of the curves of susceptible, infected and recovered individuals. this is often done by using the MATLAB program.

Keywords: SIRS Model , Infectious Rate , Infection Disease , Mathematical Modeling.

تأثير التغيير في معدل الإصابة على نموذج الامراض لمرض العدوى باستخدام أدوات الماتلاب

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الملخص:

في هذا البحث درسنا في هذا العمل نموذج SIRS لمعدل العدوى ، حيث نستخدم طريقة أويلر لكشف المعادلات التفاضلية ، ويحتوي النموذج على 3 معادلات تفاضلية عادية ، وسنقوم بمحاكاة عددية بافتراض قيم المعاملات ، ثم نقوم بتغيير معدل الإصابة ودراسة تأثير ذلك على سلوك منحنيات الأفراد المعرضين للإصابة والمصابين والمتعافين. يتم ذلك باستخدام برنامج ماتلاب.

الكلمات المفتاحية: نموذج SIRS للأمراض ، معدل العدوى ، مرض العدوى ، النمذجة الرياضية.

1- Introduction

Epidemiology is the study of patterns, causes, and effects of health and disease conditions in a population, epidemiology has helped develop methodology used in clinical research and public health studies, mathematical modeling has been increasingly recognized in the public health community as an important research tool for infectious diseases control, the disease is infectious if the causative agent, whether a virus, bacterium, protozoa, or toxin. Simulation models offer such tools for estimating the characteristics of a specific disease outbreak [4], Ross was interested in the incidence and control of malaria, so he developed differential equation models for malaria as a host-vector disease in 1911. Other deterministic epidemiology models were then developed in papers by Ross, Ross and Hudson, Martini, and Lotka, Starting in 1926 Kermack and McKendrick published papers on epidemic models and obtained the epidemic threshold result that the density of susceptible must exceed a critical value for an epidemic outbreak to occur [5]. Mathematical epidemiology seems to have grown exponentially starting in the middle of the 20th century, in recent years we have seen many papers for infectious diseases using the SIR model such as the study of the spread of Corona virus in India, as well as the analysis of leukemia in Bangladesh.

1- In this paper, we will present the SIRS model of infectious diseases, this model is one of the necessary basics that were used by the world health organizations to predict the spread of the disease in the future and take the necessary measures to contain and eliminate it, we will also study the effect of the change in the infection rate on this model and note the effect of this on the behaviour of the curves of susceptible, infected and recovered individuals[1].

2- Methodology of SIRS model :

The SIRS model that's characterised by system of nonlinear fractional order equations with standard incidence rate. this sort of compartmental structure susceptible-infected-recovered-susceptible[2],we describe how the essential Kermack - McKendrick model is modified so as to explain how a disease during a population can persist when the immunity of recovered individuals are temporary. We shall get thinking about the evolution of an epidemic during a closed hast population of total size N. the entire population is split into three classes,

susceptible S, infective I, removed R. On the above diagram, we found out the subsequent system of differential equations [3]:

$$\frac{dS}{dt} = A - \beta SI + \gamma R - \mu S \qquad (1)$$

$$\frac{dI}{dt} = \beta SI - vI - \mu I \qquad (2)$$

$$\frac{dR}{dt} = vI - \gamma R - \mu R \qquad (3)$$

This is illustrated with the SIRS compartments, Where: β = infection rate, μ = death rate, the same for all individuals, v = recovery rate, A = birth rate (the growth of susceptible), γ = rate by which recovered individuals have lost their immunity and became susceptible to the disease[5].

The equilibrium points

3- The disease free equilibrium point (DFE):

The SIRS model has an equilibrium point that is disease-free[5], namely

$$(S_0, I_0, R_0) = (\frac{A}{\mu}, 0, 0) \qquad (4)$$

It is called the disease-free equilibrium (DFE)

Hence the DFE is stable if

$$\beta \frac{A}{\mu} < v + \mu \qquad (5)$$

On the other hand if

$$\beta \frac{A}{\mu} > v + \mu \qquad (6)$$

the DFE is unstable

The infection rate at $\beta = 0.9$:-

We perform numerical simulations of our model proposed (1),(3),we consider the initial values as S(0)=1.8 ,I(0)=0.2 ,R(0)=0 and all parameters as (A=2, $\beta = 0.9, \gamma = 1, \mu = 1, v = 1$)

Where $\Delta t = \frac{1}{4}\ day$, for 20 days where $0 < t < 20$, putting the values of $\Delta t, \beta, v, \mu, \gamma, S_0, I_0, R_0$

In equation (4),(5),(6) to get the next generation values population S_1, I_1 and R_1

$S(1) = 1.769$

I(1) =0.181

R(1)=0.050

We can calculate other iteration by the ordinary differential equation _solver using MATLAB programming

The Basic reproduction number when $\beta = 0.9: -$

$$R_0 = \frac{\beta S_0}{v+\mu} = 0.81 < 1 \qquad (7)$$

So the epidemic disease will die out

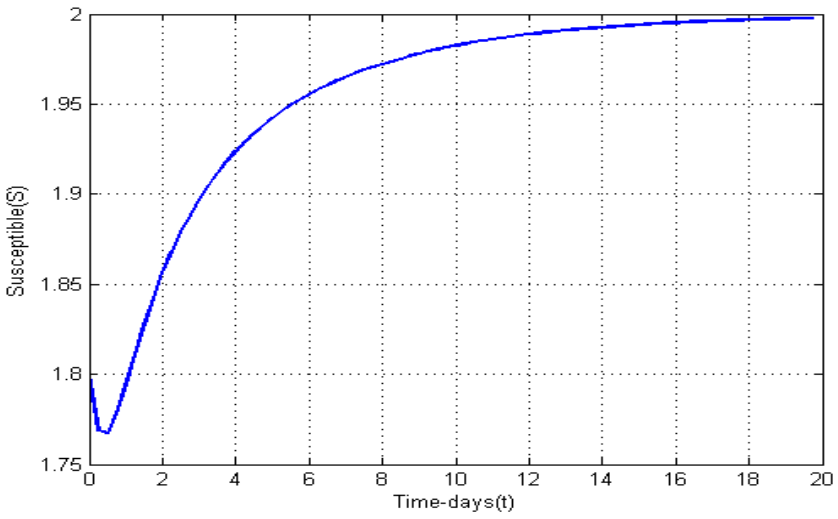
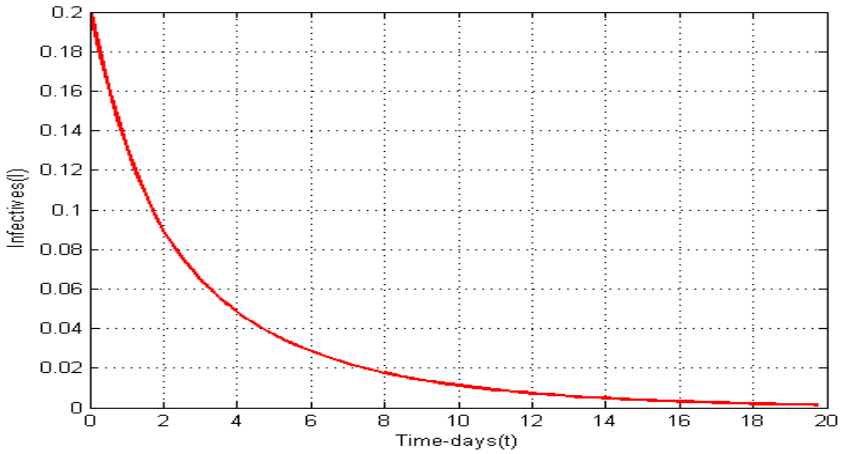


Figure (1): The susceptible individuals at

$\beta = 0.9, v = \gamma = \mu = 1, A = 2, S(0) = 1.8, I(0) = 0.2, R(0) = 0, R_0 = 0.81, \Delta t = 1/4 day$



Figure(2) : The infected individuals at $\beta = 0.9$

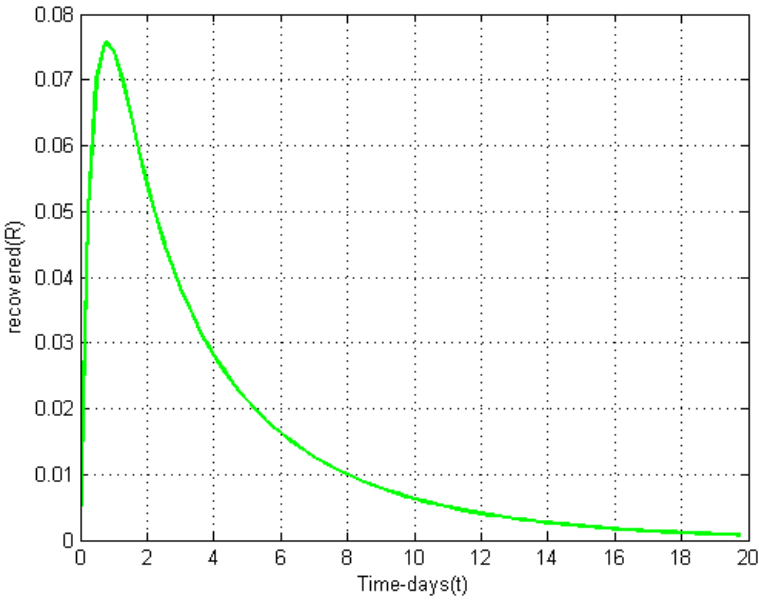


Figure (3): The removed individuals at $\beta = 0.9$

For infection rate $\beta = 1.5$

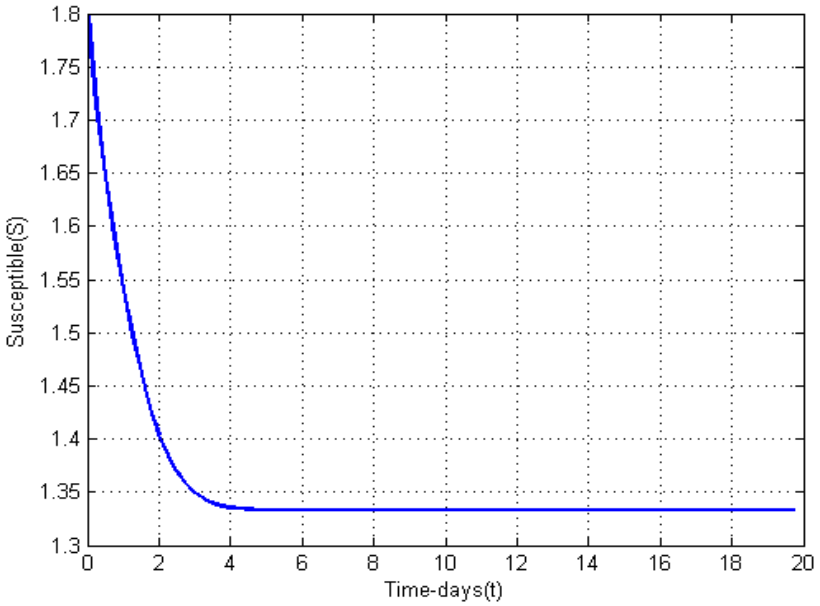
Also, we run the program when the rate of infection rate changes to $\beta = 1.5$ and all the values of parameters do not change

The Basic reproduction number at $\beta = 1.5$:

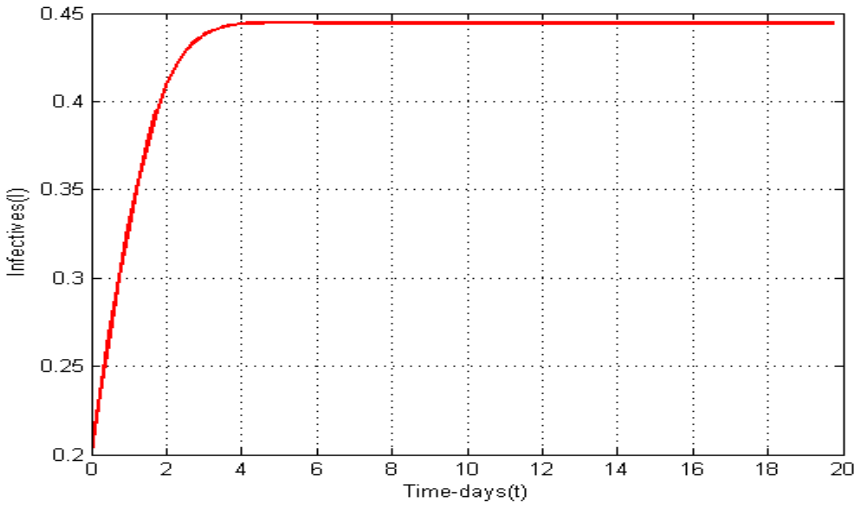
Well as we calculate the basic reproduction number R0 of the model (1)-(3) is

$$R_0 = \frac{\beta s_o}{v+\mu} = 1.35 > 1$$

So the disease will spread in society.



Figure(4): The susceptible individuals at $\beta = 1.5$



Figure(5): The infected individuals at $\beta = 1.5$

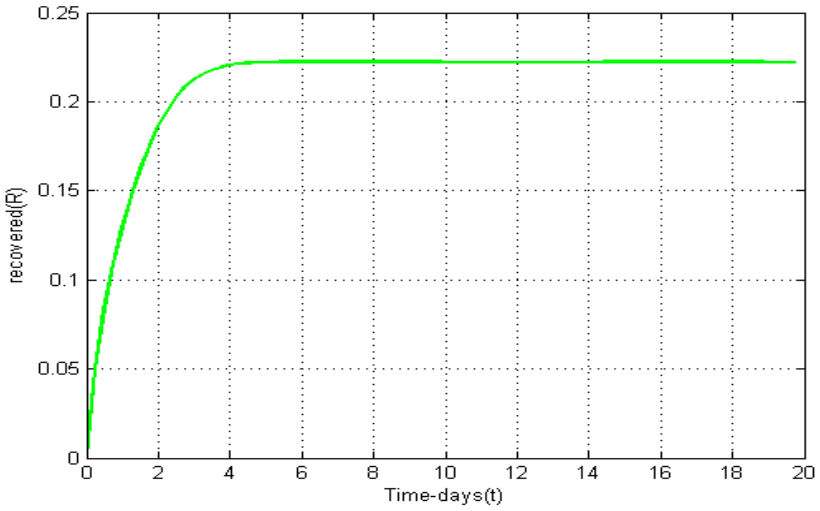


Figure (6): The removed individuals at $\beta = 1.5$

4- Conclusion

- In case of infection rate $\beta = 0.9$ i.e $\beta < 1$, the observe that the number of susceptible individuals are initially decreasing but gradually increasing as shown in Figure (1), whereas the number of removed individuals are initially increasing but gradually decreasing as shown in Figure (2), as well as infected individuals are decreasing very fast over time ,as shown as in Figure (3) .
- In case of infection rate $\beta = 1.5$ i.e $\beta > 1$, the observe that the number of susceptible individuals is decreasing as shown in Figure (4) , while the number of infected individuals is increasing as shown in Figure (5) , as well as removed individuals, are increasing very fast over time, as shown in Figure (6) .

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